April 15, 1997

Cubic intermodulation coupling in junction circulators

H. How

L. Sun

C. Vittoria
Northeastern University

Journal of Applied Physics

Cubic intermodulation coupling in junction circulators

H. How, L. Sun, and C. Vittoria

Citation: J. Appl. Phys. **81**, 5073 (1997); doi: 10.1063/1.364510
View online: http://dx.doi.org/10.1063/1.364510
View Table of Contents: http://jap.aip.org/resource/1/JAPIAU/v81/i8
Published by the American Institute of Physics.

Cubic intermodulation coupling in junction circulators

H. How
ElectroMagnetic Applications, Inc., Boston, Massachusetts 02109

L. Sun and C. Vittoria
Northeastern University, Boston, Massachusetts 02115

Nonlinear intermodulation coupling in ferrite junctions have been solved analytically for the first time. Primary microwave excitations of the ferrite junction at frequencies f_1 and f_2 will induce effective rf current at frequency $2f_1 - f_2$ which generate intermodulation field within the ferrite satisfying the boundary conditions at the junction boundary. The static demagnetizing field has been shown to have an adverse effect in increasing intermodulation coupling. Our calculations compare reasonably well with experiments.
[S0021-8979(97)60708-2]

INTRODUCTION

Under high power excitations, the motion of the magnetization vector can no longer be described by a linearized set of equations. By considering next-order (cubic) terms in the equation of motion echo signals in the time domain[1,2] and intermodulation coupling in the frequency domain[3] must be included in a nonlinear theory. In the past How *et al.*[1,2] have formulated the mechanism for echoing in a ferrite junction. Prediction from our mechanism was in full agreement with experimental data.[1,2] However, the intermodulation coupling is less understood, although an initial treatment can be found in Ref. 3.

Intermodulation signals grow rapidly with power. For input power below 1 W, the level of the induced intermodulation is in general -100 dB below the input signals.[3] However, when signal levels are increased to 30 W, the intermodulation noises acquire a power of -70 dB below the input signals. They are identified as clicking noises in a telephone line. Reduction of nonlinear intermodulation noises in ferrite junctions is therefore of crucial importance for cellular phone communications, not only to improve the quality of the telephone signals, but also to allow fewer radio relay stations to be distributed in the transmission network.

Previous theoretical formulation[3] on intermodulation was derived *ad hoc* from the equation of motion of the magnetization vector which gives rise to at best an order of estimate about this coupling effect. In Ref. 3 the equation of motion was iterated to high orders without restricting the magnetization vector to be of constant magnitude. The resultant nonlinear magnetic equation was then solved by assuming the induced intermodulation magnetic field to be identically zero. This assumption negates the coupling between the magnetization vector and other rf electromagnetic fields via Maxwell equations. In fact, by setting the intermodulation magnetic field to zero, it is not feasible to generate intermodulation power. Also, the previous theory[3] excluded a treatment on the demagnetizing field of the ferrite junction. It was pointed out by Suhl[4] that the static demagnetization is closely related to the microwave instability occurring at the subsidiary absorption for ferrimagnetic resonance measurements. Also, How *et al.*[1,2] have shown that the static demagnetization can substantially reduce the measured ferrimagnetic echo gain to zero. In this article we will show that the demagnetizing field of the ferrite junction can significantly enhance the intermodulation coupling in a ferrite circulator junction, and hence it shall be minimized in a power circulator design. Detailed discussions on intermodulation calculation will be published elsewhere.[5]

FORMULATION

Within a ferrimagnetic substance the tangential component of the magnetization vector, $\mathbf{m}$, satisfies the following equation of motion up to the third order in rf excitation[5]

$$\frac{-1}{\gamma}\frac{d\mathbf{m}}{dt} = \mathbf{e}_z \times \left(H_{\mathrm{in}}\mathbf{m} - M_s\mathbf{h} + (4\pi N_z\mathbf{m} + \mathbf{h})\,\frac{\mathbf{m}\cdot\mathbf{m}}{2M_s} \right), \tag{1}$$

