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Gilbert ferromagnetic damping theory and the fluctuation-dissipation theorem

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The phenomenological Gilbert–Landau–Lifshitz theory of magnetic damping in ordered magnetic materials has long served as the central model employed in the analysis of considerable experimental magnetic resonance data. The problem undertaken here is to formulate the microscopic basis of the model in terms of usual theory of irreversible transport embodied in the fluctuation-dissipation-response theorems. As an application of the general method we establish a rigorous relationship between magnetic damping and acoustic damping implicit in magnetoelastic models. © 2010 American Institute of Physics. [doi:10.1063/1.3330646]

I. INTRODUCTION

The theory of magnetic resonance damping in ordered magnetic media has long been of both physics and engineering interest. The rate of magnetization dynamics within a ferromagnetic domain is determined by the loss of energy from the macroscopic motion of the local magnetization via the transfer of energy to microscopic thermal motion.^{1–4} These microscopic processes may involve spin waves, lattice vibrations and/or eddy currents.⁵ The calculation of the magnetic relaxation dynamics from a microscopic viewpoint involves the theory of transport processes as embodied in the fluctuation-dissipation-response theorems.⁶ Our purpose is to present this theory. As a simple application of the microscopic theory, we consider the magnetoelastic model of the interaction between lattice vibrations and spin waves.⁷ A rigorous relationship for this model is derived wherein the magnetic relaxation times are related to viscous sound wave attenuation.

In Sec. II, the thermodynamic description of magnetically ordered crystal lattices is reviewed along with the definitions of magnetic and elastic coefficients. The phenomenological Gilbert dynamical equations of motion are discussed and the expression for the heating rate per unit volume is exhibited in Sec. III. In Sec. IV, fluctuation-dissipation-response theorems are employed to obtain microscopic expressions describing how the macroscopic motion of the local magnetization transfers energy into thermal motion. The central results of the present work concern the Kubo formulas for the resulting transport coefficients. In Sec. V the standard magnetoelastic model is discussed. The process in which sound waves are damped is described in terms of the crystal viscosity tensor. In previous treatments^{8,9} by other workers, the damping of the sound waves by crystal viscosity was omitted. While previous normal mode couplings formally leading magnetic moment damping were invoked, these modes were not further damped and thus could not be compared to experimental sound wave attenuation. By com-

paring the heat dissipation per unit volume for elastic sound waves to the heat dissipation per unit volume for magnetic energy, a rigorous relationship is established between the magnetic resonance damping and sound wave absorption. The relationship persists in the model even in its nonlocal form as is proved via the Kubo formalism. In the concluding Sec. VI we summarize the advantages of the general Kubo correlation function formalism in discussing magnetic relaxation in magnetically ordered crystals.

II. THERMODYNAMICS

If u denotes the energy per unit volume of a magnetically ordered crystal, then the thermodynamic description of the crystal is based upon

$$du = Tds + \mathbf{H} \cdot d\mathbf{M} + \sigma : d\mathbf{e}, \quad (1)$$

wherein T , s , \mathbf{H} , \mathbf{M} , σ , and \mathbf{e} represent, respectively, the temperature, entropy per unit volume, magnetic intensity vector, magnetization vector, stress tensor, and strain tensor. Alternatively, one may employ the crystal enthalpy per unit volume

$$w = u - \sigma : \mathbf{e}, \quad (2)$$

which obeys

$$dw = Tds + \mathbf{H} \cdot d\mathbf{M} - \mathbf{e} : d\sigma. \quad (3)$$

Important quantities which can be computed from the thermodynamic energy and/or enthalpy per unit volume include the adiabatic magnetic susceptibility

$$\chi_{ij} = \left(\frac{\partial M_i}{\partial H_j} \right)_{s,\sigma}, \quad (4)$$

the adiabatic elastic moduli¹⁰

$$\mathcal{E}_{ijkl} = \left(\frac{\partial \sigma_{ij}}{\partial e_{kl}} \right)_{s,\mathbf{M}}, \quad (5)$$

and the adiabatic magnetoelastic coefficients¹¹

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$$2\left(\frac{\Lambda_{ijkl}N_l}{M}\right) = \left(\frac{\partial e_{ij}}{\partial M_k}\right)_{s,\sigma} = -\left(\frac{\partial H_k}{\partial \sigma_{ij}}\right)_{s,\mathbf{M}}, \quad (6)$$

wherein $\mathbf{N}=\mathbf{M}/M$ is a unit vector in the direction of the magnetization.

