

January 01, 2004

Effective action and soft supersymmetry breaking for intersecting D-brane models

Boris Körs

Massachusetts Institute of Technology

Pran Nath

Northeastern University

Recommended Citation

Körs, Boris and Nath, Pran, "Effective action and soft supersymmetry breaking for intersecting D-brane models" (2004). *Physics Faculty Publications*. Paper 150. <http://hdl.handle.net/2047/d20000761>

This work is available open access, hosted by Northeastern University.

Effective Action and Soft Supersymmetry Breaking for Intersecting D-brane Models

Boris Körs^{1,*} and Pran Nath^{2,†}

**Center for Theoretical Physics
Laboratory for Nuclear Science
and Department of Physics
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139, USA*

*†Department of Physics
Northeastern University
Boston, Massachusetts 02115,
USA*

Abstract

We consider a generic scenario of spontaneous breaking of supersymmetry in the hidden sector within $\mathcal{N} = 1$ supersymmetric orientifold compactifications of type II string theories with D-branes that support semi-realistic chiral gauge theories. The soft breaking terms in the visible sector of the models are computed in a standard way without specifying the breaking mechanism, which leads to expressions that generalize those formerly known for heterotic or type I string models. The elements of the effective tree level supergravity action relevant for this, such as the Kähler metric for the matter fields, the superpotential of the visible sector and the gauge kinetic functions, are specified by dimensional reduction and duality arguments. As phenomenological applications we argue that gauge coupling unification can only occur in special regions of the moduli space; we show that flavor changing neutral currents can be suppressed sufficiently for a wide range of parameters, and we briefly address the issues of CP violation, electric dipole moments and dark matter, as well.

¹e-mail: kors@lns.mit.edu

²e-mail: nath@neu.edu

1 Introduction

String theory has produced a variety of approaches to construct models that come rather close to the qualitative features of low energy particle physics. While the first such models were based on the heterotic string [1], the advent of D-branes has allowed for a broader perspective and more direct bottom-up attempts to meet the desired gauge group, chiral matter spectrum etc. An approach that is maybe distinguished by its computability and simplicity together with very appealing phenomenological possibilities is that of type IIA orientifold compactifications (see [2] for a general review) on Calabi-Yau (CY) manifolds with intersecting D6-branes. It can produce a plethora of models with finite effective four-dimensional Planck-scale, with $\mathcal{N} = 1$ supersymmetry in four dimensions, gauge groups that include or consist of products of unitary groups, and chiral fermionic matter in the form of bifundamental representations - all the ingredients the supersymmetric extensions of the Standard Model (we shall often just refer to the MSSM) need [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. The first semi-realistic supersymmetric models that come close to the MSSM spectrum and gauge group were presented in [12, 13], which we will frequently refer to as prominent examples.¹ A point of contact between string models and testable physics comes from the soft breaking terms and we will follow here the hidden sector scenarios of SUGRA models in deducing these [20, 21, 22, 23].

In a first approach the attention in intersecting brane models has been concentrated on models which break supersymmetry already at the string scale [24, 25, 26, 27, 28, 29, 30, 31, 32], and may thus be considered phenomenologically relevant only in the framework of low string scale or large extra dimension scenarios [33, 34]. This is mainly due to the fact that supersymmetry imposes extra conditions, as it turns out conditions that correspond to the vanishing of Fayet-Iliopolous-terms in the effective theory [35, 36, 14]. These make it much harder to construct interesting models in an economical fashion, but some progress has been made, see e.g. [37, 38, 39, 40, 41, 42]. Of course, this also leaves the question open, how supersymmetry gets finally broken in these models. Circumventing this question for the moment, we examine the consequences of a rather generic scenario where a hidden sector breaks supersymmetry in some unspecified fashion. To do so, we shall have to determine various elements of the effective four-dimensional low energy Lagrangian, such as parts of the Kähler potential, the superpotential, gauge kinetic functions etc. We do this partly by direct dimensional reduction and partly by invoking the perturbative duality between the type I and heterotic string in four dimensions. Ultimately, the goal of this program is to put the phenomenological models based on type I strings and D-branes on the same footing with heterotic

¹An extensive review of the CY-orbifold models with intersecting D-branes, that contains a more complete introduction and nice illustrations has recently appeared in [19].

models studied in the past.

Concretely, we assume the model to contain two sectors, a visible sector that contains at least the MSSM matter fields, and one or more hidden sectors left unspecified. Supersymmetry is then assumed to be broken in a fashion that is standard in supergravity models (see [43, 44, 45] and references therein): through some unknown, possibly non-perturbative, mechanism in the hidden sector a potential is generated at some intermediate scale, usually taken to be around $M_{\text{sb}} \sim 10^{13}\text{GeV}$, and supersymmetry breaks down spontaneously. This induces explicit but soft breaking in the visible sector. As an example, in [46] the possibility of gaugino condensation in the hidden sector was considered,² but in principle one may also want to allow mechanisms that are not based on the dynamics of gauge fields and thus do not involve further hidden D-branes. An alternative example could be background fluxes that generate a perturbative scalar potential. We shall in fact leave the concrete breaking mechanism open, much in the spirit of [45], and only parametrize the breaking. We then compute the soft breaking parameters and discuss various issues that arise. There are many phenomenological restrictions that apply to the soft breaking terms in the effective action, and just to show this in some examples, we discuss the issues of flavor changing neutral currents (FCNC), CP violation, electric dipole moments (EDM), and dark matter. We also look at the perspectives to achieve a unification of gauge couplings in these models (see e.g. [42, 47, 48]), in order to derive constraints on the moduli space for the gauge coupling unification to occur. We find that it requires rather special conditions on the moduli to be fulfilled and is not generic.

We begin with a technical remark on the practical formulation of the models: The more intuitive type IIA version of the models describes them as orientifolds of type IIA with D6-branes and orientifold 6-planes (O6-planes), filling out 3+1 dimensional Minkowski space-time and wrapping internal 3-manifolds. The intersection of these branes contain chiral fermions in bifundamental representations allowing one to attempt Standard Model like constructions. These models are thus appropriately called the intersecting brane models. However, for our purposes it will be very helpful to rephrase the construction in terms of a type IIB orientifold with magnetic background fields on D9-branes (as in most of the original early works as [49, 7, 28]). The physics of the two pictures is identical, since both are related by a simple T-duality along three directions of the internal space, actually mirror symmetry. The advantage of the latter formulation lies in the fact that it can directly be understood as a compactification of type I string theory only including background fluxes for the Yang-Mills field strengths on the D9-brane world volumes.

²Actually, [46] relied on slightly different assumptions about the breaking patterns and the moduli sector, which e.g. lead to a large negative cosmological constant. This does not necessarily appear in our more complete treatment.

This allows to determine various quantities either by dimensional reduction from known ten-dimensional type I expressions or from heterotic-type I duality. In the main text we shall refer to the two pictures as type IIA and type IIB versions and frequently employ both points of view interchangeably, since they are ultimately equivalent and hope that this will not confuse the reader.

The complete picture that we put forward consists of the following elements: We compactify type I strings on a CY-orbifold space (type IIB version), add magnetic world volume fluxes, which introduces a new mass scale, and finally break supersymmetry spontaneously at the breaking scale M_{sb} . The effective action which is obtained thereafter in principle implies that we have integrated out the massive string excitations, the massive Kaluza-Klein (KK) modes, those fields that get massive upon introducing the magnetic flux, and finally the hidden sector fields that decouple when supersymmetry is broken. The four relevant mass scales are given by the string scale M_s , the KK scale $1/R$, with R an average radius, the magnetic flux scale $1/(M_s R^2)$ [50], and the breaking scale M_{sb} . For a space that is not “isotropic” it could also become necessary to include various different KK scales, such as for large transverse volume models. Since we are working in the supergravity approximation of large R , the splitting

$$M_s \gg \frac{1}{R} \gg \frac{1}{M_s R^2} \gg 1\text{TeV} \quad (1)$$

is automatically implied for the self-consistency of the expansion of the effective action in derivatives. On the other hand, supersymmetry could be broken either below or above $1/(M_s R^2)$. The relevance of Eq.(1) for writing an effective action lies in the fact that it allows to treat the effects of the world volume fluxes as rather small perturbations of the background geometry, the solution to the vacuum equations of motion without fluxes. It would of course be very interesting, if one could go beyond this approximation in some controllable manner, but we shall stick to the “probe limit” in the following, neglecting any backreaction on the geometry.

Before going into a detailed discussion of the analysis, we describe below the main results of this paper. In previous works on intersecting brane models the Kahler potential of models has not been fully specified. In this paper this potential is obtained and Eqs.(47) and (57) constitute two of the important results of this paper. This potential is then utilized to compute soft breaking for a generic class of D brane models. The soft breaking formula obtained in the paper are general and encompass a large class of D brane models and thus have a wide range of applicability. Remarkably the full information on soft breaking in these formulae is encoded in only a few indices which are determined in terms of the wrapping numbers. As a check on our results it is shown that the formula obtained reproduce the previously known results for soft breaking in the parallel brane case under specific limiting conditions. Further, an application of the general formulae for a

specific intersecting D brane model involving D9 branes and D5 brane which contains perturbative, nonperturbative and interpolating sectors is also made. Thus the soft breaking formulae given in Sec.4 encode information on soft breaking on a variety of D brane models. Further, a variety of other phenomena associated with intersecting D brane models are also discussed. Thus a significant question in D brane models centers around unification of gauge coupling constants. This issue has been addressed in the context of specific models in several recent papers. In this paper we show that the unification of gauge couplings is intimately tied to the constraints on the moduli needed to achieve $\mathcal{N} = 1$ supersymmetry in intersecting D brane models. The new results of the paper are contained in Secs. 3, 4 and 5 and in Appendix A.

