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Electronic Detection of Gravitational Disturbances and Collective Coulomb Interactions

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Abstract

The cross section for a gravitational wave antenna to absorb a graviton may be directly expressed in terms of the non-local viscous response function of the metallic crystal. Crystal viscosity is dominated by electronic processes which then also dominate the graviton absorption rate. To compute this rate from a microscopic Hamiltonian, one must include the full Coulomb interaction in the Maxwell electric field pressure and also allow for strongly non-adiabatic transitions in the electronic kinetic pressure. The view that the electrons and phonons constitute ideal gases with a weak electron phonon interaction is not sufficiently accurate for estimating the full strength of the electronic interaction with a gravitational wave.

1 Introduction

Resonant acoustic modes in massive metallic bars have long been used as a probe for detecting possible gravitational wave sources. The interaction of gravitational waves with such antennae was thought by many to be dominated by the heavy nuclear masses within the metal. The interaction with the lighter electron masses was considered to be negligible. This view is valid only for static Newtonian gravity, which couples to the mass density ρ . We have recently shown[1] that gravitational waves couple into the pressure tensor P, and the pressure is dominated by electronic motions. Thus, the electronic coupling to the gravitational wave cannot be ignored.

The condensed matter Hamiltonian required to understand the coupling of the electrons to the gravitational wave is the sum of kinetic energies of the electrons and the nuclei plus the Coulomb interactions between all of the charged particles. Some standard models of the solid state which employ free electron and free phonon gases together with a weak electron-phonon interaction are inadequate[2] for the description of electron-graviton interactions since terms leading to highly excited virtual electronic states have been incorrectly thrown away. For example, the electron-electron Coulomb interactions[3] are not considered in the usual electron-phonon interaction model. Model Hamiltonians are considered in Sec.2. From the Hamiltonian one can compute the pressure tensor which determines the interaction between the gravity wave and the antenna. The exact result is then the kinetic pressure plus the Maxwell field pressure. The Coulomb terms in the Maxwell stress tensor are crucially important for understanding the mutual interactions between electrons, phonons and gravitons as discussed in Sec.3. In Sec.(4) the rigorous expression for the single graviton absorption cross section is exhibited and shown to be expressed directly and rigorously in terms of the non-local viscosity of the crystal. Since the electrons dominate the viscosity, they also dominate the absorption cross section.

2 Condensed Matter

The metal bar may be described by a Hamiltonian \mathcal{H} which consists of the kinetic energy \mathcal{K}_n of the nuclei and the kinetic energy \mathcal{K}_{el} of the electrons all interacting with the Coulomb law \mathcal{U} ,

$$\mathcal{H} = \mathcal{K} + \mathcal{U},$$

$$\mathcal{K} = \mathcal{K}_{el} + \mathcal{K}_n = -\sum_j \left(\frac{\hbar^2}{2m}\right) \nabla_j^2 - \sum_a \left(\frac{\hbar^2}{2M_a}\right) \nabla_a^2,$$

$$\mathcal{U} = e^2 \left\{ \sum_{i < j} \frac{1}{r_{ij}} + \sum_{a < b} \frac{Z_a Z_b}{R_{ab}} - \sum_{i,a} \frac{Z_a}{|\mathbf{r}_i - \mathbf{R}_a|} \right\}.$$
(1)

The pressure implied by Eq.(1) is dominated by the electronic motions rather than the nuclear motions so that the electronic coupling to the gravitational wave becomes crucial for determining the detection efficiency. We have found that the efficiency induced by including the electronic coupling to gravity is considerably enhanced above the efficiency found by including only the nuclear coupling to gravity.

The considerations above have been recently criticized[2]. The microscopic Hamiltonian employed[2] for the crystal is a standard electron-phonon *approximation*[4] to our Hamiltonian Eq.(1) given (in first quantized notation) as

$$\mathcal{H}_{eff} = \mathcal{K}_{el} + \mathcal{H}_{ph} + \mathcal{H}_{el-ph},$$

$$\mathcal{K}_{el} = -\left(\frac{\hbar^2}{2m}\right) \sum_{j} \nabla_{j}^{2},$$

$$\mathcal{H}_{ph} = -\frac{\hbar^2}{2} \sum_{k} \left(\frac{\partial}{\partial Q_k}\right)^{2} + \frac{1}{2} \sum_{k} \omega_{k}^{2} Q_{k}^{2},$$