where $\mathbf{h}$ denotes the transverse component of the rf magnetic field, M_s the saturation magnetization, N_z the (axial) demagnetizing factor, γ the gyromagnetic ratio, H_{in} the internal field given as $H_{\mathrm{in}} = H_0 - 4\pi N_z M_s$, and H_0 the externally applied dc field, which may be expressed as $H_0 - (i\Delta H/2)\, f/f_r$ if magnetic damping effect is to be included in the formulation.[6] Here f is the frequency, and ΔH is the ferromagnetic resonance (FMR) linewidth measured at f_r. We must emphasize here that all of the above derivations are valid only for real (physical) quantities. The commonly used phasor representation of complex fields is meaningless under nonlinear consideration. Nevertheless, Eq. (1) can be converted into an ''effective'' complex from at the single intermodulation frequency.[5] That is, let the ferrite junction be excited by two monochromatic signals of angular frequency ω_1 and ω_2. As long as we are considering terms relating to intermodulation excitation at angular frequency $\omega_3 (= 2\omega_1 - \omega_2)$ in Eq. (1), complex phasor notations can be resumed in Eq. (1) provided that the inner product $\mathbf{m}\cdot\mathbf{m}$ is replaced by $\mathbf{m}_1 \cdot \mathbf{m}_2^*/8$. As such, the equation of motion, Eq. (1), becomes

$$\frac{-1}{\gamma}\frac{d\mathbf{m}_3}{dt} = \mathbf{e}_z \times [H_{\mathrm{in}}\mathbf{m}_3 - M_s(\mathbf{h}_3 \pm \boldsymbol{\alpha})], \tag{2}$$

where the inhomogeneous term $\boldsymbol{\alpha}$ is given as

$$\boldsymbol{\alpha} = \frac{\mathbf{m}\cdot\mathbf{m}_2^*}{16M_s^2}\,(4\pi N_z\mathbf{m}_1 + \mathbf{h}_1). \tag{3}$$

Here subscripts 1, 2, and 3 refer to quantities at angular frequencies ω_1, ω_2, and ω_3, respectively. The use of complex fields in Eq. (2) can largely simplify the following derivations.

For the $\exp(-i\omega_3 t)$ time dependence Eq. (2) gives the following magnetic constituent equation

$$\mathbf{h}_3 = \mathbf{h}_3 + \mathbf{m}_3 = \boldsymbol{\mu}_3(\mathbf{h}_3 - \boldsymbol{\beta}), \tag{4}$$

where

$$\boldsymbol{\beta} = (\mathbf{I} - \boldsymbol{\mu}_3^{-1})\boldsymbol{\alpha}, \tag{5}$$

and $\boldsymbol{\mu}_3$ is the regular Polder permeability tensor given by[7]

$$\boldsymbol{\mu}_3 = \begin{pmatrix} \mu_3 & i\kappa_3 & 0 \\ -i\kappa_3 & \mu_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{6}$$

with Polder elements $\mu_3 = 1 + \omega_0\omega_m/(\omega_0^2 - \omega_3^2)$, $\kappa_3 = \omega_3\omega_m/(\omega_0^2 - \omega_3^2)$, and ω_m and ω_0 are defined as $\omega_m = 4\pi\gamma M_s$, and $\omega_0 = \gamma H_{in}$.

By assuming $h_{3z} = 0 = e_{3p} = e_{3\phi}$, from Maxwell equations one derives[5]

$$(\nabla^2 + k_{3e}^2)e_{3z} = \frac{-i\omega_3\mu_{3e}}{c}(\nabla \times \boldsymbol{\beta})_z, \tag{7}$$

$$h_{3\rho} = \frac{-ic}{\omega_3\mu_{3e}}\left[\frac{1}{\rho}\frac{\partial e_{3z}}{\partial\phi} + \frac{i\kappa_3}{\mu_3}\frac{\partial e_{3z}}{\partial\rho}\right] + \beta_\rho, \tag{8}$$

$$h_{3\phi} = \frac{ic}{\omega_3\mu_{3e}}\left[\frac{-i\kappa_3}{\mu_3}\frac{1}{\rho}\frac{\partial e_{3z}}{\partial\phi} + \frac{\partial e_{3z}}{\partial\rho}\right] + \beta_\phi, \tag{9}$$

where $\mu_{3e} = (\mu_3^2 - \kappa_3^2)/\mu_3$ is the Voigt permeability, $k_{3e} = \omega_3(\mu_{3e}\epsilon)^{1/2}/c$ is the effective propagation constant, c denotes the speed of light in vacuum, and the complex dielectric constant $\epsilon = \epsilon_f(1 + i\tan\delta)$. Here ϵ_f and $\tan\delta$ are the dielectric constant and dielectric loss tangent of the ferrite material, respectively.