III. GILBERT EQUATIONS OF MOTION

The Gilbert equations of motion¹ envisage a dynamical driving magnetic intensity \mathbf{H}_d which yields a torque on the magnetic moments; i.e.,

$$\frac{\partial \mathbf{M}}{\partial t} = \gamma \mathbf{M} \times \mathbf{H}_d, \quad (7)$$

wherein γ is the gyromagnetic ratio. The driving magnetic intensity

$$\mathbf{H}_d = \mathbf{H} - \left(\frac{\partial w}{\partial \mathbf{M}}\right)_{s,\sigma} - \tau \cdot \frac{\partial \mathbf{M}}{\partial t}. \quad (8)$$

Equations (7) and (8) constitute the Gilbert equations of motion. The transport coefficient tensor τ is conventionally described in terms of a dimensionless tensor α as

$$\alpha_{ij} = \gamma M \tau_{ij}. \quad (9)$$

Note that the driving magnetic intensity \mathbf{H}_d vanishes when the equilibrium condition $\partial \mathbf{M} / \partial t = 0$ holds true. Equations (7) and (8) yield

$$\mathbf{H}_d = 0 \Rightarrow \mathbf{H} = \left(\frac{\partial w}{\partial \mathbf{M}}\right)_{s,\sigma} \quad (10)$$

in agreement with the thermal equilibrium Eq. (3). Finally,

$$\dot{q} = \frac{\partial \mathbf{M}}{\partial t} \cdot \tau \cdot \frac{\partial \mathbf{M}}{\partial t} \equiv \tau_{ij} \dot{M}_i \dot{M}_j \quad (11)$$

represents the heating rate per unit volume which occurs when the macroscopic motion of the local magnetization transfers energy into microscopic thermal motion. The heating Eq. (11) is quadratic in the rate of change in the local magnetization and thereby determines τ as a conventional tensor transport coefficient. The conventional fluctuation-dissipation-response theorems then yield the central microscopic expressions for τ .

IV. KUBO FORMALISM

We suppose that the dissipative torque exerted on the ordered magnetic moments in a crystal may be described by a random magnetic intensity $\mathbf{h}(\mathbf{r}, t)$ via the interaction Hamiltonian

$$\delta \mathcal{H}(t) = - \int \mathbf{h}(\mathbf{r}, t) \cdot \delta \mathbf{M}(\mathbf{r}, t) d^3 \mathbf{r}. \quad (12)$$

Linear response theory for the magnetic intensity may thereby be written as

$$\begin{aligned} \delta \mathbf{h}(\mathbf{r}, t) &= \int \left[\int_0^\infty \mathbf{K}(\mathbf{r}, \mathbf{r}', s) \cdot \delta \mathbf{M}(\mathbf{r}', t - s) ds \right] d^3 \mathbf{r}', \quad \mathbf{K}_{ij}(\mathbf{r}, \mathbf{r}', t) \\ &= \frac{i}{\hbar} \langle [h_i(\mathbf{r}, t), h_j(\mathbf{r}', 0)] \rangle. \end{aligned} \quad (13)$$

With $\beta = (\hbar / k_B T)$, one finds

$$\mathbf{K}_{ij}(\mathbf{r}, \mathbf{r}', t) = \frac{i}{\hbar} \langle [h_j(\mathbf{r}', -i\beta) - h_j(\mathbf{r}', 0)] h_i(\mathbf{r}, t) \rangle,$$

$$\mathbf{K}_{ij}(\mathbf{r}, \mathbf{r}', t) = \frac{i}{\hbar} \int_0^\beta \frac{\partial}{\partial \lambda} \langle h_j(\mathbf{r}', -i\lambda) h_i(\mathbf{r}, t) \rangle d\lambda,$$

$$\mathbf{K}(\mathbf{r}, \mathbf{r}', t) = - \frac{\partial}{\partial t} \mathbf{G}(\mathbf{r}, \mathbf{r}', t),$$

$$\mathbf{G}_{ij}(\mathbf{r}, \mathbf{r}', t) = \frac{1}{\hbar} \int_0^\beta \langle h_j(\mathbf{r}', -i\lambda) h_i(\mathbf{r}, t) \rangle d\lambda. \quad (14)$$

