

April 02, 2004

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### Recommended Citation

Ibrahim, Tarek and Nath, Pran, "Weak isospin violations in charged and neutral Higgs couplings from supersymmetric loop corrections" (2004). *Physics Faculty Publications*. Paper 510. <http://hdl.handle.net/2047/d20004287>

# Weak isospin violations in charged and neutral Higgs couplings from supersymmetric loop corrections

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(Received 19 November 2003; published 2 April 2004)

The supersymmetric QCD and the supersymmetric electroweak loop corrections to the violations of weak isospin to Yukawa couplings are investigated. Specifically it involves an analysis of the supersymmetric (SUSY) loop corrections to the Higgs couplings to the third generation quarks and leptons. Here we analyze the SUSY loop corrections to the charged Higgs couplings which are then compared with the supersymmetric loop corrections to the neutral Higgs couplings previously computed. It is found that the weak isospin violations can be quite significant, i.e., as much as 40–50% or more of the total loop correction to the Yukawa coupling. The effects of  $CP$  phases are also studied and it is found that these effects can either enhance or suppress the weak isospin violations. We also investigate the weak isospin violation effects on the branching ratio  $BR(H^- \rightarrow \bar{t}b)/BR(H^- \rightarrow \bar{\nu}_\tau \tau^-)$  and show that the effects are sensitive to  $CP$  phases. Thus an accurate measurement of this branching ratio along with the branching ratio of the neutral Higgs boson decays can provide a measure of weak isospin violation along with providing a clue to the presence of supersymmetry.

DOI: 10.1103/PhysRevD.69.075001

PACS number(s): 12.60.Jv, 14.80.Cp

## I. INTRODUCTION

In this paper we investigate the effects of supersymmetric QCD and the supersymmetric electroweak corrections to the violations of weak isospin in the couplings of Higgs bosons to quarks and leptons [1]. Specifically, we compute in this paper the supersymmetric loop effects to the couplings of the charged Higgs boson with quarks and leptons. These are then compared with the supersymmetric corrections to the couplings of the neutral Higgs boson. We also study the effects of  $CP$  phases on the charged Higgs couplings. The  $CP$  phases that appear in the soft parameters, however, are subject to strong constraints from the electric dipole moments (EDMs) of the electron [2], and of the neutron [3] and of the  $Hg^{199}$  atom [4]. The mechanisms available for the suppression of the EDMs associated with large phases consist of the cancellation mechanism [5,6], the mass suppression [7] and phases just in the third generation [8] among others [9,10]. Large  $CP$  phases affect a variety of low energy phenomena including the Higgs sector. One such phenomenon is the  $CP$ -even– $CP$ -odd mixing among the neutral Higgs bosons which has been studied in great detail [11–13]. Here we study the effects of  $CP$  phases on the charged Higgs couplings to quarks and leptons and also the effect of  $CP$  phases on the breakdown of the weak isospin invariance. The outline of the rest of the paper is as follows: In Sec. II we give the basic formalism. In Sec. III we compute the SUSY QCD and SUSY electroweak loop correction to the charged Higgs couplings to third generation quarks and leptons. In Sec. IV we analyze the loop corrections to the charged Higgs decays

$H^- \rightarrow \bar{t}b$  and  $H^- \rightarrow \bar{\nu}_\tau \tau^-$ . In Sec. V we give a numerical analysis of the loop effects and estimate the sizes of the loop corrections to violations of weak isospin. We also study the effects of  $CP$  phases on the charged Higgs couplings and on the ratio of the decay branching ratios of the charged Higgs boson to  $\bar{t}b$  and  $\bar{\nu}_\tau \tau^-$ . Finally, the conclusions are given in Sec. VI.

## II. THE BASIC FORMALISM

We will use the framework of the minimal supersymmetric standard model (MSSM) which contains two doublets of Higgs bosons and for the soft breaking sector we will use the extended SUGRA framework [14] with nonuniversality and with  $CP$  phases. Thus for the Higgs sector we have

$$(H_1) = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix}, \quad (H_2) = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix}. \quad (1)$$

The components of  $H_1$  and  $H_2$  interact with the quarks and the leptons at the tree level through [15]

$$-\mathcal{L} = \sum_{f=b,\tau} h_f \bar{f}_R f_L H_1^1 + h_t \bar{t}_R t_L H_2^1 - h_b \bar{b}_R t_L H_1^2 - h_t \bar{t}_R b_L H_2^1 - h_\tau \bar{\tau}_R \nu_L H_1^2 + \text{H.c.} \quad (2)$$

The loop corrections produce shifts in these couplings and generate new ones as follows:

\*Permanent address.

$$\begin{aligned}
-\mathcal{L}_{eff} = & \sum_{f=b,\tau} (h_f + \delta h_f) \bar{f}_R f_L H_1^1 + \Delta h_f \bar{f}_R f_L H_2^{2*} + (h_t + \delta h_t) \bar{t}_R t_L H_2^2 + \Delta h_t \bar{t}_R t_L H_1^{1*} - (h_b + \overline{\delta h_b}) \bar{b}_R t_L H_1^2 + \overline{\Delta h_b} \bar{b}_R t_L H_2^{1*} \\
& - (h_t + \overline{\delta h_t}) \bar{t}_R b_L H_2^1 + \overline{\Delta h_t} \bar{t}_R b_L H_1^{2*} - (h_\tau + \overline{\delta h_\tau}) \bar{\tau}_R \nu_L H_1^2 + \overline{\Delta h_\tau} \bar{\tau}_R \nu_L H_2^{1*} + \text{H.c.}
\end{aligned} \quad (3)$$

where “\*” has been used to get a gauge invariant  $\mathcal{L}_{eff}$ . We rewrite Eq. (3) in a form which is illustrative,

$$\begin{aligned}
-\mathcal{L}_{eff} = & \epsilon_{ij} [(h_b + \delta h_b) \bar{b}_R H_1^i Q_L^j + (h_\tau + \delta h_\tau) \bar{\tau}_R H_1^i l_L^j + (h_t + \delta h_t) \bar{t}_R Q_L^i H_2^j] + [\Delta h_b \bar{b}_R Q_L^i H_2^{j*} + \Delta h_\tau \bar{\tau}_R l_L^i H_2^{j*} + \Delta h_t \bar{t}_R Q_L^i H_1^{j*}] \\
& - \mathcal{L}_{violation} + \text{H.c.}
\end{aligned} \quad (4)$$

where

$$\begin{aligned}
-\mathcal{L}_{violation} = & \{ -(\overline{\delta h_b} - \delta h_b) \bar{b}_R t_L H_1^2 + (\overline{\Delta h_b} - \Delta h_b) \bar{b}_R t_L H_2^{1*} - (\overline{\delta h_\tau} - \delta h_\tau) \bar{\tau}_R \nu_L H_1^2 + (\overline{\Delta h_\tau} - \Delta h_\tau) \bar{\tau}_R \nu_L H_2^{1*} \\
& - (\overline{\delta h_t} - \delta h_t) \bar{t}_R b_L H_2^1 + (\overline{\Delta h_t} - \Delta h_t) \bar{t}_R b_L H_1^{2*} \}.
\end{aligned} \quad (5)$$

The corrections  $\delta h_b$ ,  $\delta h_\tau$ ,  $\delta h_t$ ,  $\Delta h_b$ ,  $\Delta h_\tau$ , and  $\Delta h_t$  have been calculated in Refs. [16–18] with the inclusion of  $CP$  phases. Their effects on the decay of neutral Higgs couplings have been studied in Ref. [18]. In this paper we analyze the SUSY QCD and the SUSY electroweak loop corrections to the couplings of the charged Higgs bosons with quarks and leptons in the MSSM. Specifically we calculate the corrections  $\overline{\delta h_b}$ ,  $\overline{\delta h_t}$ ,  $\overline{\delta h_\tau}$ ,  $\overline{\Delta h_b}$ ,  $\overline{\Delta h_t}$ , and  $\overline{\Delta h_\tau}$  from exchange of sparticles at one loop. We then study their effects on the decay of the charged Higgs boson into quarks and leptons for the third family. We note that in the approximation

$$\overline{\delta h_{b,t,\tau}} = \delta h_{b,t,\tau}, \quad \overline{\Delta h_{b,t,\tau}} = \Delta h_{b,t,\tau} \quad (6)$$

one finds that the right hand side of Eq. (5) vanishes and SUSY loop correction preserves weak isospin. This is the approximation that is often used in the literature [1]. However, in general, the equalities of Eq. (6) will not hold and there will be violations of weak isospin given by Eq. (5). In this paper we will investigate the size of these violations and their implications on phenomena.