$$\mathcal{H}_{el-ph} = \sum_{j} \Phi(\mathbf{r}_{j}) \quad \text{where} \quad \Phi(\mathbf{r}) = \sum_{k} Q_{k} \phi_{k}(\mathbf{r}),$$
(2)

or in second quantized notation

$$\mathcal{H}_{eff} = \int \psi^{\dagger}(\mathbf{r}) \left\{ -\frac{\hbar^2}{2m} \nabla^2 + \Phi(\mathbf{r}) \right\} \psi(\mathbf{r}) d^3 \mathbf{r} + \sum_k \left(b_k^{\dagger} b_k + \frac{1}{2} \right) \hbar \omega_k.$$
(3)

Starting from the *exact* Coulomb Hamiltonian \mathcal{H} one may derive the low energy effective Hamiltonian \mathcal{H}_{eff} only by employing a sequence of approximations[5]: (i) The electron-electron Coulomb interactions are ignored. (ii) Phonon modes are derived in the adiabatic approximation. (iii) The non-adiabatic excitation interaction matrix Hamiltonian is replaced by a local electron deformation potential $\Phi(\mathbf{r})$. These three approximations make the effective Hamiltonian in-adequate for discussing the gravitational wave interaction. In our work[1] we avoided these three approximations by employing rigorously exact sum rules. Any other work[2] which starts from the antenna Hamiltonian \mathcal{H}_{eff} is bound to miss the electronic-gravitational enhanced efficiency because of an inadequate approximation in the Hamiltonian from which the computation begins.

3 Pressure and Gravitational Interactions

For a small gravitational wave strain $u(\mathbf{r}, t)$ described by the space-time metric

$$c^2 d\tau^2 = c^2 dt^2 - |d\mathbf{r}|^2 - 2d\mathbf{r} \cdot \mathbf{u}(\mathbf{r}, t) \cdot d\mathbf{r}, \tag{4}$$

the interaction between the gravitational wave and condensed matter is described by

$$\mathcal{H}_{int} = -\int (\mathsf{P}:\mathsf{u}) d^3\mathbf{r},\tag{5}$$

wherein P is the pressure tensor of the condensed matter. For the exact Coulomb Hamiltonian the pressure tensor is given by the sum of the kinetic pressure and the Maxwell field pressure[7]

$$\mathsf{P}(\mathbf{r}) = \mathsf{P}_{\mathcal{K}}(\mathbf{r}) + \mathsf{P}_{Maxwell}(\mathbf{r}). \tag{6}$$

The electron and nuclear momenta will be denoted, respectively by $\mathbf{p}_i = -i\hbar\nabla_i$ and $\mathbf{P}_a = -i\hbar\nabla_a$. Spatial indices will be denoted by μ and ν which may be x, y or z. The kinetic pressure tensor

$$\begin{aligned} \mathsf{P}_{\mathcal{K}}(\mathbf{r}) &= \mathsf{P}_{\mathcal{K}_{el}}(\mathbf{r}) + \mathsf{P}_{\mathcal{K}_{n}}(\mathbf{r}), \\ \mathsf{P}_{\mathcal{K}_{el}}(\mathbf{r})_{\mu\nu} &= \frac{1}{4m} \sum_{j} \left(\mathbf{p}_{j\mu} \mathbf{p}_{j\nu} \delta(\mathbf{r} - \mathbf{r}_{j}) + \mathbf{p}_{j\mu} \delta(\mathbf{r} - \mathbf{r}_{j}) \mathbf{p}_{j\nu} \right) \\ &+ \frac{1}{4m} \sum_{j} \left(\mathbf{p}_{j\nu} \delta(\mathbf{r} - \mathbf{r}_{j}) \mathbf{p}_{j\mu} + \delta(\mathbf{r} - \mathbf{r}_{j}) \mathbf{p}_{j\mu} \mathbf{p}_{j\nu} \right), \\ \mathsf{P}_{\mathcal{K}_{n}}(\mathbf{r})_{\mu\nu} &= \sum_{a} \frac{1}{4M_{a}} \left(\mathbf{P}_{a\mu} \mathbf{P}_{a\nu} \delta(\mathbf{r} - \mathbf{R}_{a}) + \mathbf{P}_{a\mu} \delta(\mathbf{r} - \mathbf{R}_{a}) \mathbf{P}_{a\nu} \right) \\ &+ \sum_{a} \frac{1}{4M_{a}} \left(\mathbf{P}_{a\nu} \delta(\mathbf{r} - \mathbf{R}_{a}) \mathbf{P}_{a\mu} + \delta(\mathbf{r} - \mathbf{R}_{a}) \mathbf{P}_{a\mu} \mathbf{P}_{a\nu} \right), \end{aligned}$$
(7)

