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ENTROPY AND AREA IN LOOP QUANTUM GRAVITY

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Black hole thermodynamics suggests that the maximum entropy that can be contained in a region of space is proportional to the area enclosing it rather than its volume. I argue that this follows naturally from loop quantum gravity and a result of Kolmogorov and Bardzin' on the the realizability of networks in three dimensions. This represents an alternative to other approaches in which some sort of correlation between field configurations helps limit the degrees of freedom within a region. It also provides an approach to thinking about black hole entropy in terms of states inside rather than on its surface. Intuitively, a spin network complicated enough to imbue a region with volume only lets that volume grow as quickly as the area bounding it.

Keywords: quantum gravity; entropy; loop quantum gravity.

1. General Appearance

In quantum field theory without gravity and without any fundamental length scale, the entropy which can be contained in a region R of volume $V(R)$ is proportional to volume times a formally infinite factor.

To see this in a simple model, let us follow Susskind [1a], and think of the region R as a lattice of spins separated by some minimum length (*i.e.* the Planck length). The number $N(V)$ of orthogonal (distinct, independent) degrees of freedom then grows as $N(V) = 2^n$ where n is the number of lattices sites in R. n is clearly proportional to the volume $V(R)$. The maximum entropy S_{max} which can be contained in R is then the logarithm of this, so we have $S_{max} = n \log(2) \propto V$. In the absence of a minimum length, $n \to \infty$ and the entropy is infinite, so gravity already helps by introducing a cutoff. However, a problem still remains.

Arguments from black hole thermodynamics suggest that the maximum entropy that can be contained in R should be proportional not to its volume $V(R)$, but rather

[a]This is also a good reference to the ideas of holography where the idea is that our 3 dimensional world is somehow, more fundamentally, 2 dimensional.

the area of its boundary $A(\partial R)$. Were this not the case, one could violate the second law of thermodynamics, and the argument is well-known: Many states contributing to the entropy described above for a lattice of spins correspond to so much energy in $V(R)$ that a black hole would form. One can pick a configuration close to being a black hole, add a little matter, and form a lower entropy black hole, in violation of the second law.

How then can one resolve this problem? Detailed calculations in string theory certainly have had some successes[2], but string theory is a very complicated structure containing much besides gravity, and I want to concentrate here on what gravity alone can do. Note that loop quantum gravity [3] also provides a means of understanding black hole entropy in terms of surface degrees of freedom but a horizon is needed.

Gravity, at least intuitively, provides a fundamental length and thus a means for regularizing infinities in energies. Indeed, this is the case for finiteness of ADM mass for a charged sphere of radius tending to zero in Einstein-Maxwell theory (as well as in the simple Newton-Coulomb analog described by Ashtekar in [4]). Could gravity also provide a way to not only *regularize an infinite entropy*, but also *change the scaling behaviour for entropy* from following volume to following area?

A very simple argument by Yurtsever[5] which comes close in spirit to what I propose, is that a maximum entropy proportional to area comes about naturally from the assumption that there is a constraint on the *total energy* of the Fock space states which are allowed to contribute. Essentially what this boils down to is that some agent, which we could take to be gravity, steps in to ensure that the energy in R cannot grow without limit and constrains not one mode or another singly, but the entire set of modes that can be allowed.

Here we start to see a hint at what a fundamental underlying mechanism might be: one cannot get the correct entropy if one thinks in terms of modes of free fields which do not interact with one another, and in some sense some of these modes must get "in the way" of each other in the sense that they cannot grow in energy without limit inside some finite volume since their maximum sum is bounded.

Here I argue that within loop quantum gravity (LQG)[3]there is also a natural regularization of entropy, and that this regularization constrains the maximum entropy of a region of space to grow no faster than its area. This is perhaps a surprising result since one might imagine that even with a discrete spacetime of the type with which this essay started (with a cutoff provided by gravity) one would still find a number of lattice points inside a macroscopic volume which grows like that volume. The argument is very generic and relies only thinking about what a spin network means in terms of geometric observables.

Recall that a spin network is a graph whose edges are labelled with $SU(2)$ representations j and whose vertices are intertwiners. For a region of space R bounded by ∂R, the area spectrum is discrete and is given by a sum of contributions proportional to $\sqrt{j(j+1)}$ from each edge with label j that punctures it. The spectrum of

the volume is discrete and given by a more complex (and still not fully understood[6]) expression with contributions from each vertex with more than 3 edges. Here the details are unimportant and all that matters is the fact that the spectrum is discrete so that $V(R)$ is bounded by some constant times the number of vertices inside – that is, the vertices basically represent little elements of volume. This is all we'll need of loop quantum gravity.