Next we compare and contrast the main results achieved in this paper with those of the heterotic strings. Concretely, we will derive formulas for the Kähler metric of the open string fields with ends on the various stacks of D-branes, labelled by letters a, b , which turn out as

$$\begin{aligned}\tilde{\mathcal{K}}_{m\bar{m}}^{[aa]} &= \frac{1}{(s + \bar{s})(t_m + \bar{t}_{\bar{m}})(u_m + \bar{u}_{\bar{m}})} \frac{4\Re(f_a)}{1 + \Delta_a^{(m)}} , \\ \tilde{\mathcal{K}}_{\alpha\bar{\beta}}^{[ab]} &= \delta_{\alpha\bar{\beta}}(s + \bar{s})^{\nu_{ab}/2-1} \prod_{m=1}^3 (t_m + \bar{t}_{\bar{m}})^{\nu_{ab}^{(m)} - \nu_{ab}/2} \prod_{m=1}^3 (u_m + \bar{u}_{\bar{m}})^{\nu_{ab}^{(m)} - \nu_{ab}/2} .\end{aligned}$$

Here $\{s, t_m, u_m\}$ are the moduli fields, f_a gauge kinetic functions, $\Delta_a^{(m)}$ some function of the moduli, and ν_{ab} numerical parameters. These two expressions are clearly distinguished from any Kähler metric known formerly for heterotic or type I compactifications. The first line refers to the m -component of open strings with both ends on the brane stack a , and it generalizes the known metric for untwisted heterotic fields, which reads

$$\tilde{\mathcal{K}}_{m\bar{m}}^{\text{het}} = \frac{1}{(t_m + \bar{t}_{\bar{m}})(u_m + \bar{u}_{\bar{m}})} ,$$

while the limit $4\Re(f_a)/(1 + \Delta_a^{(m)}) = (s + \bar{s})$ in which it reduces to this expression corresponds to a non-chiral limit of the D-brane models. The second line stands for open strings that connect branes a and b . It has strong resemblance to twisted heterotic fields, but is as well clearly identified by the appearance of the dilaton $s + \bar{s}$ in the Kähler metric, which is impossible for the heterotic string. So for instance, a scenario of total dilaton dominance in the soft breaking, which leads to great simplification for heterotic models, is by far not as simple in the present class of D-brane compactifications. To make the above expressions practically useful, we further compute the soft breaking terms, where the novel Kähler metric leads to some new effects, such as for example “interference effects” of various moduli fields in the squark masses. In any case, our formulas allow a straightforward phenomenological interpretation for any brane model of the present type,

just plugging in the parameters that characterize the particular model. The well-known phenomenological consequences of the soft breaking terms in the effective Lagrangian lead finally to restrictions on these parameters in order not to be in contradiction with current experimental bounds, which we exemplify by analyzing the appearance of flavour changing neutral currents. As a by product, we also give a systematic discussion of the perspectives to achieve a unification of gauge couplings, which turns out to be completely different approach than for the heterotic string once more, where it was rather automatic to achieve grand unification.

The organization of the paper is as follows: In section 2 we give an introduction to the relevant aspects of orientifolds with intersecting branes or branes with magnetic world volume fluxes, which we (not quite successfully) tried to keep short, and also introduce our conventions and notations for the effective Lagrangian. In section 3 we determine the necessary ingredients for using the effective action, parts of the Kähler potential, the superpotential, D-terms and FI parameters, as well as axionic couplings within Stückelberg mass terms. In section 4 we use these to compute the soft breaking terms in a rather straightforward manner, and discuss some implications. Finally, in section 5 we address the mentioned phenomenological issues, applying the expressions for the soft breaking terms.

2 Intersecting brane models on CY-orbifolds

We first like to specify the type of model we are considering and set up some notations and conventions. The reader who is familiar with the literature on intersecting brane models may even want to skip the section.

2.1 Definition of the class of models

Specifically, we are discussing toroidal CY-orbifold compactifications of type I string theory, or CY-orientifolds of type IIA, the latter version featuring D6-branes that intersect each other in points on an internal six-dimensional space [35]. These models are known as intersecting brane world orbifolds. Their massless chiral fermion spectra can be engineered to match the Standard Model spectrum or that of grand unified theories. We shall actually concentrate on models that allow vacua that preserve exactly $\mathcal{N} = 1$ supersymmetry in the effective four-dimensional action. The examples discussed explicitly in the literature have orbifold groups $\mathbb{Z}_2 \times \mathbb{Z}_2$ [13], \mathbb{Z}_4 [15], or $\mathbb{Z}_4 \times \mathbb{Z}_2$ [16], but all groups of even order are believed to allow supersymmetric vacua (based on the observation of [28]).³ As mentioned in

³There has also been a simplified “local” approach to supersymmetric model building, that consists in relaxing the tadpole constraints, which we mention later, and just constructing a supersymmetric subsector of the whole model to produce the supersymmetric Standard Model, e.g. recently in [51]. The full model would however violate supersymmetry in these settings

the introduction, we will not be directly using the language of intersecting brane models as type IIA orientifolds, but a T-dual description in the form of generalized orientifolds of type IIB string theory. The main modification compared to standard orientifolds [2] then consists in allowing for non-trivial background fields on the world volume of the D-branes by including constant background values for gauge field strengths \mathcal{F} . Roughly speaking, T-duality relates the background gauge fields to the relative angles φ among intersecting D6-branes in the type IIA intersecting brane model by a formula symbolically $\mathcal{F} = \tan(\varphi)$.

The full orientifold group $\{\Omega\mathcal{R}\Theta^k, \Theta^l\}_{k,l \in \mathbb{Z}_N}$, that is divided out of the type IIA theory version of the scenario is given by the generator Θ of a the standard \mathbb{Z}_N (or by two generators Θ_i in $\mathbb{Z}_N \times \mathbb{Z}_M$, $i = 1, 2$) toroidal orbifold group with crystallographic action on the background torus \mathbb{T}^6 [52, 53], and by the modified world sheet parity $\Omega\mathcal{R}$. We assume the background torus to factorize into $\mathbb{T}^6 = (\mathbb{T}^2)^3$, so that the internal metric G splits into

$$G = \text{diag}(G^{(1)}, G^{(2)}, G^{(3)}) , \quad (2)$$

with each $G^{(m)}$ defining a 2×2 metric on \mathbb{T}_m^2 , $m = 1, 2, 3$. This ansatz excludes $\{\mathbb{Z}_3, \mathbb{Z}_4, \mathbb{Z}'_6\}$ from our analysis, which do have off-diagonal moduli. We denote the co-ordinates in the three complex planes by $z_m = (ix_{2m-1} + U_m x_{2m})/\sqrt{2}$ and the standard complex moduli of any one of the three \mathbb{T}_m^2 by

$$T_m = \text{vol}(\mathbb{T}_m^2) - i \int_{\mathbb{T}_m^2} B_2 = \sqrt{G^{(m)}} - ib^{(m)} , \quad U_m = -i \frac{R_2^{(m)}}{R_1^{(m)}} e^{i\vartheta^{(m)}} = \frac{\sqrt{G^{(m)}} - iG_{12}^{(m)}}{G_{11}^{(m)}} . \quad (3)$$

The slightly unconventional factor of $-i$ in the definition of the complex structure is introduced to make conventions compatible with [54].⁴ The T_m and U_m capture the two radii $R_i^{(m)}$ and the tilting angle $\vartheta^{(m)}$ as well as the component of the NSNS 2-form B -field $B_{12}^{(m)} = b^{(m)}$ along the respective torus. For the metric of the four-dimensional space-time we use $g_{\mu\nu}$, reserving indices $g_{ij} = G_{ij}$ for internal components. In terms of the moduli parameters the metric G is expressed through the zwei-bein

$$\mathbf{e}_{1a} = (R_1, 0)_a , \quad \mathbf{e}_{2a} = (R_2 \cos(\vartheta), R_2 \sin(\vartheta))_a \quad (4)$$

as $G_{ij}^{(m)} = \delta^{ab} \mathbf{e}_{ia} \mathbf{e}_{jb}$, or

$$G_{ij}^{(m)} = \frac{\Re(T_m)}{\Re(U_m)} \begin{pmatrix} 1 & -\Im(U_m) \\ -\Im(U_m) & |U|^2 \end{pmatrix}_{ij} \quad (5)$$

without the orbifold.

⁴The mapping of coordinates actually is $x = -ix'$, $y = iy'$, $U = -iU'$, the primed quantities referring to the standard torus conventions.

With these conventions the operation \mathcal{R} that appears in $\Omega\mathcal{R}$ can be taken the reflection of the real parts of the complex coordinates z_m along \mathbb{T}_m^2 , $\mathcal{R} : z_m \mapsto -\bar{z}_m$. The T-duality that translates this scenario into the more standard type I language, the type IIB version, is an inversion of the radii $R_1^{(m)}$ (actually the mirror symmetry transformation) and takes $\Omega\mathcal{R} \mapsto \Omega$ and $\Theta \mapsto \hat{\Theta}$, the latter being an asymmetric rotation that rotates left- and right-moving world sheet fields with opposite phases [49]. The duality also swaps the moduli, $U_m \mapsto T_m$, $T_m \mapsto U_m$. Therefore, whenever the requirement to have a crystallographic action of Θ_i has fixed the complex structure modulus U_m of the type IIA background, $\hat{\Theta}_i$ now fixes the corresponding Kähler parameter T_m in type IIB. Note that we will not use different notation for the type IIA and type IIB moduli, and our later notations will always refer to the type IIB version. The above implies that the dual model is an asymmetric orientifold which is mirror symmetric to the type IIA model and thus has swapped numbers of complex structure and Kähler moduli fields in its spectrum. For the untwisted fields, this means we are dealing with models that have the generic 3 complex structure moduli U_m and 0, 1 or 3 Kähler moduli T_m , the latter case referring to $\mathbb{Z}_2 \times \mathbb{Z}_2$ (see [55] for a list of orbifold moduli spaces). Note also that the imaginary parts of the T_m are fixed through the orientifold projection Ω anyway such that $b^{(m)} = 0$ or $1/2$ [56, 28]. The axionic imaginary parts of the scalar fields s and t_m in the chiral multiplets are in fact RR scalars that descend from reducing the RR potentials C_2 or C_6 instead of B_2 [36].

It has been shown that supersymmetric ground states in the effective theory require the presence of orientifold 9-planes (O9-planes) together with O5-planes in order to be able to achieve complete tadpole cancellation. This is possible whenever the order of the orbifold group generators is even, and the most interest has by now been paid to the simplest example of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ model [13]. As mentioned above, this case is the most generic one in the sense that we will have to deal with completely generic \mathbb{T}_m^2 each with non-trivial complex structure and Kähler modulus.