and the Maxwell field pressure

$$P_{Maxwell}(\mathbf{r}) = \frac{1}{8\pi} \sum_{(i,j\neq)} [\mathbf{E}_i(\mathbf{r}) \cdot \mathbf{E}_j(\mathbf{r}) \mathbf{1} - 2\mathbf{E}_i(\mathbf{r})\mathbf{E}_j(\mathbf{r})] + \frac{1}{8\pi} \sum_{(a,b\neq)} [\mathbf{E}_a(\mathbf{r}) \cdot \mathbf{E}_b(\mathbf{r}) \mathbf{1} - 2\mathbf{E}_a(\mathbf{r})\mathbf{E}_b(\mathbf{r})] + \frac{1}{8\pi} \sum_{(a,i)} [\mathbf{E}_a(\mathbf{r}) \cdot \mathbf{E}_i(\mathbf{r}) \mathbf{1} - \mathbf{E}_a(\mathbf{r})\mathbf{E}_i(\mathbf{r}) - \mathbf{E}_i(\mathbf{r})\mathbf{E}_a(\mathbf{r})], \quad (8)$$

wherein the electric fields due to an electron or a nucleus are given, respectively, by

$$\mathbf{E}_{i}(\mathbf{r}) = -\frac{e(\mathbf{r} - \mathbf{r}_{i})}{|\mathbf{r} - \mathbf{r}_{i}|^{3}}, \\
\mathbf{E}_{a}(\mathbf{r}) = \frac{eZ_{a}(\mathbf{r} - \mathbf{R}_{a})}{|\mathbf{r} - \mathbf{R}_{a}|^{3}}.$$
(9)

Employing the *approximate* Hamiltonian in Eq.(2), Branchina et. al.[2] imply a pressure (via the linear Hamiltonian coupling in $h_{\mu\nu} = 2u_{\mu\nu}$) which is much simpler than the rigorous Eqs.(6), (7) and (8) of our work; i.e.

$$\mathsf{P}_{eff}(\mathbf{r}) \approx \mathsf{P}_{\mathcal{K}_{el}}(\mathbf{r}) + \mathsf{P}_{ion}(\mathbf{r}). \tag{10}$$

The simplicity of the above approximate $P_{eff}(\mathbf{r})$ compared with the exact $P(\mathbf{r})$ arises because so very many of the terms in the Maxwell pressure tensor Eq.(8) have been simply thrown away. For example, all Coulomb electric field interactions between electrons have been ignored.

To see what is involved, consider a process in which a graviton g with energy $\hbar \omega_g$ is absorbed by an antenna producing a phonon ϕ with energy ϵ_{ϕ} ; i.e.

$$g + I \to F$$
 where $\hbar \omega_g = \epsilon_\phi$ (11)

With the exact Coulomb Pressure of Eqs.(6)-(8), but *not* with the approximate pressure of Eq.(10), an electronic process can occur with an intermediate electron-hole pair state N

$$g + I \to N \to F \tag{12}$$

as shown in Fig.(1).

For example, with the help of the full Maxwell stress tensor one computes the pressure matrix elements including highly excited virtual non-adiabatic excited states,

$$\langle F | \mathsf{P} | I \rangle = \frac{1}{8\pi} \sum_{N} \left\{ \langle F | \mathbf{E} | N \rangle \cdot \langle N | \mathbf{E} | I \rangle \mathbf{1} - 2 \langle F | \mathbf{E} | N \rangle \langle N | \mathbf{E} | I \rangle \right\} + \dots, \quad (13)$$

in which the electron pressure $\mathsf{P}_{\mathcal{K}_{el}}$ included in (...) also has a similar structure for the intermediate states.



Figure 1: An incident graviton of energy $\hbar \omega_g$ excites the antenna from an initial state I to a final state F containing a phonon with energy ε_{ϕ} . The reaction requires the virtual electron-hole pair N present only if the full pressure P is used to compute the matrix element $\langle F | \mathcal{H}_{int} | g, I \rangle = -\int \langle F | \mathsf{P} | I \rangle : \mathsf{u}_g d^3 \mathbf{r}$ for the process.