### III. SUSY QCD AND SUSY ELECTROWEAK CORRECTIONS TO THE CHARGED HIGGS COUPLINGS

Figure 1 gives the SUSY QCD loop correction through gluino exchange and SUSY electroweak correction via neutralino and chargino exchanges. In the analysis of these graphs we use the zero external momentum approximation [19,1,18,17] rather than the alternate technique of Refs. [20,21]. We will work within the framework of MSSM and SUGRA models allowing for nonuniversalities. We will characterize the parameter space of these models by using the parameters  $m_A$ ,  $m_0$ ,  $\tilde{m}_i = m_{1/2} e^{i\xi_i}$  ( $i=1,2,3$ ),  $A_t^0$ ,  $A_b^0$ ,  $A_\tau^0$  and  $\tan\beta$ . Here  $m_A$  is the mass parameter in the Higgs sector,  $m_0$  is the universal scalar mass,  $\tilde{m}_i$  ( $i=1,2,3$ ) are the gaugino masses corresponding to the gauge groups  $U(1)$ ,  $SU(2)$  and  $SU(3)_C$ ,  $A_{t,b,\tau}^0$  are the trilinear couplings in top quarks, sbottoms, and staus, and  $\tan\beta = \langle H_2 \rangle / \langle H_1 \rangle$  where  $H_2$  gives mass to the up quark and  $H_1$  gives mass to the

down quark and the lepton. We discuss now the various contributions in detail. For  $\overline{\Delta h_b}$  we need  $\bar{b}\bar{t}\bar{H}$  interaction which is given by

$$\mathcal{L}_{\bar{b}\bar{t}\bar{H}} = H_1^2 \bar{b}_j^* \tilde{t}_i \eta'_{ji} + H_2^1 \bar{b}_i^* \tilde{t}_j^* \eta_{ij} + \text{H.c.} \quad (7)$$

where

$$\begin{aligned}
\eta'_{ji} = & \frac{g m_b}{\sqrt{2} m_W \cos \beta} m_0 A_b D_{b2j}^* D_{t1i} + \frac{g m_t}{\sqrt{2} m_W \sin \beta} \mu D_{b1j}^* D_{t2i} \\
& + \frac{g m_b m_t}{\sqrt{2} m_W \cos \beta} D_{b2j}^* D_{t2i} + \frac{g m_b^2}{\sqrt{2} m_W \cos \beta} D_{b1j}^* D_{t1i} \\
& - \frac{g}{\sqrt{2}} m_W \cos \beta D_{b1j}^* D_{t1i}
\end{aligned} \quad (8)$$

and

$$\begin{aligned}
\eta_{ij} = & \frac{g m_t}{\sqrt{2} m_W \sin \beta} m_0 A_t D_{b1i} D_{t2j}^* + \frac{g m_b}{\sqrt{2} m_W \cos \beta} \mu D_{b2i} D_{t1j}^* \\
& + \frac{g m_b m_t}{\sqrt{2} m_W \sin \beta} D_{b2i} D_{t2j}^* + \frac{g m_t^2}{\sqrt{2} m_W \sin \beta} D_{b1i} D_{t1j}^* \\
& - \frac{g}{\sqrt{2}} m_W \sin \beta D_{b1i} D_{t1j}^*.
\end{aligned} \quad (9)$$

Using the above one finds from Fig. 1

$$-\overline{\Delta h_b^g} = - \sum_{i=1}^2 \sum_{j=1}^2 \frac{2\alpha_s}{3\pi} e^{-i\xi_3} D_{b2j} D_{t1i}^* \eta_{ji}^* m_g^{-2} f(m_g^2, m_{t_i}^2, m_{b_j}^2) \quad (10)$$

where  $f(m^2, m_i^2, m_j^2)$  is defined so that

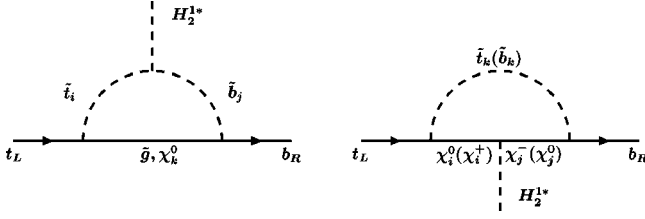


FIG. 1. Exhibition of the supersymmetric loop contribution to the charged Higgs couplings with third generation quarks. All particles in the loop are heavy supersymmetric partners with  $\tilde{t}_i(\tilde{b}_j)$  being heavy top quarks (sbottoms),  $\tilde{g}$  the gluino, and  $\chi_k^0(\chi_i^-)$  neutralinos (charginos).

$$f(m^2, m_i^2, m_j^2) = \frac{1}{(m^2 - m_i^2)(m^2 - m_j^2)(m_j^2 - m_i^2)} \times \left( m_j^2 m^2 \ln \frac{m_j^2}{m^2} + m^2 m_i^2 \ln \frac{m^2}{m_i^2} + m_i^2 m_j^2 \ln \frac{m_i^2}{m_j^2} \right) \quad (11)$$

for the case  $i \neq j$  and

$$f(m^2, m_i^2, m_i^2) = \frac{1}{(m_i^2 - m^2)^2} \left( m^2 \ln \frac{m_i^2}{m^2} + (m^2 - m_i^2) \right). \quad (12)$$

Further in Eq. (10)  $D_{bij}$  is the matrix that diagonalizes the b squark  $mass^2$  matrix so that

$$\tilde{b}_L = \sum_{i=1}^2 D_{b1i} \tilde{b}_i, \quad \tilde{b}_R = \sum_{i=1}^2 D_{b2i} \tilde{b}_i \quad (13)$$

where  $\tilde{b}_i$  are the b squark mass eigenstates. Similarly  $D_{tij}$  is the matrix that diagonalizes the t squark  $mass^2$  matrix so that

$$\tilde{t}_L = \sum_{i=1}^2 D_{t1i} \tilde{t}_i, \quad \tilde{t}_R = \sum_{i=1}^2 D_{t2i} \tilde{t}_i \quad (14)$$

where  $\tilde{t}_i$  are the t squark mass eigenstates. The SUSY electroweak correction  $\overline{\Delta h_b^{EW}}$  arising from the neutralino exchange and from the chargino exchange (see Fig. 1) is given by the following:

$$\begin{aligned} -\overline{\Delta h_b^{EW}} &= \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^4 2\eta_{ji}^* (\alpha_{bk} D_{b1j} - \gamma_{bk} D_{b2j}) \\ &\times (\beta_{tk}^* D_{t1i} + \alpha_{tk} D_{t2i}) \frac{m_{\chi_k^0}}{16\pi^2} f(m_{\chi_k^0}^2, m_{\tilde{t}_i}^2, m_{\tilde{b}_j}^2) \\ &+ \sqrt{2} g \xi_{ki} \frac{m_{\chi_k^0} m_{\chi_i^-}}{16\pi^2} [-\kappa_b U_{i2}^* D_{t1j} (\beta_{tk}^* D_{t1j} + \alpha_{tk} D_{t2j}^*) \\ &\times f(m_{\tilde{t}_j}^2, m_{\chi_i^-}^2, m_{\chi_k^0}^2) + (\alpha_{bk} D_{b1j} - \gamma_{bk} D_{b2j}) \end{aligned}$$

$$\times (U_{i1}^* D_{b1j}^* - \kappa_b U_{i2}^* D_{b2j}^*) f(m_{\tilde{b}_j}^2, m_{\chi_i^-}^2, m_{\chi_k^0}^2)] \quad (15)$$

where

$$\xi_{ki} = -g X_{4k} V_{i1}^* - \frac{g}{\sqrt{2}} X_{2k} V_{i2}^* - \frac{g'}{\sqrt{2}} X_{1k} V_{i2}^*. \quad (16)$$

Here U and V diagonalize the chargino mass matrix, X diagonalizes the neutralino mass matrix, and  $k_b$  is defined by

$$k_{b(t)} = \frac{m_{b(t)}}{\sqrt{2} m_W \cos \beta (\sin \beta)}. \quad (17)$$

Finally,  $\alpha_{bk}$ ,  $\beta_{bk}$  and  $\gamma_{bk}$  for the b quark are defined so that

$$\begin{aligned} \alpha_{bk} &= \frac{g m_b X_{3k}}{2 m_W \cos \beta}, \\ \beta_{bk} &= e Q_b X'_{1k} + \frac{g}{\cos \theta_W} X'_{2k} (T_{3b} - Q_b \sin^2 \theta_W) \\ \gamma_{bk} &= e Q_b X'_{1k} - \frac{g Q_b \sin^2 \theta_W}{\cos \theta_W} X'_{2k} \end{aligned} \quad (18)$$

and

$$\begin{aligned} \alpha_{tk} &= \frac{g m_t X_{4k}}{2 m_W \sin \beta}, \\ \beta_{tk} &= e Q_t X'_{1k} + \frac{g}{\cos \theta_W} X'_{2k} (T_{3t} - Q_t \sin^2 \theta_W) \\ \gamma_{tk} &= e Q_t X'_{1k} - \frac{g Q_t \sin^2 \theta_W}{\cos \theta_W} X'_{2k} \end{aligned} \quad (19)$$

where  $Q_{b(t)} = -\frac{1}{3}(\frac{2}{3})$  and  $T_{3b(t)} = -\frac{1}{2}(\frac{1}{2})$  and where

$$\begin{aligned} X'_{1k} &= X_{1k} \cos \theta_W + X_{2k} \sin \theta_W \\ X'_{2k} &= -X_{1k} \sin \theta_W + X_{2k} \cos \theta_W. \end{aligned} \quad (20)$$

