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Yukawa couplings and quark and lepton masses in an $SO(10)$ model with a unified Higgs sector

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Fermion mass generation for the third generation is considered within a unified Higgs model with a single pair of vector-spinor $144 + \overline{144}$ of Higgs multiplets. Extending a previous work it is shown that much larger masses can arise for all the third generation fermions from mixing with 45 and 120 matter multiplets via the cubic couplings $16 \cdot 45 \cdot \overline{144}$ and $16 \cdot 120 \cdot 144$ in addition to the quartic couplings. Further, it is found that values of $\tan\beta$ as low as 10 can allow for a $b - \tau - t$ unification consistent with the current data. The quartic and cubic couplings naturally lead to Dirac as well as Majorana neutrino masses necessary for the generation of a seesaw neutrino mass of the right size for the tau neutrino.

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I. INTRODUCTION

Recently a new formalism was proposed which has a unified Higgs structure, i.e., a $144 + \overline{144}$ multiplets of Higgs [1,2]. It was shown that the above Higgs structure can break $SO(10)$ and reduce the rank of the gauge group at the same time such that $SO(10) \to SU(3)_C \times SU(2) \times U(1)_Y$ at the GUT scale M_G. How this comes about can be easily seen by examining the $SU(5) \times U(1)$ decomposition of the 45 plet and the 144 plet representations of $SO(10)$ so that

$$\begin{aligned} 45 &= 1(0) + 10(4) + \overline{10}(-4) + 24(0), \\ 144 &= \bar{5}(3) + 5(7) + 10(-1) + 15(-1) + 24(-5) \\ &\quad + 40(-1) + \overline{45}(3), \end{aligned} \tag{1}$$

where the quantity in the parentheses represents the $U(1)$ quantum numbers. For the usual model building where the 45 plet of Higgs is utilized to break $SO(10)$ it is a combination of the singlet 1(0) and the 24(0) plets of Higgs that develop VEVs. Since each of these carry no $U(1)$ quantum numbers, giving a VEV to the 45 plet does not reduce the rank of the group. In the decomposition of 144 one finds that all the components carry $U(1)$ quantum numbers. Specifically in 144 we have $24(-5)$ and in $\overline{144}$ we have $\overline{24}(5)$. Thus when the $24(-5)$ in 144 and the $\overline{24}(5)$ in $\overline{144}$ develop VEVs so that[1]

$$\begin{aligned} \langle 24(-5)\rangle &= q\,\mathrm{diag}(2, 2, 2, -3, -3), \\ \langle \overline{24}(5)\rangle &= p\,\mathrm{diag}(2, 2, 2, -3, -3), \end{aligned} \tag{2}$$

one not only breaks $SO(10)$ but also reduces the rank of the gauge group and consequently $SO(10)$ breaks directly to the standard model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. Further, it was shown in [1,2] that a pair of light doublets can be gotten which are necessary for the breaking of the electroweak symmetry which occurs when 5, $\bar{5}$, $\overline{45}(45)$ components of $144(\overline{144})$ develop vacuum expectation values. With the $144 + \overline{144}$ Higgs sector, mass generation for the fermions requires at least the quartic interactions $(1/\Lambda)(16 \cdot 16 \cdot 144 \cdot 144)$ and $(1/\Lambda)(16 \cdot 16 \cdot \overline{144} \cdot \overline{144})$, where Λ is typically of size the string scale $M_{\rm st}$. After spontaneous breaking at the GUT scale the 144 and $\overline{144}$ will develop a VEV of size M_G and the above quartic coupling will lead to Yukawa interactions of matter fields with the Higgs doublets. These Yukawas will be $O(M_G/M_{\rm st})$, which are the appropriate size for the Yukawas for the first and for the second generation fermions. To generate larger Yukawa couplings for the third generation, it was proposed in [2] that one include additional 10 and 45 of matter. These allow couplings at the cubic level of the form $16 \cdot 144 \cdot 10$, $16 \cdot \overline{144} \cdot 45$, where the $SU(5) \times U(1)$ decomposition of 45 plet is given in Eq. (1), while the $SU(5) \times U(1)$ decomposition of the 16 plet and the 10 plet are given by