4 Viscosity

In an infinite medium, the linear approximation to the Einstein field equation is

$$\left\{\frac{1}{c^2}\left(\frac{\partial}{\partial t}\right)^2 - \nabla^2\right\} \mathbf{u} = \left(\frac{8\pi G}{c^4}\right)\mathbf{p}.$$
 (14)

The transverse traceless part \boldsymbol{p} of the pressure P is related to the strain \boldsymbol{u} by the constitutive relation

$$\mathbf{p} = -2\left(\mu\mathbf{u} + \eta\frac{\partial\mathbf{u}}{\partial t}\right),\tag{15}$$

wherein μ is a Lamé elastic constant and η is the crystal viscosity. The gravitational wave then travels through the elastic media as described by the wave equation

$$\left\{\frac{1}{c^2}\left(\frac{\partial}{\partial t}\right)^2 + \frac{16\pi G\eta}{c^4}\left(\frac{\partial}{\partial t}\right) + \frac{16\pi G\mu}{c^4} - \Delta\right\} \mathbf{u} = 0.$$
(16)

The energy in the wave attenuates at rate Γ (per unit time) as determined by Eq.(16). The absorption rate Γ is completely determined by the viscosity η via

$$\Gamma = \left(\frac{16\pi G}{c^2}\right)\eta\tag{17}$$

as has been proved in previous work[6].

For a finite size medium, such as a gravitational wave antenna, the rigorously exact result for a graviton to be absorbed at temperature T is determined by the non-local viscosity. With $\beta = (\hbar/k_BT)$, the microscopic Green-Kubo formula for viscosity is

$$\eta_{ijkl}(\mathbf{r},\mathbf{r}',\zeta) = \int_0^\infty e^{i\zeta t} \left\{ \frac{1}{\hbar} \int_0^\beta \left\langle p_{kl}(\mathbf{r}',-i\lambda)p_{ij}(\mathbf{r},t) \right\rangle d\lambda \right\} dt.$$
(18)

Theorem: The (LSZ reduction) formula for the *total cross section* for a graviton with polarization **e** at frequency $\omega = c|\mathbf{k}|$ to be absorbed by an antenna of volume Ω is given by

$$\sigma(\omega) = \frac{16\pi G}{c^3} \Re e \int_{\Omega} \int_{\Omega} e^{i\mathbf{k}\cdot(\mathbf{r'}-\mathbf{r})} \{e^*_{ij}\eta_{ijkl}(\mathbf{r},\mathbf{r'},\omega+i0^+)e_{kl}\} d^3\mathbf{r} d^3\mathbf{r'}.$$
 (19)

There can be no difference of opinion as to whether the quantum pressure fluctuations producing the viscosity in Eq.(18) determines the total graviton cross section as in Eq.(19). That the *electrons dominate the viscosity* η in low temperature metals has been experimentally well established[8]. Since the cross section for the absorption of gravity waves is rigorously determined by the viscosity response function, it is then evident that electrons dominate over the nuclear motions with regard to gravity wave absorption.

5 Conclusion

The gravitational wave is absorbed by an antenna via the quantum fluctuations in the pressure, which describe the viscous response function. In a metallic antenna, the electronic motions control the pressure and thereby control the viscosity. The electronic nature of viscous electronic damping of sound has been experimentally verified. Since viscosity also damps gravitational waves, it seem natural that electronic motions should play a key role for gravitational wave detection.

To understand the importance of electronic-gravitational wave interactions from a microscopic viewpoint, one must keep the correct virial theorem pressure contributions for both the Coulomb energy and kinetic energy. These contributions have been correctly computed in previously discussed viscosity sum rules[1]. The sum rules dictate that the full inclusion of the Coulomb interactions be present in an adequate microscopic theory. An inadequate model of pressure matrix elements consists of electrons and phonons as two ideal gas components weakly interacting with one another. In such weakly coupled electron-phonon models, the electronic Coulomb interactions are ignored, removing the strongly non-adiabatic matrix elements required for proper strength of electron-hole virtual excitations.

Finally, if the Coulomb interactions are *properly* included, then the electronic enhancement of the gravitational wave detection efficiency is theoretically substantial.

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