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J. B. Sokoloff
Northeastern University

C. Vittoria
Northeastern University

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Possible low-loss Faraday rotation and phase shifts using multilayer ferrite structures

J. B. Sokoloff and C. Vittoria
Northeastern University, Boston, Massachusetts 02115

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It is suggested that it should be possible to make low insertion loss devices (e.g., Faraday rotators, phase shifters, and broad-band filters) to operate in the millimeter wave range using a multilayer array of thin films magnetized perpendicular to the films with waves of frequency below the magnon continuum normally incident.

I. INTRODUCTION

In order to design devices which can conveniently operate in the millimeter wave region, it is necessary to use a material such as barium ferrite which possesses high enough anisotropy so that the ferrimagnetic resonant (FMR) frequency falls in the millimeter range.¹ Estimates of the intrinsic FMR linewidth of barium ferrite give values below 10 Oe.¹ The smallest experimentally observed linewidth is about 30 Oe.² Typical linewidths for samples of this material are, however, much larger, most likely due to two magnon processes associated with surface scattering,³ which are expected to be particularly large in barium ferrite because of its large saturation magnetization, which may imply an increase in insertion loss beyond the intrinsic limit. Two magnon surface scattering can be avoided by operating at frequencies below the magnon continuum.⁴ This is equivalent to the magnetic polariton effect discussed in Ref. 5. Although the imaginary part of the permeability is small at these frequencies, the real part will be significant over a wide range of frequencies below the resonant frequency, since the permeability, if the imaginary part of its self-energy is neglected, is proportional to $(\omega_{\text{res}} - \omega)^{-1}$, where ω and ω_{res} are the operating and FMR resonant frequency, respectively. For thin ferrite films, magnetized perpendicular to the film, the FMR resonance occurs at the bottom edge of the continuum.³ The basic idea is to avoid the loss due to two-magnon scattering by using thin ferrite films magnetized perpendicular to the film and operating just below the magnon continuum where energy conservation for the decay of the uniform precession magnon into other magnons in the continuum cannot be satisfied. This idea is illustrated schematically in Fig. 1. For instance, it was shown in Ref. 6 that the FMR linewidth due to two-magnon scattering is accounted for by including a complex self-energy term in the denominator of the frequency-dependent magnetic susceptibility, whose imaginary part contains a sum over all wave vectors of a delta function which is only nonzero when the energy conservation condition for two-magnon scattering is satisfied (i.e., when the uniform magnon frequency is equal to the frequency of one of the other magnons in the magnon continuum).

Since a single thin film perpendicular to the propagation direction of the incident waves is not expected to provide sufficient phase shift or Faraday rotation to be useful

for most device applications, we suggest the use of an array of such barriers in order to enhance the rotation and phase shift. (It is easily shown that for an array of very thin films, the demagnetizing factor inside a single film in the array remains nearly equal to that of an isolated film, and hence, the uniform magnon remains at the bottom of the magnon continuum.) We will see in the next section how a periodic multilayer with the films perpendicular to the wave propagation direction and magnetization (i.e., Kittel configuration) can be used to accomplish this goal. Previous work by How and Vittoria has considered other configurations for multilayer structures.⁷

II. PROPAGATION OF WAVES THROUGH MULTILAYER STRUCTURES

It is well known that wave propagation through periodic structures occurs in frequency bands separated by gaps, in which the waves decay exponentially.⁸ It is convenient to formulate this problem using the method of transfer matrices,^{9,10} which allows us to calculate the fraction of the intensity which is transmitted (i.e., the transmission coefficient), as well as the Faraday rotation and phase shift quite easily. Our discussion follows that given by Yeh, Yariv, and Hong.⁹ Throughout the discussion we will concentrate on wave propagation perpendicular to a periodic array of infinite thin permeable plane barriers with magnetization normal to the planes (i.e., the Kittel configuration). The modifications to this treatment needed in order to treat wave propagation in wave guides and slot-line devices will be discussed later on in this section. Furthermore, in order to simplify the discussion, it will be assumed that the insertion loss is sufficiently small so that it may be neglected in the present calculations on multilayer structures (whose purpose is to determine whether multilayer structures can be used to enhance the Faraday rotation and phase shift without losing most of the wave's intensity to reflection from the structure). The conditions that must be met in order to achieve this are discussed in the Appendix where it is shown that it is possible to operate in a frequency range for which the distance over which the wave decays can be made quite long (i.e., long compared to the thickness of our multilayer structure). Let us assume that we have circularly polarized electromagnetic