$$16 = 1(-5) + \bar{5}(3) + 10(-1), \qquad 10 = \bar{5}(-2) + 5(2). \tag{3}$$

In the analysis of [1,2] it was shown that there are three pairs of Higgs doublets in $144 + \overline{144}$ and one can label these as $\mathsf{D}_1(\pm 7)$, $\mathsf{D}_2(\pm 3)$ or $\mathsf{D}_3(\pm 3)$ where the ± 7 and ± 3 indicate the $U(1)$ quantum numbers of the pairs.[2] For the cases D_2, D_3 mass generation occurs for the b quark, the tau lepton and for the top quark. However, no Dirac mass is generated for the neutrino and thus a neutrino mass generation from a type I seesaw cannot occur. For the case D_1 one finds that the b quark, and the tau lepton get masses and a Dirac mass is generated for the neutrino, but there is no generation of mass for the top quark. In short it is not possible with a 10 plet and a 45 plet of matter to give

[1]In Eq. (2) the VEVs q and p are *a priori* different. However, as discussed in [1] the D-flatness condition at the GUT scale sets $p = q$.

[2]The doublets $\mathsf{D}_1(7)$ [$\mathsf{D}_1(-7)$] arise from $5_{144}(7)$ [$\bar{5}_{\overline{144}}(-7)$]. The doublets $\mathsf{D}_2(3)$ or $\mathsf{D}_3(3)$ [$\mathsf{D}_2(-3)$ or $\mathsf{D}_3(-3)$] arise from a linear combination of $\bar{5}_{144}(3)$ and $\overline{45}_{144}(3)$ [$5_{\overline{144}}(-3)$ and $45_{\overline{144}}(-3)$].

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masses to the up and to the down quarks, to the charged lepton and also give a Dirac mass to the neutrino all at once. To overcome this problem we analyze in this paper the alternate possibility of including a 45 plet and a 120 plet of matter fields and consider the couplings

$$W = m_F^{(45)}(45)^2 + m_F^{(120)}(120)^2 + \frac{1}{2!}f^{(45)}16\cdot\overline{144}\cdot 45 + \frac{1}{3!}f^{(120)}16\cdot 144\cdot 120, \tag{4}$$

where the $SU(5)\times U(1)$ decomposition of 120 is given by

$$120 = 5(2) + \bar{5}(-2) + 10(-6) + \overline{10}(6) + 45(2). \tag{5}$$

II. YUKAWA COUPLINGS FROM $16_M\cdot\overline{144}_H\cdot 45_M$ AND $16_M\cdot 144_H\cdot 120_M$

An explicit analysis of these couplings relies on oscillator method [3] with special techniques developed in [1,2,4,5](for an alternative technique see [6]). Here we discuss the result of the analysis without going into the details of the computation. To show that the couplings of Eq. (4) and (5) can generate all the third generation masses we consider the $16\cdot 45\cdot\overline{144}$ coupling first and exhibit its contribution to the quark, charged lepton and neutrino masses depending on whether they receive masses from D_1, D_2 or D_3 as exhibited below

$$\begin{aligned}
b,\tau&:\ \bar{5}_{16}(3)\cdot\bar{5}_{45}(4)\cdot\langle\bar{5}_{\overline{144}}(-7)\rangle(\mathsf{D}_1),\\
t&:\ 10_{16}(-1)\cdot 10_{45}(4)\cdot\langle 5_{\overline{144}}(-3)\rangle(\mathsf{D}_2,\mathsf{D}_3),\\
&\ \ 10_{16}(-1)\cdot 10_{45}(4)\cdot\langle 45_{\overline{144}}(-3)\rangle(\mathsf{D}_2,\mathsf{D}_3),\\
&\ \ 10_{16}(-1)\cdot 10_{45}(4)\cdot\langle 45_{\overline{144}}(-3)\rangle(\mathsf{D}_2,\mathsf{D}_3),\\
&\ \ 10_{16}(-1)\cdot 10_{45}(4)\cdot\langle 5_{\overline{144}}(-3)\rangle(\mathsf{D}_2,\mathsf{D}_3).
\end{aligned} \tag{6}$$