We note that all of the above formulas derived for the induced cubic intermodulation fields at angular frequency ω_3 can be equally applied to the primary excitation fields at ω_1 and ω_2 provided that the driving terms described by $\boldsymbol{\alpha}$ or $\boldsymbol{\beta}$ are set to zero. From Eq. (7) the forced oscillation for cubic intermodulation results in an effective current distribution within the ferrite junction,

$$\mathbf{j}_3 = \frac{c}{4\pi}(\nabla \times \boldsymbol{\beta})_z, \tag{10}$$

which generates outgoing electromagnetic waves propagating toward the boundary of the ferrite disk. Meanwhile, the imposed magnetic-wall boundary conditions on the disk boundary not adjacent to the circulator ports are effective to reflect the incident electromagnetic waves such as to cancel the tangential components of the rf magnetic fields there; only at the circulator ports the magnetic-wall boundary conditions are relaxed:

$$\begin{aligned} h_{3\phi}(R,\phi) &\neq 0, \quad \text{if } \phi_i - \theta_i \leq \phi \leq \phi_i + \theta_i, \\ &\qquad \text{for } 1 \leq i \leq 3N, \\ &= 0, \quad \text{otherwise.} \end{aligned} \tag{11}$$

Here we have assumed the circulator to possess $3N$ ports located at $\phi = \phi_i$ with half port-suspension angle θ_i. The ferrite disk is of radius R. The through-ports are 1, $1+N$, and $1+2N$, which are connected with matched loads, and the other ports are connected with open-circuited stubs of finite length. For circulation action to occur we must assume threefold symmetry on the circulator ports.[8]

Solutions to the intermodulation fields are therefore composed of two parts. The first part solutions satisfy homogeneous Helmholtz equation giving rise to the boundary conditions of Eq. (11), and the second part solutions describe outgoing electromagnetic fields generated from the current distribution of Eq. (10). We refer to the first part solutions by superscript (0) and the second part solutions by superscript (1); that is

$$h_{3\phi}(\boldsymbol{\rho}) = h_{3\phi}^{(0)}(\boldsymbol{\rho}) + h_{3\phi}^{(1)}(\boldsymbol{\rho}), \quad \text{etc.}, \tag{12}$$

which are obtainable using appropriate Green's functions:[5]

$$\begin{aligned} h_{3\phi}^{(1)}(\rho,\phi) = \Bigg[\frac{-i}{4}\int_0^R \rho' d\rho' \int_0^{2\pi} d\phi' \left(\frac{\partial\beta_\phi(\boldsymbol{\rho}')}{\partial\rho'} \right. \\ \left. - \frac{1}{\rho'}\frac{\partial\beta_\rho(\boldsymbol{\rho}')}{\partial\phi'}\right) \cdot \left(\frac{-i\kappa_3}{\mu_3}\frac{1}{\rho}\frac{\partial}{\partial\phi} + \frac{\partial}{\partial\rho}\right) H_0^{(1)} \\ \times [k_{3e}\sqrt{\rho^2 + \rho'^2 - 2\rho\rho' \cos(\phi - \phi')}] \Bigg] \\ + \beta_\phi(\boldsymbol{\rho}), \end{aligned} \tag{13}$$

$$\begin{aligned} h_{3\phi}^{(0)}(\rho,\phi) = \frac{1}{2\pi}\sum_{n=-\infty}^{\infty} \left(\int_0^{2\pi} d\phi' e^{-in\phi'} S_3(\phi')\right) \\ \times \left(\frac{J_n'(k_{3e}\rho) + \dfrac{\kappa_3}{\mu_3}\dfrac{nJ_n(k_{3e}\rho)}{k_{3e}\rho}}{J_n'(k_{3e}R) + \dfrac{\kappa_3}{\mu_3}\dfrac{nJ_n(k_{3e}R)}{k_{3e}R}} \right) e^{in\phi}, \end{aligned} \tag{14}$$