Employing Eqs. (13) and (14) together with parts integration, yields the nonlocal relation

$$\tilde{\mathbf{h}}(\mathbf{r}, t) = - \int \left[\int_0^\infty \mathbf{G}(\mathbf{r}, \mathbf{r}', s) \cdot \frac{\partial \mathbf{M}(\mathbf{r}', t-s)}{\partial t} ds \right] d^3 \mathbf{r}'. \quad (15)$$

From a nonlocal viewpoint, the relaxation of the ordered magnetic moment in the crystal obeys

$$\mathbf{H}_d = \mathbf{H} - \left(\frac{\partial w}{\partial \mathbf{M}}\right)_{s,\sigma} + \tilde{\mathbf{h}},$$

$$\frac{\partial \mathbf{M}}{\partial t} = \gamma \mathbf{M} \times \mathbf{H}_d. \quad (16)$$

If the time scale of the memory correlation function in Eq. (15) is short compared with the time scale of the magnetic moment relaxation motion, then the magnetic intensity in Eq. (15) becomes equivalent to the phenomenological Gilbert–Landau–Lifshitz equation,

$$\tilde{\mathbf{h}}(\mathbf{r}, t) \approx - \tau \cdot \frac{\partial \mathbf{M}(\mathbf{r}, t)}{\partial t}, \quad (17)$$

This is the limit of zero wave number and frequency. The phenomenological equations of motion can thereby be valid only if τ is given by the Kubo formula

$$\begin{aligned} \tilde{\mathbf{G}}(t) &= \lim_{V \rightarrow \infty} \frac{1}{V} \int_V \int_V \mathbf{G}(\mathbf{r}, \mathbf{r}', t) d^3 \mathbf{r} d^3 \mathbf{r}', \\ \tau &= \frac{\alpha}{\gamma M} = \int_0^\infty \tilde{\mathbf{G}}(t) dt, \end{aligned} \quad (18)$$

wherein V is the crystal volume. This result is similar to Eq. (41) in previous work.⁸ The Kubo Eqs. (14) and (18) are central results of this section.

V. MAGNETOELASTIC MODEL

Sound waves in crystals are absorbed converting macroscopic elastic energy into microscopic thermal motion. The heating rate per unit time is determined by a crystal viscosity tensor η via

$$\dot{q} = \eta_{lmnp} \dot{\epsilon}_{lm} \dot{\epsilon}_{np}. \quad (19)$$

From Eq. (6) it follows that

$$\dot{\epsilon}_{lm} = 2 \left(\frac{\Lambda_{lmqi} N_q \dot{M}_i}{M} \right),$$

$$\dot{\epsilon}_{np} = 2 \left(\frac{\Lambda_{nprj} N_p \dot{M}_j}{M} \right), \quad (20)$$

so that

$$\dot{q} = 4 \eta_{lmnp} \left(\frac{\Lambda_{lmqi} N_q \dot{M}_i}{M} \right) \left(\frac{\Lambda_{nprj} N_p \dot{M}_j}{M} \right). \quad (21)$$

Comparing the heating rate Eqs. (11) and (21), one rigorously relates the Gilbert damping tensor to the crystal viscosity tensor and the magnetoelastic coefficients; i.e.,

$$\alpha_{ij} = \gamma M \tau_{ij} = \left(\frac{4\gamma}{M} \right) \eta_{lmnp} \Lambda_{lmqi} N_q \Lambda_{nprj} N_p. \quad (22)$$

Thus, in a magnetoelastic model, the phenomenological Gilbert damping holds true if and only if the phenomenological viscous damping of sound waves holds true in which case Eq. (22) completely describes the Gilbert damping.

An equivalent derivation starting from a nonlocal viewpoint employs Eq. (6) according to

$$h_i(\mathbf{r}, t) = -2 \left(\frac{\Lambda_{impq} N_n}{M} \right) \sigma_{pq}(\mathbf{r}, t),$$

$$h_j(\mathbf{r}', t') = -2 \left(\frac{\Lambda_{jmrs} N_m}{M} \right) \sigma_{rs}(\mathbf{r}', t'). \quad (23)$$

The stress correlations are defined via

$$F_{rspq}(\mathbf{r}, \mathbf{r}', t) = \frac{1}{\hbar} \int_0^\beta \langle \sigma_{rs}(\mathbf{r}', -i\lambda) \sigma_{pq}(\mathbf{r}, t) \rangle d\lambda. \quad (24)$$

Equations (14), (23), and (24) imply the rigorous nonlocal correlation function relationship

$$G_{ij}(\mathbf{r}, \mathbf{r}', t) = \frac{4}{M^2} \Lambda_{impq} N_n \Lambda_{jmrs} N_m F_{rspq}(\mathbf{r}, \mathbf{r}', t). \quad (25)$$