Next we consider the coupling $16\cdot 120\cdot 144$ and its contribution to the fermion masses from D_1, D_2 or D_3 so that

$$\begin{aligned}
b,\tau&:\ 10_{16}(-1)\cdot\bar{5}_{120}(-2)\cdot\langle\bar{5}_{144}(3)\rangle(\mathsf{D}_2,\mathsf{D}_3),\\
&\ \ 10_{16}(-1)\cdot\bar{5}_{120}(-2)\cdot\langle\overline{45}_{144}(3)\rangle(\mathsf{D}_2,\mathsf{D}_3),\\
&\ \ \bar{5}_{16}(3)\cdot 10_{120}(-6)\cdot\langle\bar{5}_{144}(3)\rangle(\mathsf{D}_2,\mathsf{D}_3),\\
&\ \ \bar{5}_{16}(3)\cdot 10_{120}(-6)\cdot\langle\overline{45}_{144}(3)\rangle(\mathsf{D}_2,\mathsf{D}_3),\\
t&:\ 10_{16}(-1)\cdot 10_{120}(-6)\cdot\langle 5_{144}(7)\rangle(\mathsf{D}_1),\\
LR-\nu&:\ 1_{16}(-5)\cdot\bar{5}_{120}(-2)\cdot\langle 5_{144}(7)\rangle(\mathsf{D}_1).
\end{aligned} \tag{7}$$

From Eqs. (6) and (7) we find that if D_2 or D_3 is the light doublet which develops a VEV then b and t quarks and τ lepton develop Dirac masses but the τ neutrino does not get a mass. However, when D_1 is the light doublet then from Eq. (6) we find that the b quark and the τ lepton get a mass while from Eq. (7) we find that the top quark and the τ neutrino get masses. Thus with the cubic couplings $16\cdot 45\cdot\overline{144}$ and $16\cdot 120\cdot 144$ and with the doublet D_1 chosen to be the light doublet, *all* the third generation fermions get masses with the neutrino getting a type I seesaw mass.

In the b -quark sector one gets a (4×4) mass matrix (M_b) which has three of its eigenvalues superheavy while one is light which we identify with the b quark mass. We carry out similar analyses for the τ and for the t-quark sectors. The result of this analysis gives

$$m_b^2 \approx \epsilon_b\times(f_{33}^{(45)}\langle\mathbf{P}_5\rangle)^2,\qquad m_\tau^2\approx\epsilon_\tau\times(f_{33}^{(45)}\langle\mathbf{P}_5\rangle)^2,\qquad m_t^2\approx\epsilon_t\times(f_{33}^{(120)}\langle\mathbf{Q}^5\rangle)^2, \tag{8}$$

where $\mathbf{Q}^5\equiv 5_{144}(7)$, and $\mathbf{P}_5\equiv\bar{5}_{\overline{144}}(-7)$ and ϵ_b, ϵ_τ, ϵ_t depend on y_{45}, y_{120}[3] whose explicit forms we do not exhibit here. Rather we give the results in a numerical analysis discussed below.

III. $b-\tau-t$ UNIFICATION

In this section we will consider the light doublet case D_1 and determine the corresponding $b-\tau$ and $b-t$ unification conditions. We consider $b-\tau$ unification first and here the equality $h_b=\alpha h_\tau$ gives a relationship between y_{45} and y_{120} so that

$$(36\alpha^2-1)y_{45}^2y_{120}^4 + 8(96\alpha^2-51)y_{45}^2y_{120}^2 + 36(\alpha^2-1)y_{120}^4 + 1124(4\alpha^2-9)(y_{45}^2+y_{120}^2) = 0. \tag{9}$$

Next we look at the ratio m_t/m_b, which we write in the following form

$$\frac{m_t}{m_b} = \frac{h_t}{h_b}\tan\beta,\qquad \tan\beta = \frac{\langle\mathbf{Q}^5\rangle}{\langle\mathbf{P}_5\rangle}. \tag{10}$$

Our analysis using Eq. (8) and (9) gives for $\gamma = h_t/h_b$ at the GUT scale the result

$$\gamma = \frac{h_t}{h_b} = \frac{160}{3}\frac{f_{33}^{(120}}{f_{33}^{(45)}}\frac{y_{45}}{y_{120}}\sqrt{\frac{1+\frac{3}{32}y_{120}^2}{64+6(\frac{y_{120}}{y_{45}})^2+\frac{3}{8}y_{120}^2}}. \tag{11}$$

Since the values of y_{45} and y_{120} are constrained by Eq. (9), we solve Eqs. (9) and (11) to determine the values of h_t/h_b at the GUT scale as a function of α. These results are exhibited in Table I.