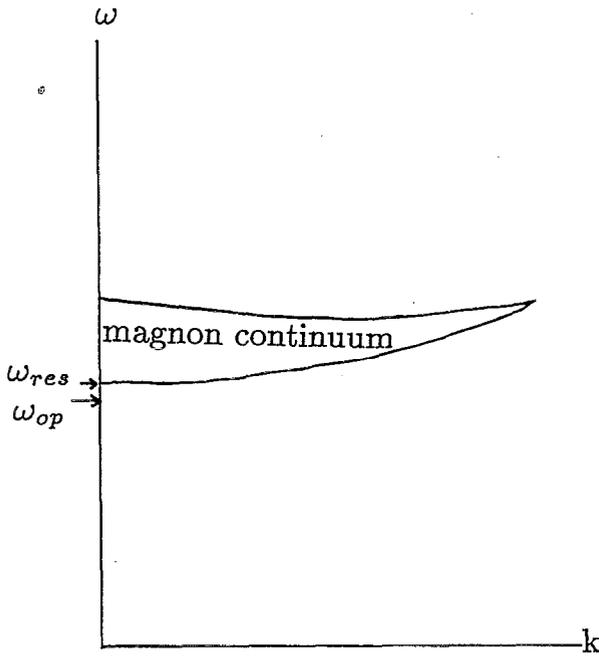


FIG. 1. A schematic diagram of the spin-wave spectrum, frequency ω vs wave vector k , is shown. The operating frequency ω_{op} and resonant frequency ω_{res} are indicated with arrows.

waves propagating along the z axis with the barriers normal to it. Then, the electric and magnetic fields are given by

$$\mathbf{E} = \text{Re } \psi(z) (\hat{x} - i\hat{y}) e^{-i\omega t}, \quad (1a)$$

$$\mathbf{B} \propto \mu^{-1} \nabla \times \mathbf{E} = \text{Re } \mu^{-1} i (\hat{x} - i\hat{y}) \frac{\partial \psi(z)}{\partial z} e^{-i\omega t}, \quad (1b)$$

where \hat{x} , \hat{y} are unit vectors in the x and y directions, μ is the permeability, and $\psi(z)$ will be given in Eq. (2). Then, the wave to the right of the n th barrier is given by

$$\psi = a_n e^{ik(z-n\Lambda)} + b_n e^{-ik(z-n\Lambda)}, \quad (2)$$

where Λ is the period or the length of the unit cell of the array, and a_n and b_n are expansion coefficients (our notation follows Ref. 9). The reflection and transmission coefficients of an N barrier array are given by $|b_1/a_1|^2$ and $|a_N/a_1|^2$, respectively, if we set b_N equal to 0 (i.e., the incident wave comes from the left). The coefficients are related by^{9,10}

$$\begin{pmatrix} a_{n+1} \\ b_{n+1} \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} \begin{pmatrix} a_n \\ b_n \end{pmatrix}. \quad (3)$$

The matrix R has a determinant of 1. As shown in Ref. 9, this relationship is obtained by applying boundary conditions on both sides of the barrier. Later on we will illustrate this procedure for a specific example. It is easily shown that $R_{1,1} = R_{2,2}^*$ and $R_{2,1} = R_{1,2}^*$, when insertion loss is neglected.^{9,10} We are specifically interested in ferrite films which are magnetized perpendicular to the plane of the film and where the c axis of the ferrite crystal is also perpendicular to the film. For this situation, the permeability