and $S_3(\phi)$ specifies the boundary conditions

$$\begin{aligned} S_3(\phi) &= 0, \quad \text{if } \phi_i - \theta_i \leq \phi \leq \phi_i + \theta_i, \quad \text{for } 1 \leq i \leq 3N, \\ &= -h_{3\phi}^{(1)}(\phi), \quad \text{otherwise.} \end{aligned} \tag{15}$$

In Eqs. (13) and (14) $J_n(x)$ denotes the Bessel function of order n and $H_0(x)$ is the Hankel function of the first kind of order 0. Therefore, provided that the first-order excitation fields $\mathbf{h}_1$ and $\mathbf{h}_2$ are known, the ϕ component of the third-order coupled intermodulation field $\mathbf{h}_3$ can now be completely calculated, Eqs. (13)–(15).

The output intermodulation power at output port j, $j = 1, 1+N, 1+2N$, is derived as[5]

$$P_{3j} = 2w^2|h_{3\phi}(R,\phi_i)|^2 Z_d, \tag{16}$$

where w denotes the width of the port, and Z_d is the wave impedance of the port, $Z_d = \epsilon_d^{-1/2}$. Here ϵ_d is the dielectric constant of the dielectric sleeve material surrounding the ferrite junction to match the circulator performance at circulation conditions.[7,8] The input signal power at ω_1 and ω_2 can

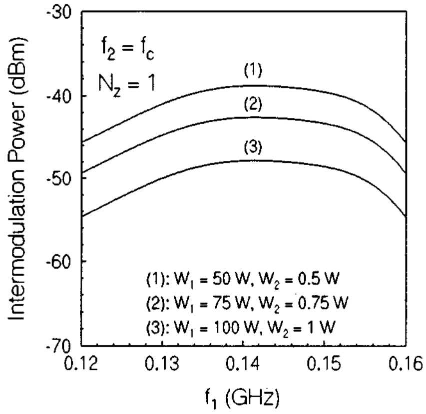

FIG. 1. Calculated intermodulation power output for $f_2 = 2f_c - f_1$ and $N_z = 1$.

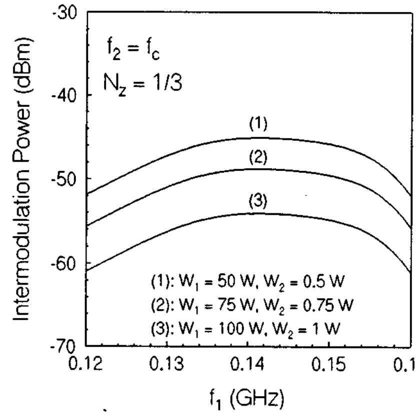

FIG. 2. Calculated intermodulation power output for $f_2 = 2f_c - f_1$ and $N_z = 1/3$.

be calculated from Eq. (19) with $h_{3\phi}(R,\phi_i)$ being replaced by the ϕ component of the incident magnetic field at port 1, $(h_{1\phi})_{\text{in}}$ or $(h_{2\phi})_{\text{in}}$, respectively.

RESULTS

We consider the three-port very high frequency (VHF) circulator design reported in Ref. 3. In Ref. 3 the circulator was given the following parameters: $4\pi M_s = 600$ G, $H_0 = 800$ Oe, $\epsilon_f = 16$. We assume the same ferrite was used as the matching sleeve material such that $\epsilon_d = \epsilon_f = 16$. In order to approximately give the center transmission frequency around 140 MHz, we further assume $R = 8$ cm. Also, we have assumed $\Delta H = 60$ Oe at $f_r = 10$ GHz, and $\tan\delta = 0.0001$. Circulation condition[8] can then be solved which requires a half port-suspension angle $\theta = 0.112$ rad, and the center transmission frequency f_c occurs at 0.1414 GHz. The calculated scattering parameters of the circulator together with a discussion on its performance can be found elsewhere.[5]