Eventually at low scales after the VEV formation, the Higgs doublets $(\mathbf{Q}^5,\mathbf{P}_5)$ will develop VEVs and lead to mass generation for the top quark, for the bottom quark and for the tau lepton so that

$$m_t = \frac{h_t v\sin\beta}{\sqrt{2}},\qquad m_b = \frac{h_b v\cos\beta}{\sqrt{2}},\qquad m_\tau = \frac{h_\tau v\cos\beta}{\sqrt{2}}, \tag{12}$$

where $v=\sqrt{\langle\mathbf{Q}^5\rangle^2+\langle\mathbf{P}_5\rangle^2}$ and where numerically $v=246$ GeV.

[3] y_{45} and y_{120} are defined so that $y_{45}=m_F^{(45)}/(\sqrt{2}f^{(45)}p)$, $y_{120}=m_F^{(120}/(\sqrt{2}f^{(120)}q)$.

TABLE I. Values of h_t/h_b vs h_b/h_τ at $Q = M_G$.

$\alpha = h_b(M_G)/h_\tau(M_G)$	.7	.75	.8	.85	.9	.95	1
$\gamma = h_t(M_G)/h_b(M_G)$	10.73	10.22	9.76	9.40	8.99	8.65	8.35
$\tan\beta$	8	8.4	8.8	9.1	9.5	9.9	10.3

In $SO(10)$ grand unification where the electroweak symmetry is broken with a 10 plet of Higgs, a $b-\tau-t$ unification typically requires a large $\tan\beta$, i.e., a value of $\tan\beta$ as large as 40–50 [7]. However, from the analysis above we find that a $b-\tau-t$ unification can occur for much smaller values of $\tan\beta$ (this was also noted in [2]). As is conventionally done, we allow for the possibility of Planck scale type corrections, and do not assume a rigid equality of the bottom and tau Yukawa couplings [8]. Typically the Planck scale corrections would involve traces over the product of the Higgs representations suppressed by the Planck scale. Thus the largeness of the Higgs representations imply that the Planck scale corrections can be significantly larger or the Planck scale effectively smaller. Specifically, as noted already just before Eq. (9), we assume $h_b(M_G) = \alpha h_\tau(M_G)$ where α can deviate from unity. With a given choice of α, the ratio h_t/h_b is then determined. In Table I we list the allowed values of h_t/h_b for α in the range 0.7–1.0. Using these boundary conditions we have carried out a one loop analysis of the coupled partial differential equations for the three Yukawa couplings h_τ, h_b, h_t and for the three gauge coupling constants g_3, g_2, g_1 corresponding to the three gauge groups $SU(3)_C$, $SU(2)_L$, $U(1)$ starting from the grand unification scale M_G and moving down to the electroweak scale. In addition to the GUT boundary conditions on the Yukawa couplings h_b, h_τ, h_t given by Table I, the assumed range of h_t is taken so that $|h_t| \leq 1$. Further, we assume that we have the GUT boundary conditions on the gauge couplings so that $g_3(M_G) = g_2(M_G) = g_1(M_G) = g_G \simeq 0.7$.

The details of our numerical computation to determine b, τ, t mass are as follows: First we do the RG analysis in the region between the GUT scale and some average weak SUSY scale M_S, i.e., $M_S \leq Q \leq M_G$ using the boundary conditions at the GUT scale as discussed above. Here we use the RG evolution equations valid for MSSM. Below the scale M_S we use the RG evolution equations appropriate for the standard model with proper matching done at the scale M_S. Numerical results for the evolution of the b, τ Yukawa couplings $h_b(Q)$, $h_\tau(Q)$ are plotted in the right panel of Fig. 1 and for $h_b(Q)$, $h_t(Q)$ are plotted in the left panel of Fig. 1 as a function of the scale $t = \ln Q/16\pi^2$. The rapid convergence of the Yukawa couplings for the top at low scale is due to the well known Landau pole singularity in the top Yukawa coupling [8,9]. In the RG analysis we compute the values of $m_t(m_t)$ which is related to the pole mass m_t^p by the relation $m_t^p = m_t(m_t)[1 + \frac{4}{3\pi}\alpha_s(m_t)]$. For the computation of $m_b(m_b)$ we use three loop QCD and one loop EM evolution between the scale M_Z and the scale m_b. The procedure of the analysis is as given in [8]. Using