In the type IIA picture the intersecting brane scenario was established by noticing that the charges and the tensions of the orientifold 6-planes, defined as the fixed locus of $\Omega\mathcal{R}$ on the orbifold, could be canceled by D6-branes which wrap on circles on any of the three \mathbb{T}_m^2 and fill out the four-dimensional Minkowski space-time [24]. On any such two torus a stack of $2N_a$ parallel D6_a-branes (a line) is defined by a set of two co-prime integers $(\mathbf{n}_a^{(m)}, \bar{\mathbf{m}}_a^{(m)})$, allowing $(1, 0)$ and $(0, 1)$ but no multiples thereof, denoting the lattice point which they meet, if drawn from the origin. The gauge group for the orbifold group $\mathbb{Z}_2 \times \mathbb{Z}_2$ is given by

$$H = \bigotimes_a U(N_a) , \quad (6)$$

with some $U(1)$ factors decoupling through axionic Green-Schwarz type couplings,

and neglecting the option of orthogonal and symplectic gauge group factors. The primordial gauge group $U(2N_a)$ on $2N_a$ brane is broken by the orientifold projection, which can be demonstrated by regarding $N_a = 1$ without loss of generality [57]. First of all, the effect of $\Omega\mathcal{R}$ is to identify the stack of branes with winding numbers $(\mathbf{n}_a^{(m)}, \bar{\mathbf{m}}_a^{(m)})$ with the stack $(\mathbf{n}_a^{(m)}, -\bar{\mathbf{m}}_a^{(m)})$ and leaves the full $U(2)$. Choosing a basis of the adjoint by using $\sigma^0 = \mathbf{1}_2$, and σ^A , $A = 1, 2, 3$, the Pauli-matrices, strings with both ends on either one of the branes can be identified with the component $(\sigma^0 \pm \sigma^3)/2$ while open strings between the two are in the directions $\sigma^{1,2}$. Now one generator of $\mathbb{Z}_2 \times \mathbb{Z}_2$ projects out the components $\sigma^{1,2}$ of the gauge bosons $A_\mu^A \sigma^A$ and the other one identifies the two $(\sigma^0 \pm \sigma^3)/2$, such that one is left with a single $U(1)_a$ gauge factor. The most general solution is obtained by tensor products of these Chan-Paton matrices [57]. For fields $\Phi^A \sigma^A$ that have opposite space-time parity under the generators of $\mathbb{Z}_2 \times \mathbb{Z}_2$, i.e. $\Theta_i : \Phi^A \mapsto -\Phi^A$, exactly the opposite mapping applies and bifundamental fields of different gauge factors can survive in the spectrum. The same reasoning can also be applied to the $\mathbb{Z}_2 \times \mathbb{Z}_4$ orbifold, while the patterns for \mathbb{Z}_4 would be slightly different. As stressed in [14], the projection on the Chan-Paton factors has a geometric interpretation in the blown-up version of the orbifolds. One should also mention that it is in fact possible to get $SO(N_a)$ and $Sp(N_a)$ gauge groups, which have played a role in some applications [42, 47, 48]. For our purposes it will mostly suffice to treat the stacks of branes as supporting the factors of H , and the bifundamental part of the chiral matter spectrum given by their intersection numbers

$$I_{ab} = \prod_{m=1}^3 \left(\mathbf{n}_a^{(m)} \mathbf{m}_b^{(m)} - \mathbf{n}_b^{(m)} \mathbf{m}_a^{(m)} \right) , \quad (7)$$

where $\mathbf{m}_a^{(m)} = \bar{\mathbf{m}}_a^{(m)} + b^{(m)} \mathbf{n}_a^{(m)}$. The realization of the Standard Model spectrum needs at least four different stacks of branes with primordial gauge group $U(3) \times U(2) \times U(1) \times U(1)$. The three generations of all the matter fields, quarks and leptons, can arise from multiple intersections of the same two stacks, respectively, except for the quark doublets, which split into two plus one from two different types of intersections in a minimal setting [29]. Beyond the bifundamental matter there are also chiral multiplets in symmetric and anti-symmetric representations, which are often completely ignored in a somewhat inconsistent manner. For a more complete treatment of this and the other models we refer to the original literature.

In the type IIB dual picture all the branes map into stacks of D9-branes (except for certain degenerate cases, which will be dealt with later) that fill out the entire ten-dimensional space-time but carry non-trivial background gauge flux on their respective world volume, given by

$$\mathcal{F}_a = \text{diag}(\mathcal{F}_a^{(1)}, \mathcal{F}_a^{(2)}, \mathcal{F}_a^{(3)}) ,$$

$$(\mathcal{F}_a^{(m)})_{ij} = \left(b^{(m)} + \frac{\bar{\mathbf{m}}_a^{(m)}}{\mathbf{n}_a^{(m)}} \right) \epsilon_{ij} = \frac{\mathbf{m}_a^{(m)}}{\mathbf{n}_a^{(m)}} \epsilon_{ij} = \mathbf{F}_a^{(m)} \epsilon_{ij}, \quad i, j = 1, 2. \quad (8)$$

The gauge flux also factorizes into 2-tori, thus \mathcal{F}_a is a (1,1) form in complex notation, and the individual D9_a-branes carry labels a . We have made the field strength dimensionless by setting $2\pi\alpha' = 1$, which otherwise appears multiplying \mathcal{F} . Through the presence of $b^{(m)}$ the effective quantum number $\mathbf{m}_a^{(m)}$ may then be integer or half-integer. The symmetry of the brane spectrum under $\Omega\mathcal{R}$ already implies the cancellation of half of all the RR charges, while the remaining conditions read

$$\begin{aligned} \sum_a N_a \prod_{m=1}^3 \mathbf{n}_a^{(m)} + N_{\text{O}9} &= 0, & \sum_a N_a \mathbf{F}_a^{(1)} \mathbf{F}_a^{(2)} \prod_{m=1}^3 \mathbf{n}_a^{(m)} + N_{\text{O}5_3} &= 0, \\ \sum_a N_a \mathbf{F}_a^{(1)} \mathbf{F}_a^{(3)} \prod_{m=1}^3 \mathbf{n}_a^{(m)} + N_{\text{O}5_2} &= 0, & \sum_a N_a \mathbf{F}_a^{(2)} \mathbf{F}_a^{(3)} \prod_{m=1}^3 \mathbf{n}_a^{(m)} + N_{\text{O}5_1} &= 0, \end{aligned} \quad (9)$$

where $N_{\text{O}9}$ denotes the total amount of 9-brane charge carried by O9-planes and $N_{\text{O}5_m}$ the 5-brane charge referring to O5-planes wrapped on the two-torus \mathbb{T}_m^2 [24, 28, 13]. The angle variables of the dual IIA picture are defined by

$$\varphi_a^{(m)} = \arctan \left(\frac{\mathbf{F}_a^{(m)}}{\Re(T_m)} \right), \quad (10)$$

which is the angle of a given stack with respect to the coordinate axis, the location of the O6-planes. We shall employ conventions to choose all $\varphi_a^{(m)}$ and also the relative angles $\varphi_{ab}^{(m)}$ between two stacks a and b modulo 2π in $[0, 2\pi]$. This makes a distinction of cases $n_a^{(m)} \geq 0$ and $n_a^{(m)} \leq 0$ necessary, such that in the latter case $\varphi_a^{(m)}$ lies in $[\pi/2, 3\pi/2]$ and in the complement otherwise. The condition for preserving supersymmetry in any open string sector reads [35, 58]

$$\varphi_a^{(1)} + \varphi_a^{(2)} + \varphi_a^{(3)} = 2\pi. \quad (11)$$

Since one can always redefine the angles by flipping the orientations of a brane on two two-tori simultaneously, shifting the two angles by π , we have adopted conventions such that the sum is always 2π . In terms of the magnetic background fields Eq.(11) takes the form

$$\sum_{m=1}^3 \frac{\mathbf{F}_a^{(m)}}{\Re(T_m)} = \prod_{m=1}^3 \frac{\mathbf{F}_a^{(m)}}{\Re(T_m)}. \quad (12)$$

This set of conditions actually fixes generically all three Kähler parameters $\Re(T_m)$. If some of these are already fixed by the asymmetric orbifold projection, as happens if the group is not $\mathbb{Z}_2 \times \mathbb{Z}_2$, it is just a further condition that necessarily has

to be met by the gauge field background for the given $\mathfrak{R}(T_m)$. Since the number of brane stacks in any model will at least be four, not counting the hidden sectors, and the number of moduli which may vary to satisfy Eq.(12) is only three, the system is already overdetermined, and thus supersymmetry in the effective action below the mass scale induced by the fluxes should not be considered a generic feature in orbifold models. Nevertheless interesting models can be found, and we shall assume Eq.(12) to be fulfilled.

As opposed to the conventions of e.g. [24] we have now chosen a normalization which is more adapted to the notation in the effective field theory where gauge fluxes are independent of the geometric moduli, since they are quantized through the Dirac quantization (see [59] as a useful reference). The integers now have an interpretation of $\mathbf{n}_a^{(m)}$ denoting the winding number of the brane stack a on the torus \mathbb{T}_m^2 and $\mathbf{m}_a^{(m)}$ denoting the first Chern number of its $U(1)_a$ world volume gauge bundle. For $b^{(m)} = 0$ the limiting case $(\mathbf{n}_a^{(m)}, \mathbf{m}_a^{(m)}) = (1, 0)$ has $\varphi_a^{(m)} = 0$ and describes a “pure” D-brane wrapping \mathbb{T}_m^2 once and without flux, while the degenerate case $(\mathbf{n}_a^{(m)}, \mathbf{m}_a^{(m)}) = (0, 1)$ with $\varphi_a^{(m)} = \pi/2$ maps to a D-brane with Dirichlet boundary conditions on \mathbb{T}_m^2 after T-duality, i.e. a point like brane of lower dimension. This case needs actually some extra care, since $\mathbf{F}_a^{(m)}$ diverges formally.

Since the Dirac-Born-Infeld (DBI) action is in its non-abelian version only reliably known to leading order in the gauge field strength, we shall actually have to refer to the limit of weak fields eventually, which we define as $\mathcal{F} \rightarrow 0$. More generally, we would favor to consider a general perturbation around an exactly solvable orientifold background with not only D9-branes with $\mathcal{F}_a = 0$ but also with D5-branes with some components of \mathcal{F}_a being infinite, or even more generic rational values of $\mathbf{F}_a^{(m)}$ values as arise in the supersymmetric orientifolds constructed e.g. in [4, 5]. This does however not appear to be feasible.

2.2 Spectrum and field theory conventions

The total massless spectrum consists of untwisted closed string fields, that include the gravity plus untwisted moduli sector, of twisted closed string fields, localized at fixed points of Θ_i , which we are mostly going to ignore throughout this paper, and finally out of open string fields. These again split naturally into states with both ends of the string on the same stack of brane, the gauge vector multiplets plus some extra matter in adjoint, symmetric and anti-symmetric representations, as well as those connecting different brane stacks in bifundamental representations, which are supposed to involve all the fermionic matter fields of the MSSM and the standard bifundamental Higgs fields. The former symmetric and anti-symmetric representations come in three copies as remnants of a “would-be” $\mathcal{N} = 4$ supersym-

metric theory, which arises in this sector upon toroidal compactification without orbifold projection, i.e. there is one complex scalar for each \mathbb{T}_m^2 . We then also denote these as $C_m^{[aa]}$. For an open string between two branes at some relative angles, the modings in the Fourier expansion of all world sheet fields are shifted by the relative angle $\varphi_{ab}^{(m)}$ and thus also the zero-point energy of the NS sector. The ground state energy becomes (for angles smaller than $\pi/2$, else see [11])

$$E_0 = \frac{1}{2} \sum_{m=1}^3 \varphi_{ab}^{(m)} - \max \{ \varphi_{ab}^{(1)}, \varphi_{ab}^{(2)}, \varphi_{ab}^{(3)} \}. \quad (13)$$

For angles that satisfy Eq.(11) exactly one of the three components stays massless, denoted $C^{[ab]}$, while the other two get masses of the order of the mass scale associated to the fluxes through a D-term in the effective action [36]. For a more complete derivation and tables that allow to compute the precise spectrum from the winding numbers we again refer to the literature, for example [13, 14, 15, 16].