In the following calculations we consider the two primary signals to be applied at port 1 and the generated cubic intermodulation signals are measured at the transmission port, port 2. Port 3 is the isolation port, which is terminated by a matched load. Figure 1 shows the calculated intermodulation output power as a function of f_1 using $f_2 = 2f_c - f_1$. The axial demagnetizing factor considered is $N_z = 1$. Three power levels have been considered, $W_1 = 50$ W and $W_2 = 0.5$ W, $W_1 = 75$ W and $W_2 = 0.75$ W, and $W_1 = 100$ W and $W_2 = 1$ W. From Fig. 1 it is seen that maximum intermodulation occurs when f_1 and f_2 are close to the center frequency f_c, which exhibits the following values: -47.8 dB m for $W_1 = 50$ W and $W_2 = 0.5$ W, -42.6 dB m for $W_1 = 75$ W and $W_2 = 0.75$ W, and -38.9 dB m for $W_1 = 100$ W and $W_2 = 1$ W. These numbers compare very well with measurements:[3] -44 dB m when $W_1 = 50$ W and $W_2 = 0.5$ W, and -42 dB m when $W_1 = 75$ W and $W_2 = 0.75$ W. However, exact comparison between theory and experiments is not possible, since the exact parameters of the circulator are not known (the center transmission frequency used in this calculational model is 144.4 MHz, whereas the center transmission frequency measured in Ref. 3 was 124 MHz). We note that in Fig. 1 intermodulation levels drop off when f_1, and hence f_2, deviate from f_c. This is due to the fact that the amplitudes of the primary fields, $\mathbf{h}_1$, $\mathbf{m}_1$, $\mathbf{h}_2$, and $\mathbf{m}_2$, decrease as a consequence of the frequency shift away from circulation conditions.

Figure 2 shows the same results as Fig. 1 except that the axial demagnetizing factor N_z has been lowered to a smaller value: $N_z = 1/3$. For this N_z value maximum intermodulation still occurs at the center transmission frequency which exhibits the following values: -54.2 dB m for $W_1 = 50$ W and $W_2 = 0.5$ W, -49.8 dB m for $W_1 = 75$ W and $W_2 = 0.75$ W, and -45.2 dB m for $W_1 = 100$ W and $W_2 = 1$ W. These values are about 5–6 dB smaller than those previous values calculated for $N_z = 1$. Therefore, we conclude that the static demagnetization of the ferrite disk has an adverse effect in generating cubic intermodulation for channel coupling. A quiet circulator can be obtained if the demagnetizing field is minimized. In the past Schloemann and Blight have applied two polycrystalline YIG domes in the shape of semispheres covering the ferrite circuit from above and below to produce uniform internal bias field within the ferrite disk.[9] As such, $N_z = 1/3$, and ultrawide bandwidth was obtained for circulator transmission.[1] Intermodulation calculations on other excitation frequencies and interference of the input signals on different phases will be reported elsewhere.[5]

[1] H. How and C. Vittoria, Phys. Rev. Lett. **66**, 1626 (1991).
[2] H. How and C. Vittoria, IEEE Trans. Microwave Theory Tech. **MTT-39**, 1828 (1991).
[3] Y.-S. Wu, W. H. Ku, and J. E. Erickson, IEEE Trans. Microwave Theory Tech. **MTT-24**, 69 (1976).
[4] H. Suhl, J. Phys. Chem. Solids **1**, 209 (1957).
[5] H. How, T.-M. Fang, C. Vittoria, and R. Schmidt, ''Nonlinear Intermodulation Coupling in Ferrite Circulator Junctions,'' (to be published).
[6] H. How, R. C. O'Handley, and F. R. Morgenthaler, Phys. Rev. B **40**, 4809 (1989).
[7] H. How, C. Vittoria, and C. Carosella, IEEE Trans. Microwave Theory Tech. **MTT-39**, 40 (1991).
[8] H. How, R. Schmidt, and C. Vittoria, IEEE Trans. Microwave Theory Tech. **MTT-42**, 1272 (1994).
[9] E. Schloemann and R. E. Blight, IEEE Trans. Microwave Theory Tech. **MTT-34**, 1394 (1986).