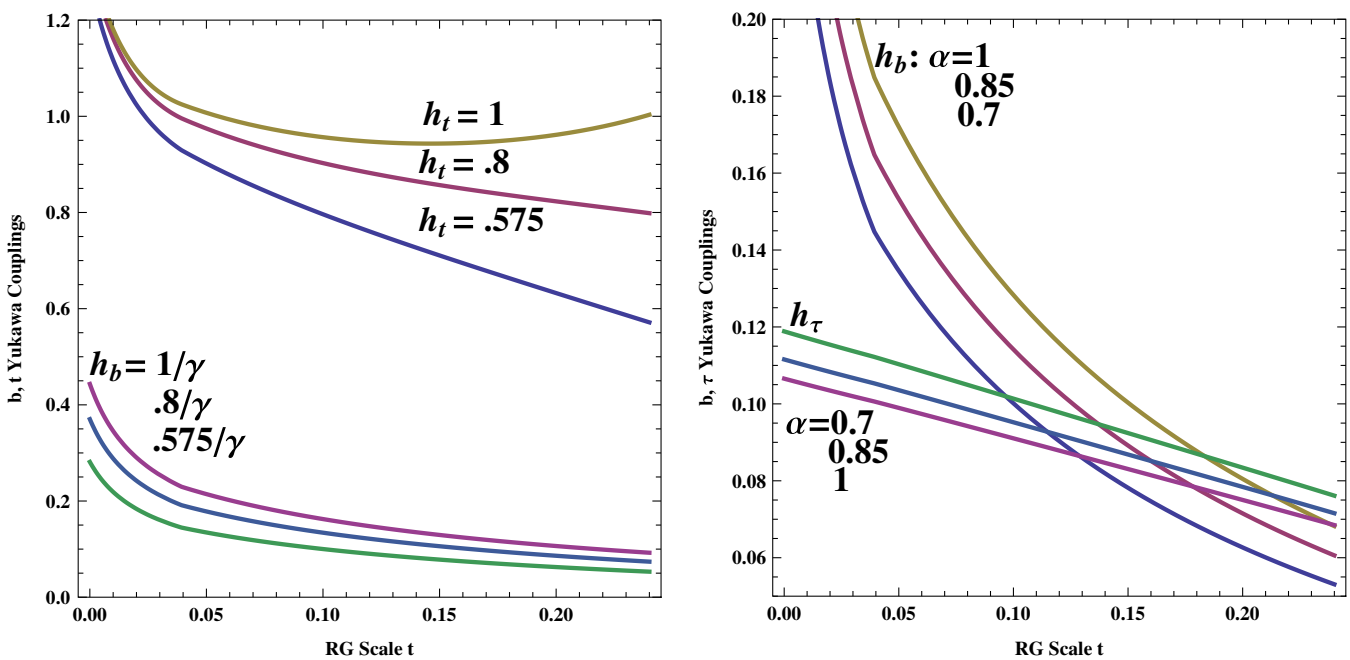

FIG. 1 (color online). Right panel: The RG evolution of the Yukawa coupling $h_b(Q)$ for the bottom quark and $h_\tau(Q)$ for the τ lepton for $\alpha = 1, 0.85, 0.7$ (see Table I) from the grand unification scale to the electroweak scale where t labeling the x-axis is defined so that $t = \ln(Q(\mathrm{GeV})/16\pi^2$. For the analysis of the figure in the right panel we have chosen $h_t(M_G) = 0.575$ and $h_b(M_G)$ and $h_\tau(M_G)$ are then determined from Table I. Left panel: The RG evolution of the Yukawa coupling $h_b(M_G)$ and of $h_t(M_G)$ for values of $h_t(M_G) = 1$, 0.8, 0.57 5 and for $h_b(M_G) = 1/\gamma$, $.8/\gamma$, $.575/\gamma$ for the case $\alpha = .7$ from the grand unification scale to the electroweak scale where $\gamma = 10.73$ as given in Table I. The best fit to the b, τ, t masses given in Eq. (13) occurs for $\alpha = .7$, $h_t(M_G) = 0.575$.

the result of Table I for $h_b/h_\tau = .7$ we find a determination of $\tan\beta$ and the following fit to the b, t pole masses using the τ lepton mass as an input $m_\tau = 1.776$ GeV, so that

$$\tan\beta = 8, \qquad m_b = 4.53 \text{ GeV}, \qquad m_t^p = 172.9 \text{ GeV}. \tag{13}$$

The above are to be compared with the experimental data on the b quark and on the top quark mass, i.e., $m_b = 4.2^{+0.17}_{-0.07}$ GeV, $m_t = 171.3 \pm 1.1 \pm 1.2$ GeV [10]. Thus our one loop analysis is within about 2σ of the b quark mass and within one sigma of the top quark mass. As seen from Eq. (13) in this model $b - \tau - t$ unification is achieved for a value of $\tan\beta$ which is much smaller than what typically appears in $b - \tau - t$ unification analyses [7].