Finally, the Kubo formula for viscosity

$$\tilde{F}_{rspq}(t) = \lim_{V \rightarrow \infty} \frac{1}{V} \int_V \int_V F_{rspq}(\mathbf{r}, \mathbf{r}', t) d^3\mathbf{r} d^3\mathbf{r}',$$

$$\eta_{rspq} = \int_0^\infty \tilde{F}_{rspq}(t) dt, \quad (26)$$

together with Eqs. (18) and (25) reaffirms the central Eq. (22) of this section. The Gilbert damping tensor α_{ij} is uniquely determined in the magnetoelastic model by the

TABLE I. Theoretical α_{th} and measured α_{exp} Gilbert damping parameters.

Materials	α_{th} ($\times 10^{-5}$)	α_{exp} ($\times 10^{-5}$)
Y ₃ Fe ₅ O ₁₂ ^a	5.56	9.0
Y ₃ Fe ₄ GaO ₁₂ ^a	51	76
Li _{0.5} Fe _{2.5} O ₄ ^b	26	50
NiFe ₂ O ₄ ^b	710	350
MgFe ₂ O ₄ ^b	120	120
MnFe ₂ O ₄ ^b	930	1040
BaFe ₁₂ O ₁₉ ^c	18	26
Ni ^d	770	2600
Fe ^d	30	220
Co ^d	530	370

^aGarnets (Ref. 12).

^bSpinel (Ref. 12).

^cHexagonal ferrite.

^dFerromagnetic materials (Refs. 13–15).

sound wave damping crystal viscosity η_{rspq} and the magnetoelastic coupling strengths Λ_{impq} .

For the mean Gilbert damping parameter,

$$\alpha = \frac{1}{3} tr \alpha = \frac{1}{3} (\alpha_{xx} + \alpha_{yy} + \alpha_{zz}), \quad (27)$$

we have analyzed a ‘‘theoretical’’ prediction α_{th} from Eqs. (26) and (27) employing experimental values for the magnetoelastic coefficients and the sound wave damping crystal viscosity. The experimental values of the Gilbert damping parameter α_{exp} obtained from direct ferromagnetic resonance data has also been analyzed. The comparison between our theoretical and experimental values may be found listed in Table I. The agreement between theory and experiment is satisfactory.

VI. CONCLUSIONS

The phenomenological Gilbert–Landau–Lifshitz theory of magnetic damping in ordered magnetic materials has been reviewed and a general microscopic expression for the Gilbert damping factor has been derived. Explicitly,

$$G_{ij}(\mathbf{r}, \mathbf{r}', t) = \frac{1}{\hbar} \int_0^\beta \langle h_j(\mathbf{r}', -i\lambda) h_i(\mathbf{r}, t) \rangle d\lambda,$$

$$\tilde{G}_{ij}(t) = \lim_{V \rightarrow \infty} \frac{1}{V} \int_V \int_V G_{ij}(\mathbf{r}, \mathbf{r}', t) d^3\mathbf{r} d^3\mathbf{r}',$$

$$\tau_{ij} = \frac{\alpha_{ij}}{\gamma M} = \int_0^\infty \tilde{G}_{ij}(t) dt. \quad (28)$$

The utility of the completely general Eq. (28) was shown by establishing within the context of the magnetoelastic model a rigorous Eq. (22) relating the Gilbert damping to sound wave damping via the crystal viscosity and the magnetoelastic coefficients.

There exists considerable work on the theory of magnetic relaxation in conducting magnetically ordered materials.^{16–18} The dominant damping mechanism is the scattering with conduction electrons. The sound wave damping

in metals is completely described via the electron viscosity by this dominant mechanism. For example, see Eq. (27) of previous¹⁹ well known sound wave attenuation work. The electron viscosity is conventionally computed by including the scattering in the collision integral of the Boltzmann transport equation for the electrons.²⁰ Since the dominant electron scattering mechanism describes the crystal viscosity in metals, it is not unreasonable to apply our model to metals as well as to insulators. If one plots the metallic FMR linewidth as a function of the square of the magnetostriction coefficients, then the linewidth scales with Λ^2 which is consistent with our model.²¹ There is one caveat to the above considerations. The problem in metals of separating intrinsic magnetic damping from Ohms law eddy current damping is not entirely trivial.

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