Thus the total correction  $\overline{\Delta h_b}$  is given by

$$\overline{\Delta h_b} = \overline{\Delta h_b^g} + \overline{\Delta h_b^{EW}}. \quad (21)$$

Similarly the SUSY QCD and SUSY electroweak correction  $\overline{\delta h_b}$  is computed to be

$$\begin{aligned}
-\overline{\delta h}_b &= \sum_{i=1}^2 \sum_{j=1}^2 \frac{2\alpha_s}{3\pi} e^{-i\xi_3} D_{b2j} D_{i1i}^* \eta'_{ji} m_{\tilde{g}} f(m_g^2, m_{\tilde{t}_i}^2, m_{\tilde{b}_j}^2) \\
&\quad - \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^4 2\eta'_{ji} (\alpha_{bk} D_{b1j} - \gamma_{bk} D_{b2j}) \\
&\quad \times (\beta_{ik}^* D_{i1i}^* + \alpha_{ik} D_{i2i}^*) \frac{m_{\chi_k^0}}{16\pi^2} f(m_{\chi_k^0}^2, m_{\tilde{t}_i}^2, m_{\tilde{b}_j}^2). \quad (22)
\end{aligned}$$

An analysis similar to the above gives for  $\overline{\Delta h}_t$  and for  $\overline{\delta h}_t$  the following:

$$\begin{aligned}
-\overline{\Delta h}_t &= - \sum_{i=1}^2 \sum_{j=1}^2 \frac{2\alpha_s}{3\pi} e^{-i\xi_3} D_{b1i}^* D_{t2j} \eta'_{ij} m_{\tilde{g}} f(m_g^2, m_{\tilde{b}_i}^2, m_{\tilde{t}_j}^2) \\
&\quad + \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^4 2\eta'_{ij} (\alpha_{tk} D_{t1j} - \gamma_{tk} D_{t2j}) \\
&\quad \times (\beta_{bk}^* D_{b1i}^* + \alpha_{bk} D_{b2i}^*) \frac{m_{\chi_k^0}}{16\pi^2} f(m_{\chi_k^0}^2, m_{\tilde{b}_i}^2, m_{\tilde{t}_j}^2) \\
&\quad + \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^4 \sqrt{2} g \xi_{ki}^* \frac{m_{\chi_k^0} m_{\chi_i^-}}{16\pi^2} \\
&\quad \times [-k_t V_{i2}^* D_{b1j} (\beta_{bk}^* D_{b1j}^* + \alpha_{bk} D_{b2j}^*) f(m_{\tilde{b}_j}^2, m_{\chi_i^-}^2, m_{\chi_k^0}^2) \\
&\quad + (\alpha_{tk} D_{t1j} - \gamma_{tk} D_{t2j}) (V_{i1}^* D_{i1j}^* - k_t V_{i2}^* D_{i2j}^*) \\
&\quad \times f(m_{\tilde{t}_j}^2, m_{\chi_i^-}^2, m_{\chi_k^0}^2)] \quad (23)
\end{aligned}$$

where

$$\xi_{ki}^* = -g X_{3k}^* U_{i1} + \frac{g}{\sqrt{2}} U_{i2} X_{2k}^* + \frac{g'}{\sqrt{2}} U_{i2} X_{1k}^*. \quad (24)$$

Similarly for  $\overline{\delta h}_t$  one has the following:

$$\begin{aligned}
-\overline{\delta h}_t &= \sum_{i=1}^2 \sum_{j=1}^2 \frac{2\alpha_s}{3\pi} e^{-i\xi_3} D_{b1i}^* D_{t2j} \eta_{ij} m_{\tilde{g}} f(m_g^2, m_{\tilde{b}_i}^2, m_{\tilde{t}_j}^2) \\
&\quad - \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^4 2\eta_{ij} (\alpha_{tk} D_{t1j} - \gamma_{tk} D_{t2j}) \\
&\quad \times (\beta_{bk}^* D_{b1i}^* + \alpha_{bk} D_{b2i}^*) \frac{m_{\chi_k^0}}{16\pi^2} f(m_{\chi_k^0}^2, m_{\tilde{b}_i}^2, m_{\tilde{t}_j}^2). \quad (25)
\end{aligned}$$

The analysis of  $\overline{\Delta h}_\tau$  and of  $\overline{\delta h}_\tau$  is free of the SUSY QCD correction while the SUSY electroweak correction gives

$$\begin{aligned}
-\overline{\Delta h}_\tau &= \sum_{j=1}^2 \sum_{k=1}^4 2\eta_j^{\tau*} [\alpha_{\tau k} D_{\tau 1j} - \gamma_{\tau k} D_{\tau 2j}] \\
&\quad \times \beta_{\nu k}^* \frac{m_{\chi_k^0}}{16\pi^2} f(m_{\chi_k^0}^2, m_{\tilde{\nu}}^2, m_{\tilde{\tau}_j}^2) \\
&\quad - \sqrt{2} g \xi_{kj} \frac{m_{\chi_k^0} m_{\chi_j^-}}{16\pi^2} [k_\tau U_{j2}^* \beta_{\nu k}^* f(m_{\tilde{\nu}}^2, m_{\chi_j^-}^2, m_{\chi_k^0}^2)] \\
&\quad + \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^4 \sqrt{2} g \xi_{ki} \frac{m_{\chi_k^0} m_{\chi_i^-}}{16\pi^2} \\
&\quad \times [(U_{i1}^* D_{\tau 1j}^* - k_\tau U_{i2}^* D_{\tau 2j}^*) \\
&\quad \times (\alpha_{\tau k} D_{\tau 1j} - \gamma_{\tau k} D_{\tau 2j}) f(m_{\tilde{\tau}_j}^2, m_{\chi_i^-}^2, m_{\chi_k^0}^2)] \quad (26)
\end{aligned}$$

where  $D_{\tau ij}$ ,  $k_\tau$ ,  $\alpha_{\tau k}$ ,  $\beta_{\tau k}$ ,  $\gamma_{\tau k}$ ,  $\beta_{\nu k}$  are defined similar to  $D_{bij}$ ,  $k_b$  etc. Finally, for  $\overline{\delta h}_\tau$  we have

$$\begin{aligned}
-\overline{\delta h}_\tau &= - \sum_{j=1}^2 \sum_{k=1}^4 2\eta_j^{\tau'} [\alpha_{\tau k} D_{\tau 1j} - \gamma_{\tau k} D_{\tau 2j}] \\
&\quad \times \beta_{\nu k}^* \frac{m_{\chi_k^0}}{16\pi^2} f(m_{\chi_k^0}^2, m_{\tilde{\nu}}^2, m_{\tilde{\tau}_j}^2) \quad (27)
\end{aligned}$$

where

$$\begin{aligned}
\eta_j^\tau &= \frac{g m_\tau}{\sqrt{2} m_W \cos \beta} \mu D_{\tau 2j} - \frac{g}{\sqrt{2}} m_W \sin \beta D_{\tau 1j} \\
\eta_j^{\tau'} &= \frac{g m_\tau}{\sqrt{2} m_W \cos \beta} m_0 A_\tau D_{\tau 2j}^* + \frac{g m_\tau^2}{\sqrt{2} m_W \cos \beta} D_{\tau 1j}^* \\
&\quad - \frac{g}{\sqrt{2}} m_W \cos \beta D_{\tau 1j}^*. \quad (28)
\end{aligned}$$

One measure of the size of the violation of the weak isospin is the deviation of the barred quantities from the unbarred quantities. Thus as a measure of violations of weak isospin in  $b$  quark couplings we define the quantity  $r_b$  where

$$r_b = \frac{\sqrt{|\overline{\Delta h}_b|^2 + |\overline{\delta h}_b|^2}}{\sqrt{|\Delta h_b|^2 + |\delta h_b|^2}}. \quad (29)$$

The deviation of this quantity from unity is an indication of the violation of weak isospin in the Higgs couplings. Similarly we can define  $r_t$  and  $r_\tau$  by replacing  $b$  with  $t$  and  $\tau$  in Eq. (29).

