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Electric Dipole Moments and Polarizability in the Quark-Diquark Model of the Neutron

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For a bound state internal wave function respecting parity symmetry, it can be rigorously argued that the mean electric dipole moment must be strictly zero. Thus, both the neutron, viewed as a bound state of three quarks, and the water molecule, viewed as a bound state of ten electrons two protons and an oxygen nucleus, both have zero mean electric dipole moments. Yet, the water molecule is said to have a *nonzero* dipole moment strength $d = e\Lambda$ with $\Lambda_{H_2O} \approx 0.385 \text{ \AA}$. The neutron may also be said to have an electric dipole moment strength with $\Lambda_{neutron} \approx 0.612 \text{ fm}$. The neutron analysis can be made experimentally consistent, if one employs a quark-diquark model of neutron structure.

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I. INTRODUCTION

Consider an internal ground state wave function Ψ bound by Hamiltonian \mathcal{H} which respects parity \mathcal{P} symmetry, i.e.

$$[\mathcal{H}, \mathcal{P}] = 0. \quad (1)$$

The ground state is expected to be in a parity eigenstate

$$\mathcal{P}\Psi = \pm\Psi \quad (2)$$

yielding a *null* ground state mean electric dipole moment

$$\begin{aligned} \bar{\mathbf{d}} &= (\Psi, \mathbf{d}\Psi) = (\mathcal{P}\Psi, \mathbf{d}\mathcal{P}\Psi), \\ \bar{\mathbf{d}} &= (\Psi, \mathcal{P}\mathbf{d}\mathcal{P}\Psi) = -(\Psi, \mathbf{d}\Psi), \\ \bar{\mathbf{d}} &= -\bar{\mathbf{d}} = 0. \end{aligned} \quad (3)$$

The vanishing mean electric dipole moment holds true for both a water molecule H_2O and a neutron $n = udd$. The water molecule is a bound state of charged constituents, i.e. ten electrons e^- , two protons $p^+ = \frac{1}{2}H$ and an oxygen nucleus 8_8O . The neutron too is a bound state of charged constituents, i.e. one up u and two down d quarks. For both of these bound states, the respect of parity symmetry requires a vanishing mean dipole moment $\bar{\mathbf{d}} = 0$. The search for a possible non-vanishing $\bar{\mathbf{d}}$ for the neutron has turned into a high precision experimental sport[1], yielding $|\bar{\mathbf{d}}_{neutron}/e| < 3 \times 10^{-13} \text{ fm}$. Albeit the vanishing of the mean dipole moment, it is possible to define a dipole strength d by

$$d^2 = \overline{\mathbf{d} \cdot \mathbf{d}} = e^2 \Lambda^2. \quad (4)$$

For the case of a water molecule, one attributes an electric dipole moment strength wherein[2]

$$d_{H_2O}/e = \Lambda_{H_2O} \approx 3.85 \times 10^{-9} \text{ cm}. \quad (5)$$

One can understand the notion of an electric dipole strength of a water molecule by examining a *snap shot* as shown in FIG. 1.

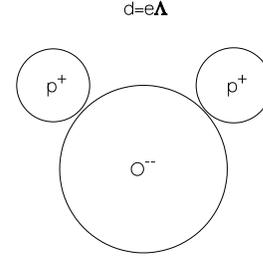


FIG. 1: Shown is a schematic picture of a water molecule with two protons each of charge $+e$ skating on the spherical electronic surface of an oxygen ion of charge $-2e$. The instantaneous dipole moment strength is $d = e\Lambda = 2e\tilde{L}\cos(\theta/2)$ wherein θ is the angle between the hydrogen bonds and \tilde{L} is the bond length. The vanishing of the mean vector electric dipole moment $\bar{\mathbf{d}} = 0$ is due to the averaged tumbling rotations of the molecule as a whole.

In Sec.II the experimental reality of FIG. 1 will be explored in terms of the rotational energy spectra of the tumbling water molecule. The energy-angular momentum levels of the water molecule form non-relativistic rotational Regge trajectories[3] from which one may deduce the molecular tensor moment of inertia I . The eigenvalues of the moment of inertia tensor are consistent with the hydrogen bonding picture as can be viewed in FIG. 1.

For the case of a neutron, we shall show in what follows that one may attribute to the neutron an electric dipole

moment strength,

$$d = e\Lambda, \quad (6)$$

wherein

$$\Lambda_{neutron} \approx 6.120 \times 10^{-14} \text{ cm}. \quad (7)$$

One can understand the notion of an electric dipole strength of a neutron by examining a *snap shot* as shown in FIG. 2.