μ is much different for the two directions of circular polarization since only the μ for one direction contains the spin-wave resonance and hence can be large. Letting the matrix $\mathbf{A}_n = (a_n, b_n)$, we have by iterating Eq. (3)

$$\mathbf{A}_n = \mathbf{R}^n \mathbf{A}_1. \quad (4a)$$

Clearly, we can always write \mathbf{R} as $\mathbf{U}\mathbf{D}\mathbf{U}^{-1}$, where \mathbf{U} is the unitary matrix which diagonalizes \mathbf{R} and \mathbf{D} is the diagonal matrix with the eigenvalues of \mathbf{R} on the diagonals. These eigenvalues of \mathbf{R} are given by

$$\lambda_{\pm} = 0.5 \text{Tr} \mathbf{R} \pm i \sqrt{1 - (0.5 \text{Tr} \mathbf{R})^2}, \quad (4b)$$

which are complex numbers of unit magnitude if $|0.5 \text{Tr} \mathbf{R}| < 1$. Then, Eq. (4a) becomes

$$\mathbf{A}_n = \mathbf{U} \mathbf{D}^n \mathbf{U}^{-1} \mathbf{A}_1. \quad (5)$$

For those special energies for which $\lambda^n = \pm 1$, this reduces to $\mathbf{A}_n = \mathbf{A}_1$. Therefore, if \mathbf{A}_1 had the form $(0, C)$, which is true if the wave is assumed to be incident from the right end and there is a transmitted wave on the left end, \mathbf{A}_n will also have that form, which implies that the reflection coefficient is zero. (Of course, it must be remembered that we are restricting ourselves to multilayer structures whose thickness is small compared to the decay distance due to the intrinsic insertion loss. This point is discussed in more detail in the Appendix.) Since λ_{\pm} has the form $e^{iQ\Lambda}$, this perfect transmission will occur when the propagation vector Q is equal to $l\pi/n\Lambda$ where l is an integer. As the number of barriers increases, the resonances get closer together. As with any periodic structure, these resonances cluster into bands. At frequencies lying between the bands, the wave decays exponentially.

Let us now consider a general value for λ_{\pm} . We make use of the fact that for elements of the matrix \mathbf{U} defined above, (U_{11}, U_{21}) and (U_{12}, U_{22}) are the eigenvectors of the matrix \mathbf{R} . These eigenvectors are given by $(R_{12}, \lambda_- - R_{11})$ and $(\lambda_+ - R_{22}, R_{21})$, where the $+$ and $-$ signs signify the two different eigenvalues. Then, applying the matrix $\mathbf{B} = \mathbf{R}^N$ to a column matrix of the form $(0, C)$, which corresponds to a transmitted wave on the left side, we obtain a transmission coefficient given by $1/|B_{22}|^2$, where B_{22} is given by

$$\frac{\lambda_-^N |R_{12}|^2 - \lambda_+^N (\lambda_- - R_{11}^*) (\lambda_+ - R_{11})}{|R_{12}|^2 - (\lambda_- - R_{11}^*) (\lambda_+ - R_{11})}, \quad (6)$$

using the fact that $R_{11} = R_{22}^*$ and $R_{12} = R_{21}^*$. Neglecting wave damping due to insertion loss to a first approximation and for values of k for which $|0.5 \text{tr} \mathbf{R}| < 1$ (since $|\lambda_{\pm}| = 1$ there), we conclude that the inverse transmission coefficient for the barriers remains of order unity no matter how large N becomes. Therefore, one way to minimize the reflection of the incident wave while still having a sufficiently large amount of Faraday rotation would be to use many barriers (but still of total thickness small compared to the decay length due to the intrinsic insertion loss discussed in the Appendix) of small reflection coefficient (e.g., by using very thin films or by using films which fill up only a small part of the cross section of the waveguide). Since the reflection coefficient does not grow with the number of bar-

riers, it can be small while we maintain a large degree of Faraday rotation by having a large number of barriers. [Of course, there can still be a good deal of reflection because of the difference in dielectric constant (i.e., impedance mismatch) between the material inside and outside the multilayer structure. This could be minimized by placing a tapered piece of the dielectric material which separates the ferrite films at the two ends of the structure.]