For the effective $\mathcal{N} = 1$ field theory we use the following conventions and notations: The set of all fields \mathcal{T}_I , moduli and matter, is split into four-dimensional dilaton field s , the closed string Kähler and complex structure moduli fields t_m and u_m , and the open string fields are denoted $C_\alpha = C_\alpha^A \lambda^A$, together $\mathcal{T}_I \in \{s, t_m, u_m, C_\alpha\}$. Just to repeat, there are generically 3 u_m and 0, 1, or 3 t_m before applying Eq.(11). The λ^A span a basis of the respective representation of the gauge group H . The C_α^A split into $C_m^{[aa]}$ fields, which, as mentioned, transform under various representations of the gauge group [13] and are often ignored, and $C^{[ab]}$, transforming as bifundamental fields and represent quark and lepton, as well as Higgs field multiplets. All gauge singlets constructed out the C_α then imply traces over gauge indices, for instance $|C_\alpha|^2 = \text{Tr} C_\alpha \bar{C}_\alpha = C_\alpha^A \bar{C}_\alpha^A$, $\text{Tr} C_\alpha C_\beta C_\gamma = \frac{i}{2} f^{ABC} C_\alpha^A C_\beta^B C_\gamma^C$. The auxiliary fields in the respective chiral multiplets are denoted $F^I \in \{F^s, F^{t_m}, F^{u_m}\}$. The relation between the string frame moduli parameters T_m, U_m and the ten-dimensional dilaton Φ and the fields in the effective four-dimensional Einstein-frame Lagrangian is [60, 61]

$$t_m + \bar{t}_{\bar{m}} = e^{-\Phi} (T_m + \bar{T}_{\bar{m}}), \quad u_m + \bar{u}_{\bar{m}} = U_m + \bar{U}_{\bar{m}}, \quad s + \bar{s} = e^{-\Phi} \prod_{m=1}^3 (T_m + \bar{T}_{\bar{m}}). \quad (14)$$

The axionic fields for the imaginary parts of the fields $\{s, t_m\}$ are provided by suitable components of RR-forms [36]. Concretely, a_0 , defined via $da_0 = *_4 dC_2$, $*_4$ denoting the four-dimensional Hodge operator, is the partner of $s + \bar{s}$ and the other axions are given by certain components of C_6 . We split $C_6 = C_2^{(m)} \wedge C_4^{(m)}$, where $C_4^{(m)}$ has components only along the two 2-tori transverse to \mathbb{T}_m^2 , i.e. $C_4^{(m)} \propto dz_n \wedge d\bar{z}_{\bar{n}} \wedge dz_p \wedge d\bar{z}_{\bar{p}}$ with $n, p \neq m$. Then $da_m = *_4 dC_2^{(m)}$ defines the partner a_m of $t_m + \bar{t}_{\bar{m}}$ [36]. The tree-level superpotential can be written

$$W_{\text{tree}}(u_m, C_\alpha) = \frac{1}{6} Y_{\alpha\beta\gamma}(u_m) C_\alpha C_\beta C_\gamma + \dots \quad (15)$$

and on general grounds only depends holomorphically on complex structure moduli u_m in perturbative type IIB string theory [62, 63, 64, 65]. The parenthesis indicate higher order terms. Additional contributions appear in the process of supersymmetry breaking in the hidden sector, such as the effective superpotential that arises by gaugino condensation. Integrating out the undetermined hidden sector a new effective superpotential can be formulated that involves corrections to W_{tree} . As an example, gaugino condensation leads to an effective superpotential that depends on the moduli through the gauge kinetic functions f_a , that turn out to depend holomorphically on s, t_m in our case. The precise form of the dependence on the matter fields relies on the structure of the W_{tree} above. It is obtained by setting the C_α in Eq.(15) that belong to the hidden sector to their moduli-dependent vacuum expectation values. Thus the trilinear term in the visible C_α should remain unmodified, and terms with only hidden fields will generate a contribution without visible matter fields. The appearance of the quadratic μ -term - in this approximation - depends on the fact, if there are fields charged under both, the hidden and the visible gauge groups, such that terms bilinear in visible C_α can be generated. In an idealized version of our model, we would want to avoid such fields, and disentangle the hidden and visible brane stacks on the internal space, such that they communicate only gravitationally. In practice, it has however not been possible to find such a model, and we keep the μ -term to be generic. Note, however, that we would favor a situation without it. Together we write

$$W_{\text{eff}}(\mathcal{T}_I) = W_{\text{tree}}(u_m, C_\alpha) + \hat{W}(\mathcal{T}_I) + \frac{1}{2}\mu_{\alpha\beta}(\mathcal{T}_I)C_\alpha C_\beta + \dots \quad (16)$$

By a slight abuse of notation, the C_α now only refer to the surviving visible matter fields. The Kählerpotential is expanded

$$\mathcal{K}(\mathcal{T}_I + \bar{\mathcal{T}}_{\bar{I}}) = \hat{\mathcal{K}}(\mathcal{T}_I + \bar{\mathcal{T}}_{\bar{I}}) + \tilde{\mathcal{K}}_{\alpha\bar{\beta}}(\mathcal{T}_I + \bar{\mathcal{T}}_{\bar{I}})C_\alpha \bar{C}_{\bar{\beta}} + \tilde{\mathcal{Z}}_{\alpha\beta}(\mathcal{T}_I + \bar{\mathcal{T}}_{\bar{I}})C_\alpha C_\beta + \dots \quad (17)$$

and the (holomorphic) gauge kinetic functions are written $f_a = f_a(\mathcal{T}_I)$, a labeling the factors of the gauge group. D-terms $D = D(\mathcal{T}_m + \bar{\mathcal{T}}_{\bar{m}}, C_\alpha, \bar{C}_{\bar{\alpha}})$ and Fayet-Iliopolous (FI) parameters, denoted $\xi_a(t_m + \bar{t}_{\bar{m}})$, may also occur. They depend only on the real parts of the Kähler moduli, due to Peccei-Quinn shift symmetries in the imaginary parts [62, 63, 64, 65]. We shall always be working at leading order in the matter fields C_α . The factorizable structure of the moduli space of the background torus implies that $\tilde{\mathcal{K}}_{\alpha\bar{\beta}} = \tilde{\mathcal{K}}_\alpha \delta_{\alpha\bar{\beta}}$. The covariant derivative with respect to \mathcal{K} is $D = \partial + (\partial\mathcal{K})$, and the Kähler metric is $\mathcal{K}_{I\bar{J}} = \partial_I \partial_{\bar{J}} \mathcal{K}$. The auxiliary fields take values $F_I \propto D_I \hat{W}$ and the scalar potential is given by the standard formula [20, 66]

$$V(\mathcal{T}_I, \bar{\mathcal{T}}_{\bar{I}}) = e^{\mathcal{K}} \left(\mathcal{K}^{I\bar{J}} D_I W_{\text{eff}} \bar{D}_{\bar{J}} \bar{W}_{\text{eff}} - 3|W_{\text{eff}}|^2 \right) \quad (18)$$

The D-terms are assumed to vanish identically in the effective theory. In the vacuum only \hat{W} will contribute and the value of V is denoted by V_0 . The gravitino mass is

$$M_{3/2} = e^{\mathcal{K}/2} |\hat{W}| \quad (19)$$

Gaugino masses are given by

$$M_a = \frac{1}{2\Re(f_a)} F^I \partial_I f_a . \quad (20)$$

The soft breaking parameters are given through the effective Lagrangian

$$\begin{aligned} \mathcal{L}_{\text{soft}} = & -m_\alpha^2 C'_\alpha \bar{C}'_{\bar{\alpha}} - \frac{1}{6} A_{\alpha\beta\gamma}^0 Y_{\alpha\beta\gamma}^0 C'_\alpha C'_\beta C'_\gamma \\ & - \frac{1}{2} (B_{\alpha\beta}^0 \mu_{\alpha\beta}^0 C'_\alpha C'_\beta + \text{h.c.}) + \dots . \end{aligned} \quad (21)$$

The primes and upper indices 0 indicate that the matter fields have been canonically normalized in their kinetic terms and suitable normalization functions have been absorbed into $\mu_{\alpha\beta}$ and $Y_{\alpha\beta\gamma}$. The desired soft parameters $m_\alpha^2, A_{\alpha\beta\gamma}^0, B_{\alpha\beta}^0$ are the functions that multiply the Yukawa-couplings $Y_{\alpha\beta\gamma}^0$ and the μ -parameter $\mu_{\alpha\beta}^0$. They are defined by [67]

$$\begin{aligned} m_\alpha^2 &= M_{3/2}^2 + V_0 - F^I \bar{F}^{\bar{J}} \partial_I \partial_{\bar{J}} \ln(\tilde{\mathcal{K}}_\alpha) , \\ A_{\alpha\beta\gamma}^0 &= \bar{c} F^I \left(\partial_I \hat{\mathcal{K}} + \partial_I \ln(Y_{\alpha\beta\gamma}) - \partial_I \ln(\tilde{\mathcal{K}}_\alpha \tilde{\mathcal{K}}_\beta \tilde{\mathcal{K}}_\gamma) \right) , \\ B_{\alpha\beta}^0 &= \bar{c} F^I \left(\partial_I \hat{\mathcal{K}} + \partial_I \ln(\mu_{\alpha\beta}) - \partial_I \ln(\tilde{\mathcal{K}}_\alpha \tilde{\mathcal{K}}_\beta) \right) + \dots , \end{aligned} \quad (22)$$

where \bar{c} is some normalization that will be put in later. In leaving out additional terms in the third line of Eq.(22) we have assumed that the term $\tilde{\mathcal{Z}}_{\alpha\beta}$ in the Kähler potential vanishes, or that and all such terms are transferred to the superpotential by a Kähler transformation.