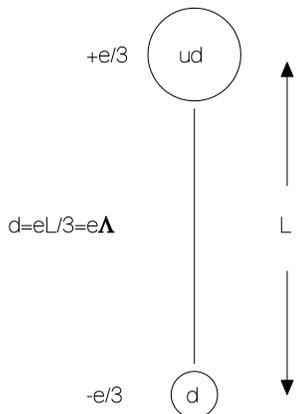


FIG. 2: Shown is a model of a neutron consisting of a bound state of three quarks $n = udd$. Two of the quarks are bound into a diquark (ud) at one end of a string which in turn is connected to the third (d) quark. The diquark has charge $e/3$ while the single quark has charge $-e/3$ yielding a dipole moment strength of $d = e\Lambda$ with $\Lambda = L/3$. The vanishing of the mean vector electric dipole moment $\bar{\mathbf{d}} = 0$ is due to the averaged tumbling rotations of the string as a whole.

The model of a neutron here of interest[4, 5] in what follows consists of a ud diquark at one end of a string of length L and tension τ and a u quark at the other end of the string. The mean electric dipole moment strength is thereby

$$d = e\Lambda_{neutron} \Rightarrow \Lambda_{neutron} = \frac{L}{3}. \quad (8)$$

The vanishing of the mean vector neutron electric dipole moment $\bar{\mathbf{d}} = 0$ is due to the quantum tumbling rotational motion of the neutron as a whole.

In Sec.III the experimental reality of FIG. 2 will be explored in terms of the rotational energy spectra of the tumbling neutron. The energy-angular momentum levels of the udd system form a relativistic rotational Regge trajectory with a linear mass squared versus angular momentum[7, 8].

For both the water molecule and the neutron, the experimental value of the electric dipole moment strength $d = e\Lambda$ may be found from measurements of the polarizability α . Experimental and theoretical values of α are discussed in the concluding Sec. IV, wherein strong evidence is presented in favor of the quark-diquark view as shown in FIG. 2. The diquark should thereby be a central notion when classifying hadrons employing quark models.

II. ROTATIONAL BANDS OF WATER

The polarizability α_T at temperature T of a classical electric dipole moment \mathbf{d} fixed in magnitude and varying in direction may be found from the classical fluctuation response theorem for Eq.(4); It is[6]

$$\alpha_T(\text{classical}) = \frac{d^2}{3k_B T}. \quad (9)$$

Eq.(9) has yielded an experimental method of defining the dipole moment strength in terms of the temperature variations of the polarizability

$$d_{H_2O}^2 = -3k_B T^2 \frac{d\alpha_T}{dT} = e^2 \Lambda_{H_2O}^2 \quad (10)$$

in a regime wherein d itself does not depend on temperature. The experimental number in Eq.(5) employed Eq.(10) in the analysis of the data. In and by itself, this analysis does not lead to the classic picture of a water molecule pictured in FIG. 1. To understand the physical picture of the molecule one must consider the rotational energy spectrum of a water molecule which amounts to a nonrelativistic Regge trajectory.

For a total angular momentum \mathbf{J} ,

$$\mathbf{J} \cdot \mathbf{J} = \hbar^2 j(j+1), \quad (11)$$

the tumbling water molecule rotational energy levels have the rigid body form

$$\mathcal{H}_{\text{rotation}} = \mathbf{J} \cdot (2\mathbf{l})^{-1} \cdot \mathbf{J} = \frac{J_1^2}{2I_1} + \frac{J_2^2}{2I_2} + \frac{J_3^2}{2I_3}, \quad (12)$$

wherein \mathbf{l} is the moment of inertia tensor with principal values $I_1 < I_2 < I_3$. The rotational energy eigenvalues

$$\mathcal{H}_{\text{rotation}} |j, n\rangle = \mathcal{E}_{jn} |j, n\rangle, \quad (13)$$

give rise to measurable molecular tumbling frequencies,

$$\omega_{jn, j'n'} = \frac{1}{\hbar} (\mathcal{E}_{jn} - \mathcal{E}_{j'n'}), \quad (14)$$

from which the principal values $I_1 < I_2 < I_3$ can be deduced. In this manner the picture in FIG. 1 can be completed with numbers for the hydrogen bonding lengths and angles consistent with the numbers for the dipole strength.

III. RELATIVISTIC REGGE TRAJECTORIES

The neutron analysis proceeds in close analogy to the water molecule case. One must consider the rotational states of the udd quark system which in the zero quark mass limit resides in the string of FIG. 2. The tumbling string frequencies obey

$$\omega = \frac{d\mathcal{E}}{dJ} = \frac{\pi c}{L}. \quad (15)$$

The string tension is the energy per unit string length,

$$\tau = \frac{\mathcal{E}}{L}, \quad (16)$$

so that

$$\frac{1}{\mathcal{E}} \left(\frac{dJ}{d\mathcal{E}} \right) = \frac{1}{\pi c \tau}. \quad (17)$$

The solution of Eq.(17)

$$J = J_0 + \frac{\mathcal{E}^2}{2\pi c \tau}. \quad (18)$$

is thereby the linear relativistic Regge trajectory for the string rotational energy levels for the udd system.