As an illustration of wave propagation through a periodic array of permeable barriers, consider the case of plane

$$\begin{pmatrix} (\cos Kb + iQ_1 \sin Kb)e^{ika} & -ie^{ika}Q_2 \sin Kb \\ ie^{-ika}Q_2 \sin Kb & (\cos Kb - iQ_1 \sin Kb)e^{-ika} \end{pmatrix}, \quad (7)$$

where k is the incident wave vector (i.e., $k = \sqrt{\epsilon_1\mu_1}\omega/c$, where ω is the wave's frequency, c is the speed of light and ϵ_1 and μ_1 are the dielectric constant and permeability outside the ferrite barriers), $K = \sqrt{\epsilon_2\mu_2}k$ (where ϵ_2 and μ_2 are the dielectric constant and permeability inside a ferrite film barrier), $Q_1 = 0.5(\mu_2 k/K\mu_1 + K\mu_1/\mu_2 k)$, $Q_2 = 0.5(K\mu_1/\mu_2 k - \mu_2 k/K\mu_1)$ and $a = \Lambda - b$. The only difference between this matrix and that given in Ref. 9 is that K/k is replaced by $K\mu_1/\mu_2 k$ because in the present problem, the permeability is discontinuous across the surface of a barrier, as opposed to the case considered in Ref. 9 in which it is continuous. Using this expression for the matrix R with the above expression for the transmission coefficient, the transmission coefficient was calculated for an array of 100 barriers with $b/\Lambda = 0.002$ as $k\Lambda$ varied from 0 to 5 and N is varied from 1 to 100 for $\epsilon_1 = \epsilon_2$ and $\mu_2 = 100\mu_1$. In these calculations, μ_2 represents the permeability for the direction of circular polarization for which the spin-wave resonance occurs in μ . For the other direction of circular polarization, it is assumed to be equal to μ_1 . The results are shown in Fig. 2.

In our case of incident circularly polarized waves, the degree of Faraday rotation is equal to half the phase shift³ (i.e., the difference between the phase of the transmission amplitude for the multilayer structure calculated above and Nka). (This is because the transmission coefficient is the ratio of the transmitted to the incident wave amplitudes, and for circular polarization the Faraday rotation of the wave's polarization direction is by definition half the difference between the phase change of the wave on going through the barriers and the phase change on traveling through the same region of space if the barriers were not present because μ is so much smaller for the other direction of circular polarization.) The N dependence of the Faraday rotation is shown in Fig. 2(b) for the example represented by the matrix R given in Eq. (7). In our example, our choice of an array of barriers whose width is only 0.2% of their spacing results in a transmission coefficient which is quite close to 1 but with a large degree of Faraday rotation. The main reason for this can be explained as follows: From the discussion under Eq. (4b), we

waves normally incident on a periodic array of infinite plane barriers of thickness b , relative permeability μ , and spacing Λ . Since the wave in Eq. (2) represents the electric field of the electromagnetic wave, the requirement that it and the H field be continuous at an interface means that the wave and μ^{-1} times its derivative must be continuous. Applying these boundary conditions to both faces of the n th barrier gives the following expression for the matrix R :⁹

can define a propagation vector Q for the periodic structure from $e^{\pm iQ\Lambda} = \lambda_{\pm}$ or $\cos(Q\Lambda) = 0.5 \text{Tr}R$. Then the Faraday rotation and phase shift come about primarily from the fact that Q and k differ in the structure. The phase shift should then be approximately equal to $N(Q - k)a$. From the definition of the matrix R and Eqs. (4b) and (7), we have for $\epsilon_1 = \epsilon_2$, $\mu_2 = \mu$, and $\mu_1 = 1$

$$\begin{aligned} \cos(Q\Lambda) &= \cos k(\Lambda - b)\cos(\sqrt{\mu}kb) \\ &\quad - 0.5(\sqrt{\mu} + \sqrt{\mu^{-1}})\sin(k(\Lambda - b))\sin(\sqrt{\mu}kb). \end{aligned} \quad (8)$$

If $\sqrt{\mu}kb \ll 1$, which is the case for the example considered above, we obtain by expanding Eq. (9) to first order in $\sqrt{\mu}kb$

$$Q - k \approx 0.5(\mu - 1)kb/\Lambda. \quad (9)$$

Using the parameters from the calculation discussed above, we find using Eq. (9) that $N(Q - k)\lambda \approx 9.9k\Lambda$, which for $k\Lambda = 1$ gives a Faraday rotation of 5.0 rad or 289°, which accounts for most of the Faraday rotation of Fig. 2(b). The occurrence of a transmission coefficient of order unity despite the existence of such a large Faraday rotation can be demonstrated analytically using the definition of the matrix B in the paragraph containing Eq. (6), as well as Eqs. (6) and (7). In the small Kb limit considered above, we obtain