#### IV. SUSY LOOP CORRECTION TO CHARGED HIGGS DECAYS: $H^- \rightarrow \bar{t}b$ AND $H^- \rightarrow \bar{\nu}_\tau \tau^-$

In this section we study the branching ratio involving the decays  $H^- \rightarrow \bar{t}b$  and  $H^- \rightarrow \bar{\nu}_\tau \tau^-$ . One may recall that in the neutral Higgs sector, the ratio  $R^{h^0} = BR(h^0 \rightarrow b\bar{b})/BR(h^0 \rightarrow \tau\bar{\tau})$  is found to be sensitive to the supersymmetric loop corrections [22] and to  $CP$  phases. In an analogous fashion in this paper we define the ratio  $R^{H^-} = BR(H^- \rightarrow \bar{t}b)/BR(H^- \rightarrow \bar{\nu}_\tau \tau^-)$  and show that this ratio is a sensitive function of the supersymmetric loop corrections, a sensitive function of the  $CP$  phases and in addition sensitive to the violations of weak isospin. To this end it is convenient to display the charged Higgs interaction

$$-\mathcal{L}_{int} = \bar{b}(B_{bt}^s + B_{bt}^p \gamma_5)tH^- + \bar{\tau}(B_{\nu\tau}^s + B_{\nu\tau}^p \gamma_5)\nu H^- + \text{H.c.}, \quad (30)$$

where

$$\begin{aligned} B_{bt}^s &= -\frac{1}{2}(h_b + \overline{\delta h_b})e^{-i\theta_{bt}}\sin\beta + \frac{1}{2}\overline{\Delta h_b}e^{-i\theta_{bt}}\cos\beta \\ &\quad -\frac{1}{2}(h_t + \overline{\delta h_t^*})e^{i\theta_{bt}}\cos\beta + \frac{1}{2}\overline{\Delta h_t^*}e^{i\theta_{bt}}\sin\beta \\ B_{bt}^p &= -\frac{1}{2}(h_t + \overline{\delta h_t^*})e^{i\theta_{bt}}\cos\beta + \frac{1}{2}\overline{\Delta h_t^*}e^{i\theta_{bt}}\sin\beta \\ &\quad +\frac{1}{2}(h_b + \overline{\delta h_b})e^{-i\theta_{bt}}\sin\beta - \frac{1}{2}\overline{\Delta h_b}e^{-i\theta_{bt}}\cos\beta \\ B_{\nu\tau}^s &= -B_{\nu\tau}^p \\ &= -\frac{1}{2}(h_\tau + \overline{\delta h_\tau})e^{-i\chi_\tau/2}\sin\beta, \\ &\quad +e^{-i\chi_\tau/2}\frac{1}{2}\overline{\Delta h_\tau}\cos\beta, \end{aligned} \quad (31)$$

where  $\theta_{bt} = (\chi_b + \chi_t)/2$  and where  $\chi_b, \chi_\tau$  and  $\chi_t$  are defined by the following:

$$\tan\chi_b = \frac{\text{Im}\left(\frac{\delta h_b}{h_b} + \frac{\Delta h_b}{h_b}\tan\beta\right)}{1 + \text{Re}\left(\frac{\delta h_b}{h_b} + \frac{\Delta h_b}{h_b}\tan\beta\right)} \quad (32)$$

and the same holds for  $\tan\chi_\tau$  with  $b$  replaced by  $\tau$  on the right hand side of Eq. (32). For  $\tan\chi_t$  an expression similar to Eq. (32) holds with  $b$  replaced by  $t$  and  $\tan\beta$  replaced by  $\cot\beta$ . The coupling  $h_b$  is related to the  $b$  quark mass by the relation

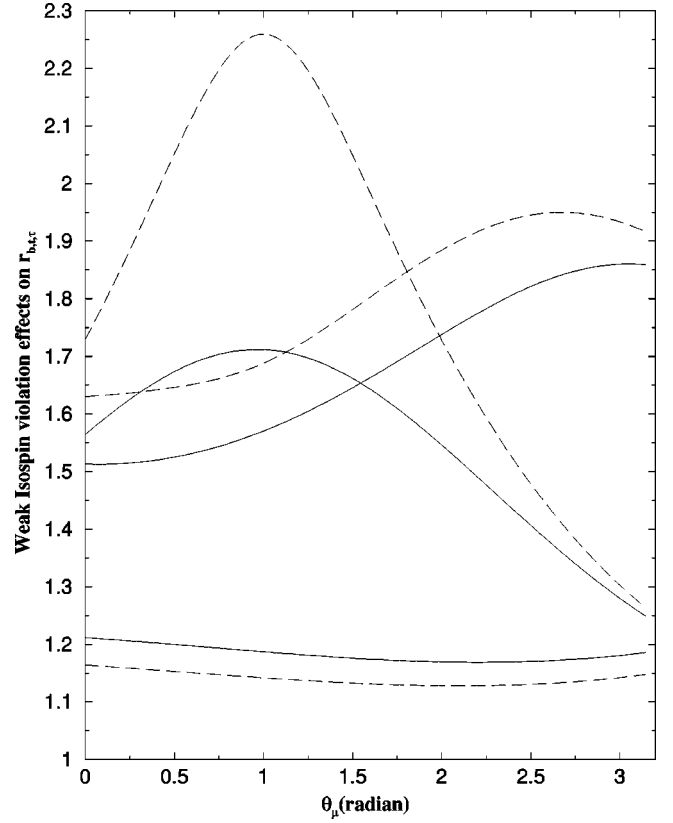


FIG. 2. Plot of  $r_b, r_t, r_\tau$  as a function of  $\theta_\mu$  for the following inputs: (solid curves)  $m_A = 200$  GeV,  $\tan\beta = 20$ ,  $m_0 = 350$  GeV,  $m_g = 300$  GeV,  $\xi_1 = .1$ ,  $\xi_2 = .2$ ,  $\xi_3 = -.3$ ,  $|A_t^0| = 3$ ,  $\alpha_{A_t^0} = 0$ ,  $|A_b^0| = 7$ ,  $\alpha_{A_b^0} = 2$ . The curves in descending order at  $\theta_\mu = 0$  correspond to  $r_b, r_\tau$  and  $r_t$ ; (dashed curves) same input as for solid curves except that  $m_0 = 375$  GeV and  $|A_b^0| = 8$ .

$$\begin{aligned} h_b &= \sqrt{2} \frac{m_b}{v_1} \left[ \left( 1 + \text{Re} \frac{\delta h_b}{h_b} + \text{Re} \frac{\Delta h_b}{h_b} \tan\beta \right)^2 \right. \\ &\quad \left. + \left( \text{Im} \frac{\delta h_b}{h_b} + \text{Im} \frac{\Delta h_b}{h_b} \tan\beta \right)^2 \right]^{-1/2} \end{aligned} \quad (33)$$

and similar relations hold for  $h_\tau$ . For  $h_t$  a similar relation holds but with  $v_1$  replaced by  $v_2$  and  $\tan\beta$  replaced by  $\cot\beta$ . Notice that  $\delta h_b$  and  $\Delta h_b$  in Eqs. (32), (33) are not barred quantities. Quantities of interest for the purpose of illustration of loop effects are  $R_{tb}$  an  $R_{\nu\tau}$  defined by

$$R_{tb} = \frac{\Gamma(H^- \rightarrow \bar{t}b)}{\Gamma(H^- \rightarrow \bar{t}b)_0} \quad (34)$$

and

$$R_{\nu\tau} = \frac{\Gamma(H^- \rightarrow \bar{\nu}_\tau \tau^-)}{\Gamma(H^- \rightarrow \bar{\nu}_\tau \tau^-)_0}, \quad (35)$$

where

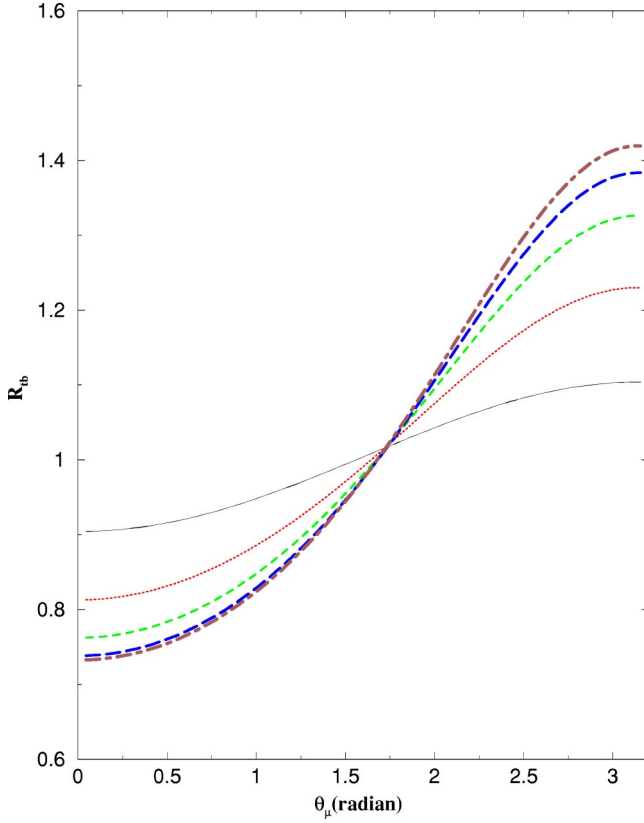


FIG. 3. Plot of  $R_{tb} = \Gamma(H^- \rightarrow \bar{t}b)_{loop} / \Gamma(H^- \rightarrow \bar{t}b)_{tree}$  as a function of the phase  $\theta_\mu$ . The input parameters are  $m_A = 200$  GeV,  $m_0 = 200$  GeV,  $m_{\tilde{g}} = 400$  GeV,  $\xi_1 = 0$ ,  $\xi_2 = \pi$ ,  $\xi_3 = \pi$ ,  $\alpha_{A_t^0} = 0 = \alpha_{A_b^0}$ , and  $|A_t^0| = |A_b^0| = 4$ . The curves in ascending order at the point  $\theta_\mu = \pi$  correspond to  $\tan \beta = 5, 10, 15, 20, 30$ .