3 Elements of the effective action

The ingredients to perform explicit calculations are obviously the functions that determine the effective Lagrangian. In order to obtain expressions for these, we have translated the intersecting brane world scenario back into the type I language of the IIB picture, which allows us to use dimensional reduction of standard ten-dimensional type I expressions, and further employ the (partly) perturbative duality to the heterotic string, whose effective action is expected to be very similar in many respects [68, 69, 70, 71]. This would not be easily possible in the type IIA picture with intersecting D6-branes and O6-planes. To keep formulas handy we specialize to the case of the $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold group, i.e. we always keep 3 t_m moduli.

3.1 $\hat{\mathcal{K}}, \tilde{\mathcal{K}}_{\alpha\beta}^{[aa]}$ and f_a

Before getting started let us actually cite previous expressions given for the type I Kähler potential in the presence of D9- and D5-branes. In [61] such were given for

the case of all tori being squares, $U_m = 1$, and $B_2 = 0$ such that T-duality can be used very simply to infer the terms for the three possible types of supersymmetry preserving D5-branes wrapped on any single \mathbb{T}_m^2 from those of D9-branes, the latter being taken from the perturbative heterotic result by use of [60]. The open string fields are now denoted $C_m^{[99]}$ and $C_m^{[5_n 5_n]}$ for the m -component of the massless bosonic excitation of an open string with both ends on a given 9-brane, or on a D5_{*n*}-brane wrapping \mathbb{T}_n^2 . Analogously, one has $C^{[5_m 5_m]}$ and $C^{[95_m]}$ as the massless bosonic NS-ground state of an open string connecting two different D5-branes wrapped on two different tori m and n or connecting a 9-brane and a 5-brane. These branes are degenerate examples of magnetic fluxes corresponding to

$$\begin{aligned} \text{D9} & : & (\mathbf{n}_a^{(m)}, \mathbf{m}_a^{(m)}) &= (\pm 1, 0) \text{ for all } m, \\ \text{D5}_n & : & (\mathbf{n}_a^{(n)}, \mathbf{m}_a^{(n)}) &= (\pm 1, 0), \quad (\mathbf{n}_a^{(p)}, \mathbf{m}_a^{(p)}) = (0, \pm 1) \text{ for } p \neq n. \end{aligned} \quad (23)$$

The branes have to be defined with appropriate orientation so that the relative angles work out to fulfill Eq.(11). The important point now is that a combined T-duality along two among the three 2-tori, say all except \mathbb{T}_n^2 , is a symmetry of the effective action, by exchanging $\text{D9} \leftrightarrow \text{D5}_n$ and $s \leftrightarrow t_n$, $t_m \leftrightarrow t_p$ for $m \neq n \neq p$. The full T-duality group of the \mathbb{T}^6 will actually no longer be a symmetry of the background.

Let us first concentrate on the fields $C_m^{[aa]}$, i.e. only regard $C_m^{[99]}$ and $C_m^{[5_n 5_n]}$. Their metric was written

$$\begin{aligned} \hat{\mathcal{K}} &= -\ln(s + \bar{s}) - \sum_{m=1}^3 \ln(t_m + \bar{t}_m), \quad (24) \\ \frac{1}{2} \tilde{\mathcal{K}}_{\alpha\bar{\beta}}^{[aa]} C_\alpha^{[aa]} \bar{C}_{\bar{\beta}}^{[aa]} &= \sum_{m=1}^3 \frac{|C_m^{[99]}|^2}{t_m + \bar{t}_m} + \sum_{m=1}^3 \frac{|C_m^{[5_m 5_m]}|^2}{s + \bar{s}} + \frac{1}{2} \sum_{m,n,p=1}^3 \gamma_{mnp} \frac{|C_n^{[5_m 5_m]}|^2}{t_p + \bar{t}_p}, \end{aligned}$$

using $\gamma_{mnp} = 1$ for $m \neq n \neq p \neq m$ and 0 else. Since the magnetic field background only affects the open string fields, the first line of Eq.(24) is the standard Kähler potential for

$$\left(\frac{SU(1,1)}{U(1)} \right)_s \times \left(\frac{SU(1,1)}{U(1)} \right)_t^3, \quad (25)$$

the scalar manifold of the untwisted moduli of a toroidal orbifold without u_m moduli. It remains unchanged and also applies to our models. The first term in the second line of Eq.(24) is identical to the heterotic potential for an untwisted matter field, corresponding to

$$\left(\frac{SU(1,1)}{U(1)} \right)_t^3 \rightarrow \left(\frac{SU(1,1+N)}{U(1)} \right)_t^3, \quad (26)$$

and the other two follow from applying T-dualities along any two among the three two-tori. Here N is the number of extra matter fields C_m . The role of D5-branes

in type I vacua is unconventional if compared to the heterotic string, and partly appears as a non-perturbative effect there. In particular, in the heterotic Kähler potential the dilaton $s + \bar{s}$ does not appear in the matter metric. The above expressions will later be reproduced as special cases by our more general formulas upon applying Eq.(23).

We now turn to computing the Kähler metric for the $C_\alpha^{[aa]}$ strings from a simple direct reduction of the (abelian) Dirac-Born-Infeld (DBI) effective Lagrangian. Since we are dealing with strings with both ends on the same brane we do not really have to invoke the full (unknown) non-abelian version of the DBI action, but can restrict ourselves to a single stack of a single brane, if necessary. Nevertheless we keep the formulas non-abelian. The general form for a Dp -brane labelled by a is

$$\mathcal{S}_{\text{DBI}} = -\mu_p \text{Tr} \int_{\mathcal{W}_a^{p+1}} d^{p+1}\xi e^{-\Phi} \sqrt{-\det(P[G+B] + F_a)}. \quad (27)$$

The same formula measures the tension of the orientifold planes, setting the gauge fields to zero. The pull-back to the world volume \mathcal{W}_a^{p+1} is trivial for $p = 9$, and world volume and space-time fields and coordinates can be identified. For the abelian case we can also just drop the trace, which contains all the non-abelian information and problems. One only needs to take account of the multiple wrapping of the branes when writing the integral over the world volume as an integral over the torus. For this reason we rescale the background fields and introduce $\mathbf{n}_a \cdot G$ and $\mathbf{n}_a \cdot \mathcal{F}_a$ by

$$\mathbf{n}_a \cdot G = \text{diag}(\mathbf{n}_a^{(1)} G^{(1)}, \mathbf{n}_a^{(2)} G^{(2)}, \mathbf{n}_a^{(3)} G^{(3)}, \dots), \quad \text{etc.} \quad (28)$$

The scalar matter fields $C_\alpha^{[aa]}$ now arise as internal components of the world volume gauge fields A_a^5 inside F_a , which split into four-dimensional scalars A_{ai} and vectors $A_{a\mu}$. From their kinetic terms we want to read off the moduli space metric. We combine $F_a + B = \tilde{\mathcal{F}}_a$ and write $\tilde{\mathcal{F}}_a = \mathcal{F}_a + \delta\mathcal{F}_a$, where

$$\begin{aligned} (\delta\mathcal{F}_a)_{\mu\nu}^A &= 2\partial_{[\mu} A_{\nu]a}^A + f^{ABC} A_{a\mu}^B A_{a\nu}^C, \\ (\delta\mathcal{F}_a)_{\mu i}^A &= D_\mu A_{ai}^A = \partial_\mu A_{ai}^A + f^{ABC} A_{a\mu}^B A_{ai}^C, \\ (\delta\mathcal{F}_a)_{ij}^A &= f^{ABC} A_{ai}^B A_{aj}^C \end{aligned} \quad (29)$$

(now letting $i, j = 1, \dots, 6$) contains only the fluctuations of the four-dimensional vector fields $A_{a\mu}$, and scalars A_{ai} . \mathcal{F}_a is the constant background in the Cartan subalgebra, or more precisely in the $U(1)_a$ subgroup explicit in $U(N_a) = (SU(N_a) \times U(1)_a)/\mathbb{Z}_N$, such that $SU(N_a)$ remains unbroken. To extract the leading terms of

⁵The upper capital gauge group indices will be suppressed most of the time, while the lower index a for the brane stack is written. This is a redundant labeling anyway, since the stacks are simultaneously counted as factors of the gauge group.

the DBI Lagrangian in the fluctuations one uses

$$\sqrt{\det(1+M)} = 1 + \frac{1}{2}\text{tr}(M) + \frac{1}{8}(\text{tr}(M))^2 - \frac{1}{4}\text{tr}(M^2) + \dots \quad (30)$$

Here the trace only refers to Lorentz indices. The kinetic terms then read

$$\begin{aligned} \mathcal{S}_{\text{DBI}} = & -\mu_9 \int d^{10}x \sqrt{-g_4} \sum_a \sqrt{\det(\mathbf{n}_a \cdot G + \mathbf{n}_a \cdot \mathcal{F}_a)} e^{-\Phi} \\ & \times \left(\frac{1}{2}(\mathbf{n}_a \cdot G + \mathbf{n}_a \cdot \mathcal{F}_a)^{ij} g^{\mu\nu} D_\mu A_{ai} D_\nu A_{aj} + \frac{1}{4}(\delta\mathcal{F}_a)_{\mu\nu}(\delta\mathcal{F}_a)^{\mu\nu} \right) + \dots \end{aligned} \quad (31)$$

We use the convention that $(G + \mathcal{F}_a)^{ij}$ is the inverse of $(G + \mathcal{F}_a)_{ij}$. The gauge coupling g_{10} enters via

$$\mu_9 = \frac{1}{g_{10}^2} (2\pi\alpha')^{-2}, \quad (32)$$

where the extra factors $2\pi\alpha'$ are absorbed by rescaling the fields, and are set to 1 in our conventions anyway. The gauge kinetic functions are defined by the prefactor in

$$-\frac{\mathfrak{R}(f_a)}{4g_{10}^2} (\delta\mathcal{F}_a)_{\mu\nu} (\delta\mathcal{F}_a)^{\mu\nu}, \quad (33)$$

and using the supersymmetry condition Eq.(11) one can show that

$$\begin{aligned} \mathfrak{R}(f_a) &= e^{-\Phi} \sqrt{\det(\mathbf{n}_a \cdot G + \mathbf{n}_a \cdot \mathcal{F}_a)} \\ &= \frac{1}{2} \mathbf{n}_a^{(1)} \mathbf{n}_a^{(2)} \mathbf{n}_a^{(3)} (s + \bar{s}) - \frac{1}{4} \sum_{m,n,p=1}^3 \gamma_{mnp} \mathbf{n}_a^{(m)} \mathbf{m}_a^{(n)} \mathbf{m}_a^{(p)} (t_m + \bar{t}_{\bar{m}}). \end{aligned} \quad (34)$$