$$B_{22} = \cos NQ\Lambda + i \sin NQ\Lambda \frac{|\lambda_+ - R_{11}|^2 + |R_{12}|^2}{|\lambda_+ - R_{11}|^2 - |R_{12}|^2}, \quad (10)$$

where

$$|R_{12}|^2 \approx 0.25(\mu_2/\mu_1 - 1)^2(kb)^2,$$

and

$$\begin{aligned} |\lambda_+ - R_{11}|^2 &\approx 4 \cos^2 k\Lambda + (\mu_2/\mu_1 - 1)^2(kb)^2 \\ &\quad + 4 \cos(k\Lambda)\sin(k\Lambda)(\mu_2/\mu_1 - 1)kb. \end{aligned}$$

Substituting in Eq. (10), we obtain

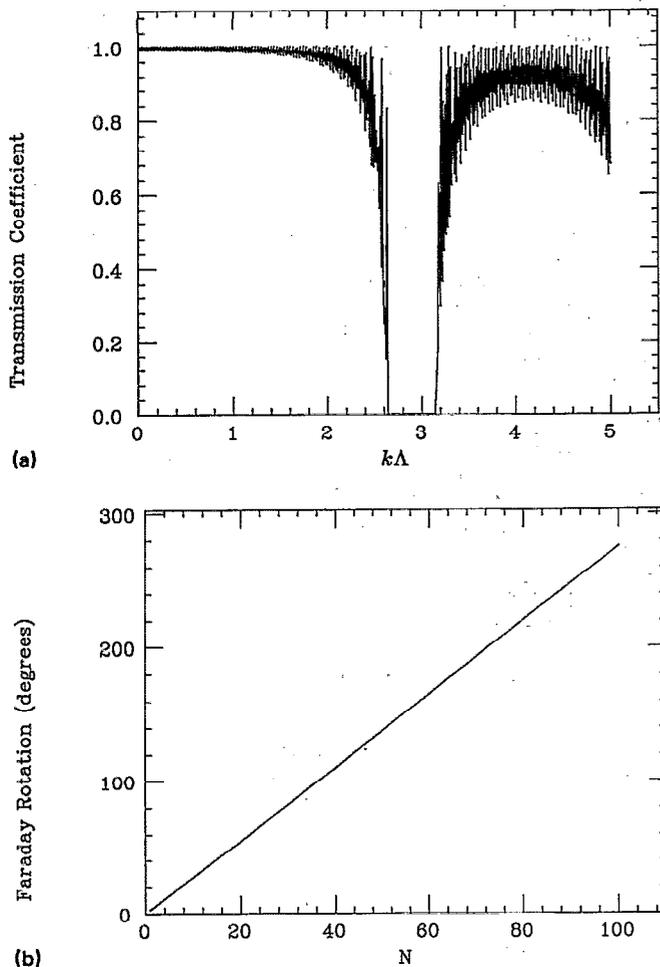


FIG. 2. (a) The transmission coefficient is shown for left circularly polarized electromagnetic waves (propagating along the direction of the applied field) incident an array of 100 barriers of permeability 100 and thickness 0.002Λ , where Λ is the spacing between the centers of the barriers, as a function of $k\Lambda$, where k is the wave vector of the incident radiation. (b) The degree of Faraday rotation is shown for circularly polarized electromagnetic waves incident an array of barriers permeability 100 and thickness 0.002Λ , where Λ is the spacing between the centers of the barriers, for $k\Lambda = 1$, where k is the wave vector of the incident radiation as a function of N , the number of barriers.

$$T \approx 1 - 0.25(\mu_2/\mu_1 - 1)^2(kb)^2 \sin^2 NQ\Lambda / \cos^2 k\Lambda, \quad (11)$$

which is valid as long as we are not too close to a value of $k\Lambda$ for which $\cos(k\Lambda)$ vanishes. For $\mu_2/\mu_1 = 100$ and $kb = 0.002$, we find that $1 - T \approx 0.04$.