$$\Gamma(H^- \rightarrow \bar{t}b) = \frac{3}{4\pi M_{H^-}^3} [(m_t^2 + m_b^2 - m_{H^-}^2)^2 - 4m_t^2 m_b^2]^{1/2} \times \left\{ \frac{1}{2} (|B_{bt}^s|^2 + |B_{bt}^p|^2) (m_{H^-}^2 - m_t^2 - m_b^2) - \frac{1}{2} (|B_{bt}^s|^2 - |B_{bt}^p|^2) (2m_t m_b) \right\} (1 + \omega). \quad (36)$$

Here  $(1 + \omega)$  is the QCD enhancement factor and is given by [23]

$$(1 + \omega) = 1 + 5.67 \frac{\alpha_s}{\pi} + 29.14 \frac{\alpha_s^2}{\pi^2} \quad (37)$$

so that  $(1 + \omega) \approx 1.25$  for  $\alpha_s \approx 0.12$ . Similarly we have

$$\Gamma(H^- \rightarrow \bar{\nu}_\tau \tau^-) = \frac{1}{8\pi M_{H^-}^3} (m_{H^-}^2 - m_\tau^2)^2 \times (|B_{\nu\tau}^s|^2 + |B_{\nu\tau}^p|^2). \quad (38)$$

The quantities  $\Gamma(H^- \rightarrow \bar{t}b)_0$  and  $\Gamma(H^- \rightarrow \bar{\nu}_\tau \tau^-)_0$  correspond to the tree value, i.e., when the loop corrections in

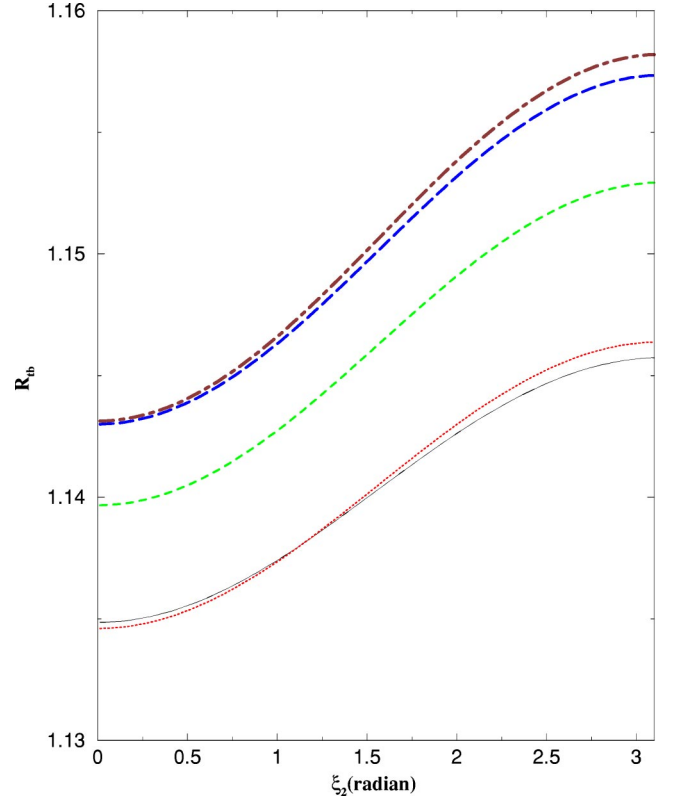


FIG. 4. Plot of  $R_{tb}$  as a function of the phase  $\xi_2$ . The input parameters are  $m_A = 200$  GeV,  $m_0 = 200$  GeV,  $\alpha_{A_t^0} = 0 = \alpha_{A_b^0}$ ,  $|A_t^0| = |A_b^0| = 2$ ,  $\xi_1 = 0$ ,  $\xi_3 = 0$ ,  $\theta_\mu = 0$  and  $\tan \beta = 10$ . The curves in ascending order at  $\xi_2 = \pi$  correspond to values of  $m_{\tilde{g}} = 300, 400, 600, 800, 1000$  GeV.

$\Gamma(H^- \rightarrow \bar{t}b)$  and  $\Gamma(H^- \rightarrow \bar{\nu}_\tau \tau^-)$  are set to zero. Finally, to quantify the breakdown of the weak isospin arising from SUSY QCD and SUSY electroweak loop effects we define the following quantities:

$$\Delta R_{tb/\nu\tau} = \frac{R_{tb/\nu\tau}^{H^-} - R_0^{H^-}}{R_0^{H^-}}, \quad (39)$$

where the first term in the numerator includes the full loop correction including the effects of weak isospin violation and the quantities with subscripts 0 are evaluated at the tree level. Further, we define

$$\Delta r_{tb/\nu\tau} = \frac{\bar{R}_{tb/\nu\tau}^{H^-} - R_0^{H^-}}{R_0^{H^-}}, \quad (40)$$

where  $\Delta r_{tb/\nu\tau}$  is defined identical to  $\Delta R_{tb/\nu\tau}$  except that no barred quantities are used, i.e., we set  $\bar{\Delta} h_{b,t,\tau} = \Delta h_{b,t,\tau}$  and  $\bar{\delta} h_{b,t,\tau} = \delta h_{b,t,\tau}$  in Eq. (31). A comparison of  $\Delta r_{tb/\nu\tau}$  and  $\Delta R_{tb/\nu\tau}$  exhibits the amount of weak isospin violation induced by SUSY loop effects.



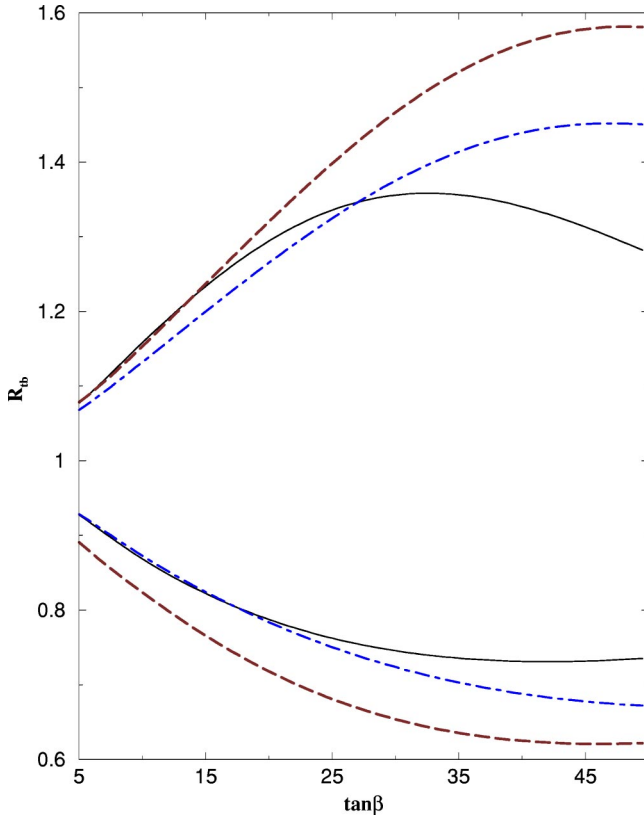


FIG. 5. An exhibition of  $R_{tb}$  as a function of  $\tan\beta$  with and without phases. The three lower curves are for the cases (a), (b) and (c) with parameters given by (a)  $m_A=200$  GeV,  $m_0=m_{1/2}=300$  GeV,  $A_0=4$ ,  $\alpha_{A_0}=1$ ,  $\xi_1=.5$ ,  $\xi_2=.659$ ,  $\xi_3=.633$ ,  $\theta_\mu=2.5$  (solid); (b)  $m_A=200$  GeV,  $m_0=m_{1/2}=555$  GeV,  $A_0=4$ ,  $\alpha_{A_0}=2$ ,  $\xi_1=.6$ ,  $\xi_2=.653$ ,  $\xi_3=.672$ ,  $\theta_\mu=2.5$  (long-dashed); (c)  $m_A=200$  GeV,  $m_0=m_{1/2}=480$  GeV,  $A_0=3$ ,  $\alpha_{A_0}=.8$ ,  $\xi_1=.4$ ,  $\xi_2=.668$ ,  $\xi_3=.6$ ,  $\theta_\mu=2.5$  (dot-dashed).  $|A_t^0|=|A_b^0|=A_0$ ,  $\alpha_{A_t^0}=\alpha_{A_b^0}=\alpha_{A_0}$  in all cases. The EDM constraints including the  $H_g^{199}$  are satisfied for the above curves at  $\tan\beta=50$  as shown in Table I. A plot of the above three cases but with the phases set to zero is given by the similar upper curves.