Under a Weyl-rescaling of the four-dimensional metric $g_{\mu\nu}$ the kinetic term for the gauge field fluctuations $A_{a\mu}$ is invariant, such that Eq.(34) is the answer in the Einstein frame. Together with the Chern-Simons (CS) action

$$\mathcal{S}_{\text{CS}} = \mu_9 \sum_a \int_{\mathcal{W}_a^{10}} d^{10}\xi e^{-\mathcal{F}_a} \wedge (C_2 + C_6), \quad (35)$$

that contains the axionic couplings of the imaginary parts in C_2 and C_6 , the real part combines into the gauge kinetic function [36]

$$f_a(s, t_m) = \mathbf{n}_a^{(1)} \mathbf{n}_a^{(2)} \mathbf{n}_a^{(3)} s - \frac{1}{2} \sum_{m,n,p=1}^3 \gamma_{mnp} \mathbf{n}_a^{(m)} \mathbf{m}_a^{(n)} \mathbf{m}_a^{(p)} t_m. \quad (36)$$

In [72] the threshold corrections to gauge couplings have been calculated as well. The matter metric for the A_{ai} transforms under the Weyl-rescaling given by $g_{\mu\nu} \mapsto$

$e^{2\Phi}\sqrt{G}^{-1}g_{\mu\nu}$, that puts the gravitational Lagrangian into standard Einstein-Hilbert form and one gets

$$e^\Phi \sqrt{\frac{\det(\mathbf{n}_a \cdot G + \mathbf{n}_a \cdot \mathcal{F}_a)}{\det(G)}} (\mathbf{n}_a \cdot G + \mathbf{n}_a \cdot \mathcal{F}_a)_{\text{sym}}^{ij} = \frac{2\Re(f_a)}{s + \bar{s}} e^\Phi (\mathbf{n}_a \cdot G + \mathbf{n}_a \cdot \mathcal{F}_a)_{\text{sym}}^{ij} \quad (37)$$

for the proper matter metric. Using the factorization of the six-torus we specialize to a single two-torus

$$e^\Phi (\mathbf{n}_a^{(m)} G^{(m)} + \mathbf{n}_a^{(m)} \mathcal{F}_a^{(m)})_{\text{sym}}^{-1} = \frac{1}{1 + \Delta_a^{(m)}} (e^{-\Phi} \mathbf{n}_a^{(m)} G^{(m)})^{-1}, \quad (38)$$

having defined

$$\Delta_a^{(m|np)} = \frac{(t_n + \bar{t}_n)(t_p + \bar{t}_p)}{(s + \bar{s})(t_m + \bar{t}_m)} (\mathbf{F}_a^{(m)})^2, \quad \Delta_a^{(m)} = \frac{1}{2} \sum_{n,p=1}^3 \gamma_{mnp} \Delta_a^{(m|np)}. \quad (39)$$

The significance of this $\Delta_a^{(m)}$ is that it effectively summarizes explicit moduli dependence that will appear in the soft breaking parameters. We see that the Kähler metric for any two-torus is equal to the metric on the manifold Eq.(26) only rescaled by a factors that depends on \mathcal{F}_a .

For the sake of comparing to Eq.(24) we specify to the simple case where all $U_m = 1$ and $b^{(m)} = 0$, such that $2e^{-\Phi} \mathbf{n}_a^{(m)} G_{ij}^{(m)} = \mathbf{n}_a^{(m)} \text{diag}(t_m + \bar{t}_m, t_m + \bar{t}_m)_{ij}$. In this limit we shall identify the $A_{a2m-1} + iA_{a2m}$ with the Kähler coordinates $C_m^{[aa]}$. The D9-brane case, $(\mathbf{n}_a^{(m)}, \mathbf{m}_a^{(m)}) = (1, 0)$ for all m , then easily reproduces the result of Eq.(24) for the $C_m^{[99]}$ fields, since $2\Re(f_a) = s + \bar{s}$ and $\Delta_a^{(m)} = 0$. Only slightly more challenging are the components of world volume gauge fields along 5-branes, the $C_m^{[5_m 5_m]}$ fields, which are reproduced upon choosing $(\mathbf{n}_a^{(m)}, \mathbf{m}_a^{(m)}) = (1, 0)$ and $(\mathbf{n}_a^{(p)}, \mathbf{m}_a^{(p)}) = (0, 1)$ along the other two two-tori with $p \neq m$. Note that then $2\Re(f_a) = t_m + \bar{t}_m$ and again $\Delta_a^{(m)} = 0$. Thus, the metric Eq.(38) is a deformation of the Kähler metric on $SU(1, 1+N)/U(1)$ and reduces to the standard metric upon switching off the flux, but also knows about the heterotically non-perturbative 5-brane sectors. As another comparison, one may easily recognize the so-called ‘‘open string metric’’ of [73] in Eq.(38), which guarantees that it describes the correct KK mass spectrum of massive excitations in the gauge field background [49], or equivalently, on a non-commutative torus. That the string spectrum coincides with the field theory approximation [74, 75] has been shown in [59, 76].

The metric for the transverse scalars of a D5-brane, the $C_n^{[5_m 5_m]}$ fields, needs to be treated separately. It would be described by choosing $(\mathbf{n}_a^{(m)}, \mathbf{m}_a^{(m)}) = (1, 0)$ and $(\mathbf{n}_a^{(p)}, \mathbf{m}_a^{(p)}) = (0, 1)$ for $p \neq m$ as before, but then Eq.(38) vanishes. This is,

however, not surprising since the fluctuations of the transverse scalars, denoted A_a^i , of any lower-dimensional brane are not internal components of world volume gauge fields, but enter the DBI action via the non-trivial (and non-abelian [77]) pull-back, once the brane is not ten-dimensional. By substituting

$$P[g_{\mu\nu}] = g_{\mu\nu} + G_{kl} D_\mu A^k D_\nu A^l + \dots \quad (40)$$

and $\mathcal{F}_a^{(m)} = 0$ into Eq.(27), one finds the kinetic term

$$\mathcal{S}_{\text{DBI}} = -\mu_9 \int d^{10}x \sqrt{-g_4} \sqrt{\det(G)} e^{-\Phi} G_{ij} g^{\mu\nu} D_\mu A_a^i D_\nu A_a^j + \dots \quad (41)$$

Here it is important that the $A_a^i = G^{ij} A_{aj}$ are used as independent Kähler coordinates, not the A_{ai} (see e.g. [77]). Therefore, the matter metric is

$$\frac{2\Re(f_a)}{s + \bar{s}} e^{2\Phi} \left(e^{-\Phi} G_{ij}^{(m)} \right) = \frac{8\Re(f_a)}{\prod_{p=1}^3 (t_p + \bar{t}_{\bar{p}})} \left(e^{-\Phi} G_{ij}^{(m)} \right). \quad (42)$$

For the scalars $C_n^{[5m5m]}$ we have to use $(\mathbf{n}_a^{(m)}, \mathbf{m}_a^{(m)}) = (1, 0)$ and $(\mathbf{n}_a^{(p)}, \mathbf{m}_a^{(p)}) = (0, 1)$ for $p \neq m$ as mentioned above, and via $2e^{-\Phi} G_{ij}^{(n)} = \text{diag}(t_n + \bar{t}_{\bar{n}}, t_n + \bar{t}_{\bar{n}})$ and $2\Re(f_a) = t_m + \bar{t}_{\bar{m}}$ just obtain the third term of Eq.(24). The same procedure would apply to the transverse scalars of D7-branes or D3-branes as well.

For the general dependence of the Kähler metric in the presence of U_m moduli as well, we just note that

$$\begin{aligned} e^\Phi (G^{(m)})^{ij} D_\mu A_i D^\mu A_j &= \frac{4}{(t_m + \bar{t}_{\bar{m}})(u_m + \bar{u}_{\bar{m}})} |u_m D_\mu A_{2m-1} + i D_\mu A_{2m}|^2, \\ e^{-\Phi} (G^{(m)})_{ij} D_\mu A^i D^\mu A^j &= \frac{t_m + \bar{t}_{\bar{m}}}{u_m + \bar{u}_{\bar{m}}} |i D_\mu A^{2m-1} - u_m D_\mu A^{2m}|^2. \end{aligned} \quad (43)$$

Therefore, the correct Kähler coordinates for the open string fields are defined by [54, 60]

$$C_m^{[aa]} = u_m A_{a2m-1} + i A_{a2m} \quad \text{or} \quad C_m^{[aa]} = i A_a^{2m-1} - u_m A_a^{2m}, \quad (44)$$

for the two longitudinal components A_i of world volume vectors or the two transverse scalars A^i along any \mathbb{T}_m^2 , respectively. This reproduces the correct Kähler metric for the moduli space

$$\left(\frac{SU(1, 1)}{U(1)} \right)_s \times \left(\frac{SO(2, 2 + N)}{SO(2) \times SO(2 + N)} \right)_{t,u}^3, \quad (45)$$

which is known to be correct for the orbifolds of the heterotic string [54]. The effect of the gauge flux is merely a moduli-dependent rescaling of this metric.

While the Kähler potential for the $SU(1,1)/U(1)$ moduli space, $\mathcal{K} = -\ln(t_m + \bar{t}_{\bar{m}} - |C_m^{[aa]}|^2) + \dots$ does not induce holomorphic or anti-holomorphic terms like $C_m^{[aa]} \bar{C}_{\bar{m}}^{[aa]}$ in the effective action, the Kähler potential of the more general case $SO(2,3)/(SO(2) \times SO(3))$ [54]

$$\mathcal{K} = -\ln \left((t_m + \bar{t}_{\bar{m}})(u_m + \bar{u}_{\bar{m}}) - \frac{1}{2}(C_m^{[aa]} + \bar{C}_{\bar{m}}^{[aa]})^2 \right) + \dots \quad (46)$$

does. In other words, already in the model undeformed by magnetic flux the coefficients $\tilde{\mathcal{Z}}_{\alpha\beta}$ in Eq.(17) are non-vanishing (even equal to the coefficients $\tilde{\mathcal{K}}_{\alpha\beta}$), and thus we expect that generically these will survive in the deformed theory. To compute their contribution in the effective potential in the same way as the metric above from a dimensional reduction we would have to use the non-abelian DBI action. For our present purposes, these coefficients are not so important, and we set them to zero henceforth. They would be relevant for discussing the μ -problem, of course, as in [67].