The calculations presented in this article were performed for an array of infinite barriers in free space. In microwave circuits, however, the waves are generally confined to a restricted region of space, e.g., to waveguides, microstrips or slot-lines. For the case of barriers in a waveguide operating in its lowest mode, which is the easiest case to consider mathematically, if the barriers are sufficiently far apart, the waves will have the form of Eq. (2) multiplied by a function of x and y at a point midway between two barriers. If the barriers are closer together, the present methods can no longer be used because in addition to the incident, reflected and scattered waves (which are lowest

mode waves), we must include in the wave a linear combination of the higher modes in the form of evanescent modes, which decay as one moves away from the barriers.¹¹ We do not, however, expect the band and gap structure that we have obtained to be significantly modified by this. If we want to include the effect of evanescent waves in our calculations, we simply must include them in the wave function in Eq. (2). The resulting transfer matrices which relate the expansion coefficients of the wave on the two sides of the barrier will now simply be higher order matrices (i.e., 3×3 , 4×4 , etc.) because there are more coefficients to relate. We will also have a boundary condition on the two sides of the multilayer structure which requires that the coefficients of the evanescent waves which grow with increasing z will be zero on the right of the structure, and the coefficients for evanescent waves on the left which grow with decreasing z will be zero. A similar method should be applicable to microstrips and slotlines, which contain qualitatively similar higher order modes,¹² but the problem is mathematically more complicated. These applications will be considered in future work.

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APPENDIX

Although the procedure suggested here of operating just below the magnon continuum in frequency will minimize the insertion loss by eliminating the contribution of the two-magnon scattering, there will still be some damping of the waves (of the order of 100 MHz) in the multilayer structures due to the intrinsic loss mechanisms suggested in Refs. 1 and 2. The main result of this damping will be to make μ_2 and hence K in Eq. (7) complex. The main consequence of this is that even in the frequency range of the transmission bands for the multilayer structure, $|\lambda_{\pm}|$ in Eq. (4b) will no longer be equal to 1. As a consequence, from Eqs. (5) and (6) we find that there will be additional damping of the waves, even in the transmission bands. As we shall see, this damping can be minimized by our choice of the parameters k , b , and a , as well as our choice of operating frequency with respect to the FMR resonant frequency. If we let $z = 0.5T\tau R = z_1 + iz_2$, if $z_2 \ll z_1$, we find using Eq. (4b),

$$|\lambda_{\pm}| \approx 1 + 2[\pm z_1^2 + 1 - z_1^2](1 - z_1^2)^{-1/2} z_2. \quad (A1)$$

Therefore, one of the $|\lambda|$'s will be greater than 1, and hence, it follows from Eqs. (5) and (6) that we may define a decay length (i.e., an approximate number of barriers N_0 which the wave must pass through to decay to e^{-1} of its original value) in the following way:

$$e^{-N/N_0} \approx |\lambda|^{-1} = (1 + \epsilon)^{-N}, \quad (A2)$$

where ϵ is the amount that $|\lambda|$ differs from 1, which is $\ll 1$ if $z_2 \ll 1$. It follows that $N_0 \approx \epsilon^{-1}$. From Eq. (7), it is easily shown by expanding to lowest order in n_2 , where $n = \sqrt{\mu_2/\mu_1} = n_1 + in_2$, that

$$z_2 \propto n_2 K b, \quad (\text{A3})$$

where $K = kn_1$, and hence, z_2 can be made small if we make b sufficiently small, even though n_1 and n_2 can be large compared to 1 if the damping is small compared to other frequencies in the problem and we are sufficiently close to the FMR resonance because

$$\mu_2/\mu_1 = 1 + \omega_m(\omega_{\text{res}} - \omega - i\gamma)^{-1}, \quad (\text{A4})$$

where γ is the damping constant. From this it is easily shown that

$$n_1 \approx \omega_m^{1/2}(\omega_{\text{res}} - \omega)^{-1/2}, \quad (\text{A5})$$

and

$$n_2 \approx 0.5\omega_m^{1/2}\gamma(\omega_{\text{res}} - \omega)^{-3/2}, \quad (\text{A6})$$

and thus, n_2 can be made $\ll n_1$ if we work in a regime where $\gamma \ll \omega_{\text{res}} - \omega \ll \omega_m$. Since for barium ferrite the intrinsic γ is

believed to be^{1,2,5} about 100 MHz, whereas ω_m is about 13 GHz and ω_{res} is about 100 GHz, it should not be difficult to satisfy this condition for a range of frequencies.

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