## V. NUMERICAL ANALYSIS

The analytical analysis of Secs. II–IV is valid for MSSM. However, the parameter space of MSSM is rather large and for this reason for the numerical analysis we use the framework of SUGRA models but allowing for nonuniversalities. Specifically in the numerical analysis we use the parameters  $m_A$ ,  $m_0$ ,  $A_t^0$ ,  $A_b^0=A_\tau^0$  (where  $A^0$ 's are in general complex),  $\tan\beta$  and  $\xi_i$  ( $i=1,2,3$ ) where  $\xi_i$  are the phases of the gaugino masses, i.e.,  $\tilde{m}_i=m_{1/2}e^{i\xi_i}$ . In addition one has the Higgs mixing parameter  $\mu$  (which appears in the superpotential as  $\mu H_1 H_2$ ) which is also in general complex, i.e.,  $\mu=|\mu|\exp(i\theta_\mu)$ , where  $|\mu|$  is determined by radiative breaking of the electroweak symmetry while  $\theta_\mu$  is arbitrary. (We note, however, that not all the phases are independent since the phases appear only in certain combinations in physical quantities [24].) We then evolve them through renormalization group equations to the low energy scales (see e.g., Ref. [25]). Now as seen from the discussion in Secs. I and II, the weak

TABLE I. EDMs at  $\tan\beta=50$  for Figs. 5 and 8.

Case	$ d_e $ (e cm)	$ d_n $ (e cm)	$C_{Hg}$ cm
(a)	$1.67 \times 10^{-27}$	$1.59 \times 10^{-27}$	$1.18 \times 10^{-27}$
(b)	$6.05 \times 10^{-28}$	$3.47 \times 10^{-27}$	$1.29 \times 10^{-26}$
(c)	$2.14 \times 10^{-27}$	$8.90 \times 10^{-28}$	$1.25 \times 10^{-26}$

isospin is a symmetry of the tree level Lagrangian but is violated at the loop level. The size of the weak isospin violation arising from loop corrections can be quantified by the  $r_b$  defined by Eq. (29) (and by  $r_t, r_\tau$  similarly defined). In Fig. 2 we give a plot of  $r_b, r_t, r_\tau$  as a function of  $\theta_\mu$ . Recalling that deviations of  $r_{b,t,\tau}$  from unity register the violations of weak isospin, we find that indeed such deviations can be as much as 50% or more depending on the region of the parameter space one is in. Thus in general the violations of weak isospin arising from Eq. (5) would be significant.

Next we investigate the question of how large the loop corrections themselves are relative to the tree values. In Fig. 3 we give a plot of  $R_{tb}$  defined by Eq. (34) as a function of  $\theta_\mu$  for values of  $\tan\beta$  ranging from 5 to 30 for the specific set of inputs given in the caption of Fig. 3. The analysis of the figure shows that the loop correction varies strongly with the phase  $\theta_\mu$  with the correction changing sign as  $\theta_\mu$  varies

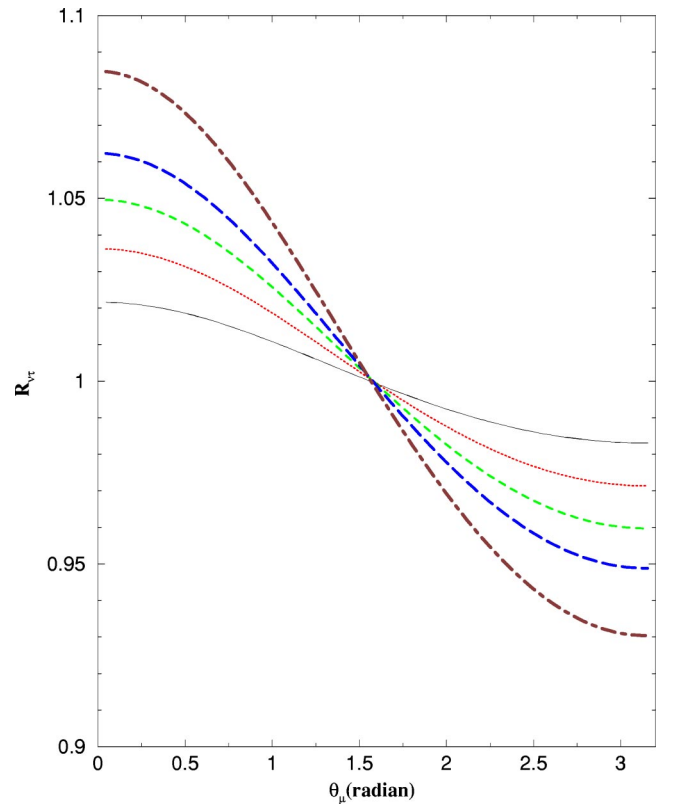


FIG. 6. Plot of  $R_{\nu\tau}=\Gamma(H^-\rightarrow\bar{\nu}\tau^-)_{loop}/\Gamma(H^-\rightarrow\bar{\nu}\tau^-)_{tree}$  as a function of the phase  $\theta_\mu$ . The input parameters are  $m_A=200$  GeV,  $m_0=200$  GeV,  $m_g=400$  GeV,  $\xi_1=0$ ,  $\xi_2=\pi$ ,  $\xi_3=\pi$ ,  $\alpha_{A_t^0}=\alpha_{A_b^0}=0$ , and  $|A_t^0|=|A_b^0|=4$ . The curves in descending order at the point  $\theta_\mu=\pi$  correspond to  $\tan\beta=5, 10, 15, 20, 30$ .



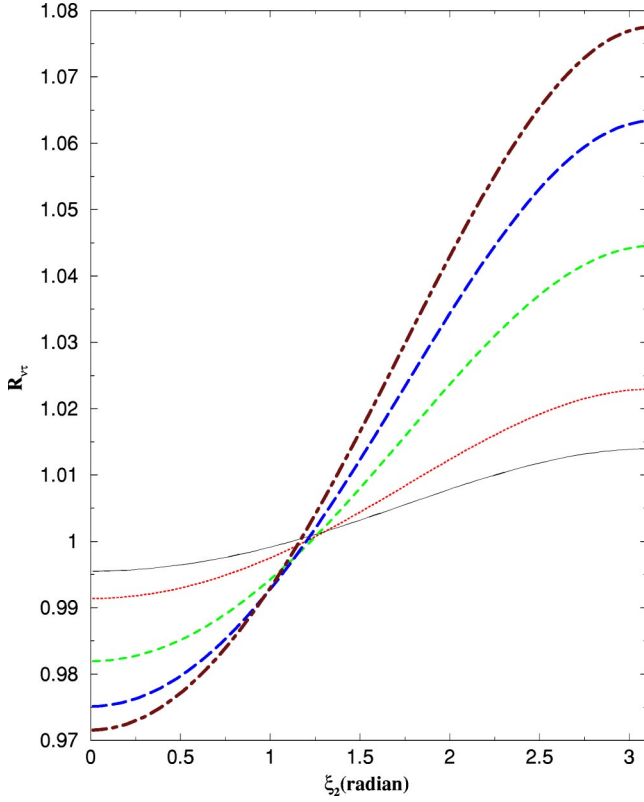


FIG. 7. Plot of  $R_{\nu\tau}$  as a function of the phase  $\xi_2$ . The input parameters are  $m_A = 200$  GeV,  $m_0 = 200$  GeV,  $\alpha_{A_t}^0 = \alpha_{A_b}^0 = 0$ ,  $|A_t^0| = |A_b^0| = 2$ ,  $\xi_1 = 0$ ,  $\xi_3 = 0$ ,  $\theta_\mu = 0$  and  $\tan\beta = 10$ . The curves in ascending order at  $\xi_2 = \pi$  correspond to values of  $m_{\tilde{g}} = 300, 400, 600, 800, 1000$  GeV.