Since the case of Dirichlet boundary conditions for D5- or even D7- or D3-branes is somewhat more special and one can always adapt to it easily, we henceforth only regard branes with mixed Neumann-Dirichlet or pure Neumann boundary conditions, i.e. D9-branes with regular gauge fluxes, to simplify the notation. Just to collect the result of this section, the Kähler potential so far reads

$$\begin{aligned} \hat{\mathcal{K}}(\mathcal{T}_I + \bar{\mathcal{T}}_{\bar{I}}) &= -\ln(s + \bar{s}) - \sum_{m=1}^3 \ln(t_m + \bar{t}_{\bar{m}}) - \sum_{m=1}^3 \ln(u_m + \bar{u}_{\bar{m}}) , \quad (47) \\ \tilde{\mathcal{K}}_{m\bar{m}}^{[aa]}(\mathcal{T}_I + \bar{\mathcal{T}}_{\bar{I}}) &= \frac{1}{(s + \bar{s})(t_m + \bar{t}_{\bar{m}})(u_m + \bar{u}_{\bar{m}})} \frac{4\Re(f_a)}{1 + \Delta_a^{(m)}} \end{aligned}$$

with $\Re(f_a)$ given in Eq.(34). In the weak field limit $\mathcal{F}_a \rightarrow 0$,

$$\frac{2\Re(f_a)}{1 + \Delta_a^{(m)}} = (s + \bar{s}) + o(\mathcal{F}_a) , \quad (48)$$

and we arrive at the metric derived from Eq.(46). While this case is well-known from the heterotic string or type I models with only D9-branes, the full expressions (47) is completely new and will lead to various novel effects when computing the soft breaking terms.

3.2 Twisted open strings: $\tilde{\mathcal{K}}_{\alpha\bar{\beta}}^{[ab]}$

We now turn to the Kähler potential for the open strings with ends on two different D-branes, two D9-branes with different gauge fields \mathcal{F}_a and \mathcal{F}_b . In the T-dual version they stretch between two D6-branes at some non-vanishing relative angle

$$\varphi_{ab}^{(m)} = \varphi_a^{(m)} - \varphi_b^{(m)} . \quad (49)$$

They can be considered twisted open strings in the sense that the oscillator modings of the world sheet fields are shifted by rational numbers in a fashion very reminiscent of twisted closed string sectors in orbifold compactifications. The zero-point energy of the NS sector is shifted and the lowest excitation is given in Eq.(13). Therefore, when turning on a relative rotation continuously, splitting a stack a into two stacks b and c , two of the three massless scalars $C_{m\bar{m}}^{[aa]}$ become massive and only one survives as $C^{[bc]}$. In the effective action, the massive fields are assumed being integrated out. Unfortunately, in a compact background the deformation cannot be done continuously for flat branes, but involves discrete jumps.

In [61] the duality to the heterotic string [79], where the metric for twisted matter fields is known [80, 81, 55], was employed to write down a proposal for their Kähler potential in the presence of D9- and D5-branes. Since any attempt to find the metric for these $C^{[bc]}$ fields directly via a dimensional reduction would involve the full non-abelian DBI action, we are unable to follow the same path as in the previous section⁶ and rely on the same duality arguments to determine the relevant terms in the effective action. In its probably best known example, the duality of the type I and heterotic string with gauge group $SO(32)$ compactified to four dimensions on a K3-orbifold space $\mathbb{T}^4/\mathbb{Z}_2 \times \mathbb{T}^2$ requires a matching of open strings with both ends on D9-branes with untwisted heterotic matter fields, and open strings between D9- and D5-branes with twisted fields [60, 83]. Finally, strings with both ends on a D5-brane are non-perturbative heterotic excitations, such as shrunken instantons, invisible in the perturbative effective action. The rank of the perturbative heterotic gauge group is then one half of the type I rank. In the simplest example of a CY-orbifold $\mathbb{T}^6/\mathbb{Z}_3$ one can combine knowledge about the tree level string spectrum and the superpotential to demonstrate the low energy theories on both sides may have identical massless degrees of freedom [84, 85]. In more general $\mathcal{N} = 1$ orbifold models more intricate structures arise, it can, for example, happen that now some twisted heterotic matter does not have a perturbative type I origin, see [85, 86, 87]. The models of interest here are even asymmetric orientifold vacua with reduced rank of the gauge group, whose heterotic duals are so far not known. The assumptions that underlie $\mathcal{N} = 1$ heterotic-type I duality on CY-orbifold spaces may thus not have the status of proven facts, but we will use the analogy between twisted heterotic matter fields and open type I strings stretching between D-branes at relative angles, relying on that there is an overlap of perturbative heterotic and perturbative type I sectors.

The explicit expressions of [61] were further based on the invariance of the

⁶It was noted in [59, 76] that a symmetrized trace prescription, proposed in [82], is not entirely sufficient to describe open strings stretching between two branes at a relative angle, producing not quite the correct mass spectrum in a general case. In fact, it is believed that the order \mathcal{F}^4 is correctly given by the DBI action with symmetrized trace, and should then be comparable to our formulas.

action under T-duality in a similar way as for Eq.(24). For $U_m = 1$ and $B_2 = 0$ this lead to

$$\begin{aligned} \frac{1}{2} \tilde{\mathcal{K}}_{\alpha\bar{\beta}}^{[ab]} C_{\alpha}^{[ab]} \bar{C}_{\bar{\beta}}^{[ab]} &= \frac{1}{2} \sum_{m,n,p=1}^3 \gamma_{mnp} \frac{|C^{[95_m]}|^2}{((t_n + \bar{t}_n)(t_p + \bar{t}_p))^{1/2}} \\ &+ \frac{1}{2} \sum_{m,n,p=1}^3 \gamma_{mnp} \frac{|C^{[5_n 5_p]}|^2}{((s + \bar{s})(t_m + \bar{t}_m))^{1/2}} . \end{aligned} \quad (50)$$

Again, the first line is the perturbative heterotic result for the three twisted sectors of a $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifold, upon applying the trivial identification of moduli fields [60, 83]

$$s_m^{\text{het}} \mapsto s_m , \quad t_m^{\text{het}} \mapsto t_m , \quad u_m^{\text{het}} \mapsto u_m . \quad (51)$$

The three terms in the first line of Eq.(50) refer to the three possible open strings between a D9-brane and wrapped D5-branes, the second line is obtained by T-duality along two among the three two-tori again. An important point is the different definition of the moduli fields in type I compared to the heterotic string: The t_m depend on the ten-dimensional dilaton, while the t_m^{het} do not [60].

We will now apply a similar reasoning to the generalized open string sectors between branes at relative angles, assuming that there the Kähler metric can be derived by translating the tree level heterotic metric for a twisted sector matter field [80, 81, 55].⁷

$$\tilde{\mathcal{K}}_{\alpha\bar{\beta}}^{\text{het}} = \delta_{\alpha\bar{\beta}} \prod_{m=1}^3 (t_m^{\text{het}} + \bar{t}_m^{\text{het}})^{v^{(m)}-1} \prod_{m=1}^3 (u_m^{\text{het}} + \bar{u}_m^{\text{het}})^{v^{(m)}-1} . \quad (52)$$

Generally, any generator of an orbifold group is defined by a twist vector $v = (v^{(1)}, v^{(2)}, v^{(3)})$ through its eigenvalues $\exp(2\pi i v^{(m)})$ when acting on the complex coordinates of \mathbb{T}_m^2 , where we choose conventions $v^{(1)} + v^{(2)} + v^{(3)} = 1$ or 2 , and $v^{(m)} \in (0, 1]$.⁸ A twisted NS or R world sheet oscillator field of lowest energy has then modings in $1/2 \pm v^{(m)} + \mathbb{Z}$ or $v^{(m)} \pm \mathbb{Z}$ respectively, and similar shifts apply to the coefficients in the OPE of vertex operators. On the other hand, an open string sector of strings stretching between two D6-branes is similarly defined by a vector of relative (oriented) angles $(\varphi_{ab}^{(1)}, \varphi_{ab}^{(2)}, \varphi_{ab}^{(3)})$, $\varphi_{ab}^{(m)} \in [0, 2\pi]$ and subject to the supersymmetry constraint Eq.(11). Since the twisted open strings that stretch

⁷In a world sheet sense the analogy between twisted open strings and true twisted closed strings intuitively boils down to taking the square root of the sphere diagrams to get the results for discs [88, 89, 90].

⁸These are slightly non-standard conventions as compared to $v^{(m)} \in [0, 1)$ [81, 55]. We allow $v^{(m)} = 1$ to avoid the distinction of two cases in the final formulas. One has $v^{(m)} = 1$ for at most one $m \in \{1, 2, 3\}$, and $v^{(1)} + v^{(2)} + v^{(3)} = 2$ if and only if this case, one $v^{(m)} = 1$, occurs. See equations (2.21) to (2.28) in [55] for a comparison.

between branes at relative angles are subject to the same kind of shifts in their oscillator modings and OPE, one would now like to just identify the relative angles with the heterotic shift vectors and apply formula Eq.(52) to the twisted open strings. There are, however, some practical subtleties concerning the translation of the parameters and the accomodation of the non-perturbative sectors connected to D5-branes.

For the shift $\nu_{ab}^{(m)}$ in the Kähler potential of the open string fields the orientation of the branes plays a role, but the orientation of the open string does not, since we want the Kähler metric for $C^{[ab]}$ and $C^{[ba]}$ to be equal. We therefore suggest to use the standard formula for an oriented angle between two vectors $(\mathbf{n}_a^{(m)} R_2, \mathbf{m}_a^{(m)} / R_1)$ in \mathbb{R}^2 ,

$$\nu_{ab}^{(m)} = \frac{1}{\pi} \arccos \left(\frac{1 + \mathbf{F}_a^{(m)} \mathbf{F}_b^{(m)} \Re(T_m)^{-2}}{\prod_{c=a,b} \sqrt{1 + (\mathbf{F}_c^{(m)})^2 \Re(T_m)^{-2}}} \right), \quad (53)$$

to define the analogue of a $\nu^{(m)}$ in the heterotic string for the shift between the stack a and b . This measures the angle only in $[0, \pi]$ and we have $\nu_{ab}^{(m)} = \nu_{ba}^{(m)}$. This means, we identify a shift vector $(\nu_{ab}^{(1)}, \nu_{ab}^{(2)}, \nu_{ab}^{(3)})$ with the relative angle by $\nu_{ab}^{(m)} = \varphi_{ab}^{(m)} / \pi$ or $2 - \varphi_{ab}^{(m)} / \pi$ depending on $\varphi_{ab}^{(m)}$ being less or bigger than π . The condition Eq.(11) for the angles now translates into

$$\nu_{ab} = \nu_{ab}^{(1)} + \nu_{ab}^{(2)} + \nu_{ab}^{(3)} \in [0, 2]. \quad (54)$$