The ground rotational state of the udd system is the neutron with angular momentum ($\hbar/2$) and mass M . The first excited rotational state of the udd system is the delta with angular momentum ($3\hbar/2$) and mass M_Δ . The experimental masses of the neutron and the delta determine the string tension τ employing Eq.(17). It is

$$\frac{2\pi\hbar\tau}{c^3} = M_\Delta^2 - M^2 = \frac{1}{\alpha'}, \quad (19)$$

wherein α' is the conventional Regge slope parameter. On the other hand, the length L of the neutron string in the quark-diquark model follows from Eq.(16) to be determined by τ via Eq.(18). It is

$$L = \frac{Mc^2}{\tau} = 2\pi \left(\frac{\hbar}{Mc} \right) \left(\frac{M^2}{M_\Delta^2 - M^2} \right) = 3\Lambda. \quad (20)$$

Thus, the dipole strength of the neutron in the quark-diquark model of FIG. 2 is given by τ via Eq.(18); It is

$$\begin{aligned} \frac{d_{neutron}}{e} &= \Lambda_{neutron} \\ \Lambda_{neutron} &= \frac{2\pi}{3} \left(\frac{\hbar}{Mc} \right) \left[\frac{1}{(M_\Delta/M)^2 - 1} \right]. \end{aligned} \quad (21)$$

Employing the experimental numbers

$$\begin{aligned} \frac{\hbar}{Mc} &= 0.21009416 \text{ fm} \quad \text{and} \quad \frac{M_\Delta}{M} = 1.311, \\ \text{yielding} \quad \Lambda_{neutron} &\approx 0.6120 \text{ fm}, \end{aligned} \quad (22)$$

one finds the neutron dipole strength as in Eq.(7).

Now let us consider the neutron polarizability α . The quantum mechanical expression for α is

$$\alpha = \frac{2}{3} \sum_n \frac{|\langle n | \mathbf{d} | 0 \rangle|^2}{\mathcal{E}_n - \mathcal{E}_0}. \quad (23)$$

For the problem at hand, $|0\rangle$ is the neutron $j = 1/2$ state in one of its two spin projection states $m_j = \pm 1/2$ and $\langle n |$ is a delta $j = 3/2$ state in one of its 4 spin projection states $m_j = \pm 1/2, \pm 3/2$. Since the electric dipole moment operator \mathbf{d} is a vector, it can only connect $j = 1/2$ states to $j = 3/2$ states in yielding a dipole moment strength

$$d^2 = \langle 0 | \mathbf{d} \cdot \mathbf{d} | 0 \rangle = \sum_n |\langle n | \mathbf{d} | 0 \rangle|^2. \quad (24)$$

The neutron polarizability is thereby

$$\begin{aligned} \alpha &= \frac{2}{3c^2} \left[\frac{d^2}{M_\Delta - M} \right], \\ \alpha &= \frac{2}{3} \left(\frac{e^2}{\hbar c} \right) \left(\frac{\hbar}{Mc} \right) \left[\frac{1}{(M_\Delta/M) - 1} \right] \Lambda^2, \\ \Lambda &= \frac{2\pi}{3} \left(\frac{\hbar}{Mc} \right) \left[\frac{1}{(M_\Delta/M)^2 - 1} \right], \\ \frac{e^2}{\hbar c} &\approx 7.29735257 \times 10^{-3}. \end{aligned} \quad (25)$$

The utility of the quark-diquark string model of fermion bound states, such as the neutron, rests on whether the neutron polarizability calculated from this model agrees with experiment. The comparison is made below.

IV. CONCLUSIONS

The theoretical prediction for the neutron polarizability follows numerically from Eqs.(22) and (25),

$$\alpha(\text{theory}) \approx 12.3 \times 10^{-4} \text{ fm}^3. \quad (26)$$

The experimental value[9] for the neutron polarizability is

$$\alpha(\text{experiment}) = (12.0 \pm 1.5 \pm 2.0) \times 10^{-4} \text{ fm}^3. \quad (27)$$

We note in passing, that in a similar quark-diquark model of the proton, $p = duu$, the polarizability is expected to have a similar value, and it does[10]. In that there are no adjustable parameters in the theoretical calculation, the satisfactory agreement between theory and experiment is unlikely to be fortuitous. Thus, the quark-diquark on a string model pictured in FIG. 2, predicting the neutron electric dipole strength in Eqs.(6) and (8) has a quantitatively adequate significance.

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