from 0 to  $\pi$ . Further, the analysis shows that the loop correction can be as large as about 40–50% of the tree contribution in this case. In Fig. 4 we give a plot of  $R_{tb}$  as a function of  $\xi_2$  for the specific set of inputs given in the caption of Fig. 4. The analysis of Fig. 4 shows that the loop corrections are substantial and further that they have some sensitivity to  $\xi_2$  though the sensitivity is significantly smaller when compared to the sensitivity to variations in  $\theta_\mu$  seen in Fig. 3. The reason for this difference is that  $\xi_2$  is the phase that appears only in the electroweak loops whose contributions are relatively smaller than those arising from SUSY QCD while the variations in  $\theta_\mu$  arise from both QCD and electroweak contributions. In Fig. 5 we give a comparison of the loop correction to  $R_{tb}$  with and without phases as a function of  $\tan\beta$ . The lower curves are for three cases (a), (b) and (c) whose inputs are given in the figure caption. These cases at  $\tan\beta = 50$  satisfy the EDM constraints including the  $H_g^{199}$  constraint as shown in Table I (taken from Ref. [18]). The upper three similar curves have all the same inputs as the lower three curves except that the phases are all set to zero. Figure 5 shows that the loop corrections to  $R_{tb}$  depend sensitively on the phases and the correction can change sign from its tree value in the presence of phases. Further, the ratio  $R_{tb}$  is sensitive to  $\tan\beta$  with or without the inclusion of phases. The analysis of Figs. 6–8 is identical to the analysis of Figs. 3–5 except that the analysis of Figs. 6–8 is for the

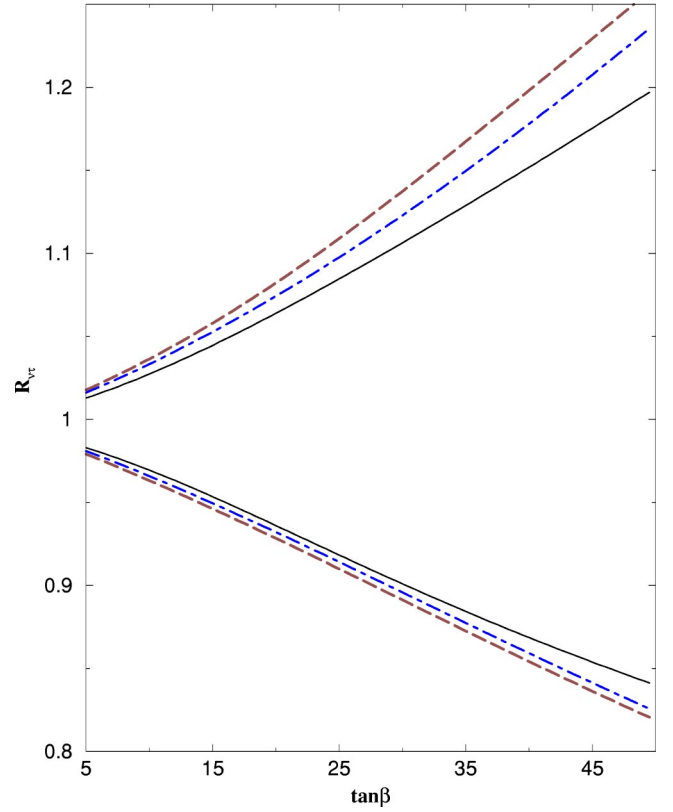


FIG. 8. An exhibition of  $R_{\nu\tau}$  as a function of  $\tan\beta$  with and without phases. The three upper curves are for the cases (a), (b) and (c) with parameters given by (a)  $m_A = 200$  GeV,  $m_0 = m_{1/2} = 300$  GeV,  $A_0 = 4$ ,  $\alpha_{A_0} = 1$ ,  $\xi_1 = .5$ ,  $\xi_2 = .659$ ,  $\xi_3 = .633$ ,  $\theta_\mu = 2.5$  (solid); (b)  $m_A = 200$  GeV,  $m_0 = m_{1/2} = 555$  GeV,  $A_0 = 4$ ,  $\alpha_{A_0} = 2$ ,  $\xi_1 = .6$ ,  $\xi_2 = .653$ ,  $\xi_3 = .672$ ,  $\theta_\mu = 2.5$  (long-dashed); (c)  $m_A = 200$  GeV,  $m_0 = m_{1/2} = 480$  GeV,  $A_0 = 3$ ,  $\alpha_{A_0} = .8$ ,  $\xi_1 = .4$ ,  $\xi_2 = .668$ ,  $\xi_3 = .6$ ,  $\theta_\mu = 2.5$  (dot-dashed).  $|A_t^0| = |A_b^0| = A_0$ ,  $\alpha_{A_t}^0 = \alpha_{A_b}^0 = \alpha_{A_0}$  in all cases. The EDM constraints including the  $H_g^{199}$  are satisfied for the above curves at  $\tan\beta = 50$  as shown in Table I. A plot of the above three cases but with the phases set to zero is given by the similar lower curves.

ratio  $R_{\nu\tau}$ . The main difference here from the  $R_{tb}$  case is that for the case of  $R_{\nu\tau}$  there are no SUSY QCD corrections. Thus the sensitivity to the variations in the electroweak parameters is larger. Thus a comparison of Fig. 4 and Fig. 7 shows that  $R_{\nu\tau}$  is more sensitive to variations in  $\xi_2$  than  $R_{tb}$  because the electroweak corrections are not masked by QCD as in the case of  $R_{tb}$ . Finally, in Fig. 9 we plot  $\Delta R_{tb/\nu\tau}$  and  $\Delta r_{tb/\nu\tau}$  defined in Eq. (39) and Eq. (40) as a function of  $\theta_\mu$ . A comparison of the two shows that the effect of weak isospin violation on the branching ratios can be in the neighborhood of 20–25%.

As mentioned in the Introduction, the technique used in this paper is different from the one in the analysis of Ref. [21]. While the main focus of the present work is to calculate the loop effects on the decay widths  $\Gamma(H^- \rightarrow \bar{t}b)$  and  $\Gamma(H^- \rightarrow \bar{\nu}_\tau \tau^-)$ , the main focus of the work of Ref. [21] is the calculation of the  $CP$  asymmetry parameter  $\delta^{CP} = [\Gamma(H^+ \rightarrow t\bar{b}) - \Gamma(H^- \rightarrow \bar{t}b)] / [\Gamma(H^+ \rightarrow t\bar{b}) + \Gamma(H^- \rightarrow \bar{t}b)]$ .

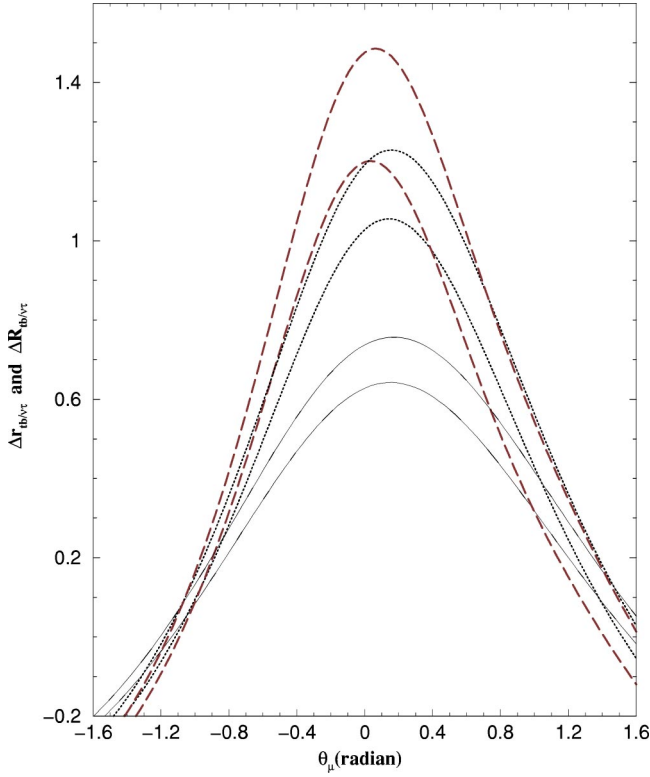


FIG. 9. Plot of  $\Delta R_{tb/\nu\tau}$  and  $\Delta r_{tb/\nu\tau}$  as a function of  $\theta_\mu$  for the following inputs: (solid curves)  $m_A=200$  GeV,  $m_0=300$  GeV,  $m_g^-=300$  GeV,  $\tan\beta=20$ ,  $\xi_1=.1$ ,  $\xi_2=.2$ ,  $\xi_3=-.3$ ,  $|A_t^0|=3$ ,  $\alpha_{A_t^0}=0$ ,  $|A_b^0|=7$ ,  $\alpha_{A_b^0}=2$ . The curves in descending order at  $\theta_\mu=0$  correspond to  $\Delta r_{tb/\nu\tau}$  and  $\Delta R_{tb/\nu\tau}$ ; (dotted curves) same input as for solid curves except that  $m_0=350$  GeV; (dashed curves) same input as solid curves except that  $m_0=375$  GeV and  $|A_b^0|=8$ .

The analysis of Ref. [21] also gives an estimate of the branching ratio for the mode  $H^- \rightarrow \bar{t}b$ . However, a comparison with this estimate requires that one compute all the decay modes of  $H^-$  including the mode  $\chi^0 + \chi^-$ . In this analysis we did not compute this decay mode but propose to do so in a future work which would allow a direct comparison with the analysis of Ref. [21].