IV. TAU NEUTRINO MASS

The model contains both heavy Majorana mass terms and Dirac mass terms. The Majorana mass terms arise from the $\sum L = -2$ mixing terms ${}^{(1_{16})}\bar{\nu}_{R\tau}{}^{(5_{120})}\nu^c_{L\tau}$, ${}^{(1_{16})}\bar{\nu}_{R\tau}{}^{(1_{16})}\nu^c_{L\tau}$, and the electroweak Dirac mass terms arise from the $\sum L = 0$ mixing terms ${}^{(1_{16})}\bar{\nu}_{R\tau}{}^{(\bar{5}_{120})}\nu_{L\tau}$, ${}^{(5_{120})}\bar{\nu}_{R\tau}{}^{(\bar{5}_{16})}\nu_{L\tau}$, ${}^{(5_{120})}\bar{\nu}_{R\tau}{}^{(\bar{5}_{120})}\nu_{L\tau}$, ${}^{(1_{45})}\bar{\nu}_{R\tau}{}^{(1_{45})}\nu^c_{L\tau}$, ${}^{(1_{16})}\bar{\nu}_{R\tau}{}^{(1_{16})}\nu^c_{L\tau}$. Together they produce the type I seesaw mass for the light tau neutrino which is given by

$$m_{\nu_\tau} \simeq \frac{m_D^2 \sin^2\beta}{m_F^{(16)}}, \qquad m_D \simeq 96(f_{33}^{(120)})^2 v\left(\frac{q}{m_F^{(120)}}\right). \tag{14}$$

It is interesting to note the dependence of the neutrino mass on $\sin\beta$ in Eq. (14) which reflects the chosen connection of the neutrino mass with electroweak physics. Now the choice $f_{33}^{(120)} \sim .2$, $q/m_F^{(120} \sim 1$ and $m_F^{(16)} = 10^{16}$ GeV, and $v = 246$ GeV, $\sin\beta \sim 1$ leads to a type I seesaw mass of $m_{\nu_\tau} \simeq 0.1$ eV. A more complete analysis of fermion masses including also the type II and type III seesaw masses in the unified Higgs framework will be given elsewhere [11].

V. CONCLUSION

To allow for significantly larger third generation fermion masses we considered cubic couplings involving 45 and 120 plets of matter fields, i.e., couplings of the type $16 \cdot 45 \cdot \overline{144}$ and $16 \cdot 120 \cdot 144$. These interactions lead to cubic couplings of size appropriate for the third generation Yukawa couplings. An important result of the analysis is that among the allowed possibilities for the light Higgs doublets, D_1, D_2, D_3, the doublet D_1 gets picked in a unique way. Thus as discussed in the introduction, there are three pairs of possibilities for the Higgs doublets in $144 + \overline{144}$ which are labeled as $\mathsf{D}_1(\pm 7)$, $\mathsf{D}_2(\pm 3)$ or $\mathsf{D}_3(\pm 3)$ where the ± 7 and ± 3 indicate the $U(1)$ quantum numbers of the pairs. However, it was shown in the analysis given here that if D_2 or D_3 is the light doublet which develops a VEV then b and t quarks and τ lepton develop Dirac masses but the τ neutrino does not get a mass. On the other hand, when D_1 is the light doublet the b quark, the τ lepton the top quark, get masses. Further, it was shown that the 144 plet Higgs couplings allow for a seesaw mechanism for the generation of the tau neutrino mass. Thus if D_1 is chosen to be the light doublet, then *all* the third generation fermions get masses. Using the doublet D_1 an analysis of the $b - \tau - t$ unification was also given. The analysis of the Yukawa interactions given here would be of significant value in the further development of the $SO(10)$ phenomenology in the unified Higgs framework.

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