Now let us appeal to a little-known theorem[b] of Kolmogorov and Bardzin'[7]. Originally stated with electronic logic circuits in mind, the idea is simple. Consider some cubical region of space R (*i.e.* a box) and try to build a computer in it. If you try to pack a circuit into it made of elements of some finite size connected by wires of some finite thickness, with some minimum distance between the elements, then provided the circuit is complicated enough, the maximum number of elements you can put inside grows not like the volume of R but of the area that encloses it. In intuitive terms, the finite-thickness wires of a sufficiently complicated circuit get in the way so much that, as you put a large number of elements in, packing as closely as you can, you only get to put in a number of elements that grows as the area of the box's surface. Essentially you lose a dimension to the wires, or to the need for elements to "communicate with each other", or be "linked" or "connected". The term "sufficiently complicated" basically removes degenerate cases like a long circuit made of elements all connected by wires in one long line.

Now consider a spin network. To each edge you must assign a cross sectional area since any surface gets its geometrical area precisely from its being crossed by spin network edges. Similarly to each vertex you must assign a volume since any region of space gets its geometrical volume precisely from the presence of vertices. Finally, the clause about "sufficiently complicated" in the previous paragraph, is taken care of by the need to have quadrivalent vertices.

In other words, it would seem that any region of spacetime (horizon present or not) bounded by some area cannot contain a spin network whose number of nodes (chunks of volume) grows asymptotically faster than the area. If one has additional fields, one expects the same argument to hold - gravity regularizes the field theoretic divergences and also changes the nature of the support for the fields (chunks of volume) so that asymptotically, in a sense, the volume over which integrations are performed only grows like area.

A point of clarification should be made here which did not appear so explicitly in the first version of this paper. One might imagine running into trouble by arguing as follows: Consider a spin network which has given volume and area to a region of space as described above. Following Perez[8] one can now imagine choosing some internal part of the spin network and adding one quadrivalent vertex with edges connecting to other internal edges. In fact, this could even be done starting with the simplest nontrivial network: one quadrivalent vertex corresponding to volume

[b] I learned about it first from V. I. Arnold's obituary of Kolmogorov in Physics Today, October 1989 where he explains that Kolmogorov was motivated by ideas from the physiology of the brain.

with the 4 edges coming out and puncturing an enclosing surface to imbue it with area. This will increase the volume inside the region without changing its area, since the quadrivalent node will carry volume but add no new edges which carry area for the region. In other words, it might seem that a given fixed area can enclose an arbitrarily large volume!

What is being argued here is precisely that this may not be the case when one thinks about spin networks physically. In other words, instead of writing down a spin network in the usual way, thinking of the edges as 1-dimensional and the nodes as 0-dimensional, one should take the physical interpretation seriously and "thicken" the edges so that they have 2-dimensional cross-sections and the nodes so that they are small 3-dimensional balls. The actual sizes are unimportant as long as there is some minimum, which we already know from the quantization of area and volume in loop quantum gravity.

Now while one might write down a spin network in the usual way (not thinking of the edges and vertices as essentially thickened as described in the previous paragraph) and get, for some area, an arbitrarily large volume, this idea does not fit with the Kolmogorov-Bardzin' result. It suggests that there must be some additional, perhaps dynamical, feature that constrains what spin networks one would have to use to specify spacetime geometries. I argue that one can already see at least part of what that must be from the results presented here in terms of replacing a spin-network by a physically-motivated thickened graph.

While naively gravity provides a cutoff at short distances, it also "gets in the way of itself" in such a way as to reduce the effective dimensionality of spacetime so that asymptotically as one tries to include more and more degrees of freedom "volume" (as counted by nodes of a spin network inside) scales like "area"! It is intriguing to think of this as a loop quantum gravity variant of an old idea of Crane and Smolin[9] on "spacetime foam as a universal regulator" where regularization comes about due to a small decrease in the effective dimension of spacetime. Here something similar happens in that the microstructure of space is modified in LQG, but now the dimension of space is not reduced by a small amount, but all the way from 3 to 2 as distances become small!

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must be given to the possible role of dynamics in an ultimate explanation of the connections between area and volume in loop quantum gravity.

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