## VI. CONCLUSIONS

In this paper we have investigated the effects of supersymmetric loop corrections on the violations of weak isospin in Yukawa couplings of the Higgs bosons to quarks and leptons. Specifically we have computed the gluino, chargino and neutralino loop corrections to the charged Higgs couplings to the third generation quarks and leptons. We find that the loop corrections to the charged Higgs couplings can be as much as 40–50% of the tree level contribution. We also compared the supersymmetric loop corrections to the charged Higgs couplings with the supersymmetric loop corrections to the neutral Higgs couplings. The disparity between the charged Higgs and the neutral Higgs couplings is a measure of the violations of weak isospin in the effective low energy Lagrangian. The analysis shows that the effects of violations of weak isospin on the Yukawa couplings can be as much as 50% or more. It is also found that such violations are in fact also sensitive to  $CP$  phases. Using these results we have investigated the charged Higgs decays  $H^- \rightarrow \bar{t}b$  and  $H^- \rightarrow \bar{\nu}_\tau \tau^-$ . It is shown that the branching ratios for these decays are sensitive to weak isospin violation effects, and the effects of the violations of weak isospin on the branching ratio can be as much as 20–25%, and thus accurate measurement of the branching ratios of the charged and neutral Higgs decays can provide a measure of such violations. The new results of this paper are contained in Secs. III–V. Specifically in Sec. III we have given computations of  $\Delta h_{b,t,\tau}$  and  $\delta h_{b,t,\tau}$  not previously computed in the literature in the current framework. The analysis of this paper will also be useful in the more complete computations of decays of the top quarks and sbottoms [26] and in the more accurate computation of charged Higgs decays [27].

## ACKNOWLEDGMENT

This research was also supported in part by NSF grant PHY-0139967.

## APPENDIX A:

For the convenience of comparison of the barred and the unbarred quantities, we exhibit below the unbarred quantities for the bottom quark [18]. First we exhibit  $\Delta h_b$ . We have

$$\begin{aligned}
 -\Delta h_b = & -\sum_{i=1}^2 \sum_{j=1}^2 \frac{2\alpha_s}{3\pi} e^{-i\xi_3} m_g^- G_{ij}^* D_{b1i}^* D_{b2j} f(m_g^2, m_{b_i}^2, m_{b_j}^2) \\
 & -\sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 g^2 E_{ij}^* \{V_{k1}^* D_{t1i}^* - k_t V_{k2}^* D_{t2ij}^*\} (k_b U_{k2}^* D_{t1j}) \frac{m_{\chi_k^+}}{16\pi^2} f(m_{\chi_k^+}^2, m_{\tau_i}^2, m_{\tau_j}^2) \\
 & -\sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 g^2 C_{ij} \{V_{t1}^* D_{t1k}^* - k_t V_{t2}^* D_{t2kj}^*\} (k_b U_{j2}^* D_{t1k}) \frac{m_{\chi_i^+} m_{\chi_j^+}}{16\pi^2} f(m_{\tau_k}^2, m_{\chi_i^+}^2, m_{\chi_j^+}^2)
 \end{aligned}$$

$$\begin{aligned}
& + \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^4 2G_{ij}^* \{ \alpha_{bk} D_{b1j} - \gamma_{bj} D_{b2j} \} \{ \beta_{bk}^* D_{b1i}^* + \alpha_{bk} D_{b2i}^* \} \frac{m_{\chi_k^0}}{16\pi^2} f(m_{\chi_k^0}^2, m_{\tilde{b}_i}^2, m_{\tilde{b}_j}^2) \\
& + \sum_{i=1}^4 \sum_{j=1}^4 \sum_{k=1}^2 2\Gamma_{ij} \{ \alpha_{bj} D_{b1k} - \gamma_{bj} D_{b2k} \} \{ \beta_{bi}^* D_{b1k}^* + \alpha_{bi} D_{b2k}^* \} \frac{m_{\chi_i^0} m_{\chi_j^0}}{16\pi^2} f(m_{\tilde{b}_k}^2, m_{\chi_i^0}^2, m_{\chi_j^0}^2)
\end{aligned} \quad (A1)$$

where

$$\begin{aligned}
\frac{E_{ij}}{\sqrt{2}} &= \frac{gM_Z}{2 \cos \theta_W} \left\{ \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) D_{t1i}^* D_{t1j} + \frac{2}{3} \sin^2 \theta_W D_{t2i}^* D_{t2j} \right\} \sin \beta \\
& - \frac{gm_t^2}{2M_W \sin \beta} [D_{t1i}^* D_{t1j} + D_{t2i}^* D_{t2j}] - \frac{gm_t m_0 A_t}{2M_W \sin \beta} D_{t2i}^* D_{t1j}
\end{aligned} \quad (A2)$$

and

$$\frac{C_{ij}}{\sqrt{2}} = -\frac{g}{\sin \beta} \left[ \frac{m_{\chi_i^+}}{2M_W} \delta_{ij} - Q_{ij}^* \cos \beta - R_{ij}^* \right] \quad (A3)$$

$$\frac{G_{ij}}{\sqrt{2}} = \frac{gM_Z}{2 \cos \theta_W} \left\{ \left( -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W \right) D_{b1i}^* D_{b1j} - \frac{1}{3} \sin^2 \theta_W D_{b2i}^* D_{b2j} \right\} \sin \beta + \frac{gm_b \mu}{2M_W \cos \beta} D_{b1i}^* D_{b2j} \quad (A4)$$

and where

$$\begin{aligned}
Q_{ij} &= \sqrt{\frac{1}{2}} U_{i2} V_{j1} \\
R_{ij} &= \frac{1}{2M_W} [\tilde{m}_2^* U_{i1} V_{j1} + \mu^* U_{i2} V_{j2}]
\end{aligned} \quad (A5)$$

$\Gamma_{ij}$  appearing in Eq. (A1) is defined by

$$\frac{\Gamma_{ij}}{\sqrt{2}} = -\frac{g}{2 \sin \beta} \left[ \frac{m_{\chi_i^0}}{2M_W} \delta_{ij} - Q''_{ij} \cos \beta - R''_{ij} \right] \quad (A6)$$

where

$$\begin{aligned}
gQ''_{ij} &= \frac{1}{2} [X_{3i}^* (gX_{2j}^* - g'X_{1j}^*) + (i \leftrightarrow j)] \\
R''_{ij} &= \frac{1}{2M_W} [\tilde{m}_1^* X_{1i} X_{1j}^* + \tilde{m}_2^* X_{2i} X_{2j}^* - \mu^* (X_{3i}^* X_{4j}^* + X_{4i}^* X_{3j}^*)].
\end{aligned} \quad (A7)$$

Next we exhibit  $\delta h_b$  [18]. We have

$$\begin{aligned}
-\delta h_b &= -\sum_{i=1}^2 \sum_{j=1}^2 \frac{2\alpha_s}{3\pi} e^{-i\xi_3} m_g^- H_{ji} D_{b1i}^* D_{b2j} f(m_g^2, m_{\tilde{b}_i}^2, m_{\tilde{b}_j}^2) \\
& - \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^2 g^2 F_{ji} \{ V_{k1}^* D_{t1i}^* - k_t V_{k2}^* D_{t2i}^* \} (k_b U_{k2} D_{t1j}) \frac{m_{\chi_k^+}}{16\pi^2} f(m_{\chi_k^+}^2, m_{\tilde{t}_i}^2, m_{\tilde{t}_j}^2) \\
& + \sum_{i=1}^2 \sum_{j=1}^2 \sum_{k=1}^4 2H_{ji} \{ \alpha_{bk} D_{b1j} - \gamma_{bk} D_{b2j} \} \{ \beta_{bk}^* D_{b1i}^* + \alpha_{bk} D_{b2i}^* \} \frac{m_{\chi_k^0}}{16\pi^2} f(m_{\chi_k^0}^2, m_{\tilde{b}_i}^2, m_{\tilde{b}_j}^2)
\end{aligned} \quad (A8)$$

where

$$\begin{aligned} \frac{H_{ij}}{\sqrt{2}} = & -\frac{gM_Z}{2\cos\theta_W} \left\{ \left( -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W \right) D_{b1i}^* D_{b1j} - \frac{1}{3}\sin^2\theta_W D_{b2i}^* D_{b2j} \right\} \cos\beta \\ & - \frac{g m_b^2}{2M_W \cos\beta} [D_{b1i}^* D_{b1j} + D_{b2i}^* D_{b2j}] - \frac{g m_b m_0 A_b}{2M_W \cos\beta} D_{b2i}^* D_{b1j} \end{aligned} \quad (\text{A9})$$

and

$$\frac{F_{ij}}{\sqrt{2}} = -\frac{gM_Z}{2\cos\theta_W} \left\{ \left( \frac{1}{2} - \frac{2}{3}\sin^2\theta_W \right) D_{t1i}^* D_{t1j} + \frac{2}{3}\sin^2\theta_W D_{t2i}^* D_{t2j} \right\} \cos\beta + \frac{g m_t \mu}{2M_W \sin\beta} D_{t1i}^* D_{t2j}. \quad (\text{A10})$$

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