Then Eq.(52) maps naively to

$$\begin{aligned} \tilde{\mathcal{K}}_{\alpha\bar{\beta}}^{\text{het}} \mapsto \tilde{\mathcal{K}}_{\alpha\bar{\beta}}^{[ab]} &= \delta_{\alpha\bar{\beta}} \prod_{m=1}^3 (t_m + \bar{t}_{\bar{m}})^{\nu_{ab}^{(m)} - 1} \prod_{m=1}^3 (u_m + \bar{u}_{\bar{m}})^{\nu_{ab}^{(m)} - 1} \\ &= \delta_{\alpha\bar{\beta}} e^{\Phi(3-\nu_{ab})} \prod_{m=1}^3 (T_m + \bar{T}_{\bar{m}})^{\nu_{ab}^{(m)} - 1} \prod_{m=1}^3 (U_m + \bar{U}_{\bar{m}})^{\nu_{ab}^{(m)} - 1}. \end{aligned} \quad (55)$$

and the first perturbative line of Eq.(50) is reproduced for $\nu_{ab}^{(m)} = (1/2, 1/2, 1)$ and permutations thereof. However, this cannot be the general answer, since the dependence on the ten-dimensional dilaton does not always match the proper power $\exp(\Phi)$, expected for the kinetic term that stems from a disc diagram and after the Weyl rescaling to the Einstein frame. Instead, the perturbative heterotic Kähler potential only leads to an acceptable perturbative Kähler metric in type I if the orientations are chosen such that $\nu_{ab} = 2$. Turning the argument around, we can start with the correct dilaton prefactor in type I and rewrite

$$e^{\Phi} \prod_{m=1}^3 (T_m + \bar{T}_{\bar{m}})^{\nu_{ab}^{(m)} - 1} = (s + \bar{s})^{\nu_{ab}/2 - 1} \prod_{m=1}^3 (t_m + \bar{t}_{\bar{m}})^{\nu_{ab}^{(m)} - \nu_{ab}/2}, \quad (56)$$

or

$$\tilde{\mathcal{K}}_{\alpha\bar{\beta}}^{[ab]} = \delta_{\alpha\bar{\beta}}(s + \bar{s})^{\nu_{ab}/2-1} \prod_{m=1}^3 (t_m + \bar{t}_m)^{\nu_{ab}^{(m)} - \nu_{ab}/2} \prod_{m=1}^3 (u_m + \bar{u}_m)^{\nu_{ab}^{(m)} - \nu_{ab}/2} \quad (57)$$

which is the form that we are going to use in the following. The expression is actually also meant to be diagonal in the multiple intersections of the branes a and b , i.e. diagonal in the generations of matter multiplets. The perturbative type I sectors are mapped to perturbative heterotic sectors, whenever $\nu_{ab} = 2$, whereas $\nu_{ab} = 1$ maps to a non-perturbative sector in the same way as the strings among D5-branes in Eq.(50) are. In this way, the Kähler potential of the heterotic orbifold models is substantially generalized, and in particular the distinguished role of the heterotic dilaton negated. With this formula, both the D9-D5 and D5-D5 sectors in Eq.(50) can be accommodated. Using for instance the shift vector $(1/2, 1/2, 1)$ reproduces the D9-D5₃ potential in Eq.(50) whereas $(1/2, 1/2, 0)$ produces the D5₁-D5₂ term. In the appendix we have computed the shift vectors for a more elaborate example used for modelling a semi-realistic supersymmetric Standard-like Model in [13], which actually contains the above D9- and D5-branes as subsectors, showing that they can be consistently implemented as special cases within our prescription.

The effective action derived for heterotic orbifolds, and in particular Eq.(52), is strictly only valid in a vicinity of the orbifold point, which is defined as the point in moduli space where the locally flat background of the orbifold conformal field theory (CFT) really solves the equations of motion of the ten-dimensional string theory. This would require the local cancellation of the RR charges and brane tension among the orientifold planes and the D-branes. In general the orientifold planes are located at the fixed loci of $\Omega\mathcal{R}\Theta^k$ (type IIA picture), with $k = 0 \bmod N$ referring to the “pure” unrotated O6-planes. Via $\Omega\mathcal{R}\Theta^k = \Theta^{-k/2}\Omega\mathcal{R}\Theta^{k/2}$ these fixed loci organize themselves into two orbits under the orbifold group \mathbb{Z}_{2N} (or products) [14]. The two orbits produce two factors in the gauge group of the effective theory, which then will correspond to the perturbative and non-perturbative dual heterotic gauge groups, as the D9-branes and D5-branes in the simpler $\mathbb{T}^4/\mathbb{Z}_2 \times \mathbb{T}^2$ example discussed above. In order to cancel charge and tension locally the D6-branes must come to lie on top of these O6-planes in appropriate numbers, which unfortunately only leads to non-chiral matter spectra as in [4]. But there are more general configurations that lead to a cancellation after integrating over the internal space and allow chiral matter. These are the ones we are interested in, such that the background that appears in our models is strictly speaking not an orbifold and corrections to the effective action may be expected anyway.

It is important to notice that at the orbifold point the dependence of the angles on the Kähler moduli of the background torus drops out. Recall that the (asymmetric) orbifold projection may leave 0, 1 or 3 of the T_m unfixed and that the

relative angles in principle depend on these via Eq.(10). In a second step, the supersymmetry condition Eq.(11) will fix some or (generically) all of these remaining T_m , such that they drop out of the effective action. If some of them should instead survive, it is straightforward to substitute a dependence into Eq.(57) but this produces rather awkward expressions when computing the soft parameters such as the scalar masses for this sector. More precisely, a modulus T_m is not projected out of the spectrum exactly if the orbifold generator Θ (or generators Θ_i) is a reflection along \mathbb{T}_m^2 , i.e. if $v^{(m)} = 1/2$. The D-branes on top of the orientifold planes on these tori, that would cancel the charge and tension locally, are then given by the degenerate cases $\mathbf{F}_a^{(m)} = 0$, pure D9-branes, or $\mathbf{F}_a^{(m)} = \infty$, localized D5-branes (type IIB picture). These two are exactly the choices where the dependence of the angle on the modulus T_m drops out, even if they should remain unfixed. Thus we see that at the orbifold point no dependence of shift vectors $\nu_{ab}^{(m)}$ on moduli exists, which appears very reasonable, since they take the role of modular weights and conformal dimensions. Given these considerations, we will actually use Eq.(57) as a trial metric on heuristic grounds, being aware that there may be corrections or modifications in the true Kähler metric,⁹ away from the orbifold point. In doing so, we treat the $\nu_{ab}^{(m)}$ as constant free parameters independent of the moduli. Together Eq.(47) and Eq.(57) define the full Kähler metric of the class of models at hand, expanded to leading order in the matter fields and subject to the caveats just explained.

3.3 The superpotential

At the orbifold point the superpotential and D-terms, before considering the deformations by magnetic fluxes and the breaking of supersymmetry, vanish identically. Therefore we are only left with the standard superpotential for the fluctuations of the ten-dimensional gauge fields, that induces Yukawa couplings but no scalar potential in the four-dimensional theory. We start with the classical superpotential known for the heterotic or type I string compactified on a CY 3-fold \mathcal{M}^6 [69, 70, 71]

$$W_{\text{tree}} = \int_{\mathcal{M}^6} \Omega_3 \wedge \omega_3 , \quad (58)$$

where

$$\Omega_3 = dz_1 \wedge dz_2 \wedge dz_3 , \quad \omega_3 = \text{Tr} \left(A \wedge dA - \frac{2i}{3} A \wedge A \wedge A \right) \quad (59)$$

are the holomorphic (3,0)-form and the Yang-Mills CS-form. Since the gauge field background \mathcal{F}_a is a (1,1) form it does not contribute to Eq.(58) and the only

⁹We would like to acknowledge the work of [91], where the twisted Kähler metric has been computed from first principles from string scattering amplitudes, see their section 5.2. It appeared some time after the present paper had been published, and corrects the above formula (57), which is only usable for $\frac{1}{2}$ BPS configurations.

relevant term in ω_3 is of the form $f^{ABC} A_i^A A_j^B A_k^C$, where the A_i^A are now the internal components of the world volume gauge fields of the D9-branes. Upon noting that

$$\begin{aligned}\Omega_3 \wedge \omega_3 &= 2f^{ABC}(u_1 A_1^A + iA_2^A)(u_2 A_3^B + iA_4^B)(u_3 A_5^C + iA_6^C) dx_1 \wedge \cdots \wedge dx_6 \\ &= 2f^{ABC} C_1^A C_2^B C_3^C dx_1 \wedge \cdots \wedge dx_6 ,\end{aligned}\tag{60}$$

one finds that the trilinear couplings in W_{tree} are actually independent of the moduli u_m , once the matter fields have been expressed in terms of the Kähler coordinates $C_m^{[aa]}$ of the first equation of Eq.(44). Though the expression Eq.(60) formally looks like the standard commutator form of $\mathcal{N} = 4$ supersymmetry, the orientifold projection on the Chan-Paton indices effectively breaks this symmetry [57]. The surviving couplings are then among the components $(\sigma^1 \sigma^2 + \sigma^2 \sigma^1) \sigma^3$ in the $U(2)$ basis introduced below Eq.(6).

For the open string fields that connect different branes the world volume gauge flux shifts the ground state energy of the NS sector such that only one of the three complex coordinate fields $C_m^{[aa]}$ survives as $C^{[ab]}$, when a stack splits into two. Therefore the only terms in Eq.(58) that contain only massless fields, but no massive scalars, are of the form

$$C^{[ab]} C^{[bc]} C^{[ca]} + C^{[ab]} C^{[ba]} C_m^{[aa]} .\tag{61}$$

A very similar form, again for D9- and D5-brane only, has been deduced from open string splitting arguments [92, 61]. Together, the trilinear couplings in W_{tree} are of the form

$$\text{Tr} \left(C_1^{[aa]} C_2^{[aa]} C_3^{[aa]} + C^{[ab]} C^{[bc]} C^{[ca]} + C^{[ab]} C^{[ba]} C_m^{[aa]} \right)\tag{62}$$

and do not appear to depend on the closed string moduli. On the contrary, in [40, 88, 89] the mirror symmetric situation with intersecting branes in type IIA orientifolds has been evaluated directly by performing a non-perturbative summing over world sheet instanton contributions, and in principle a dependence of the Yukawa couplings on the moduli of the dual torus arises, which would translate into a dependence on the u_m in our case. This appears in fact before putting the matter fields into the proper form Eq.(44) and it is not clear to us if and how the results of [40, 88, 89] are compatible with Eq.(62) or with [69, 70, 71]. Since the complete absence of moduli dependence would also cause problems in generating mass hierarchies between the quark and lepton generations, we formally keep $Y_{\alpha\beta\gamma} = Y_{\alpha\beta\gamma}(u_m)$ in Eq.(22). In particular, among the $Y_{\alpha\beta\gamma}$ only three, which we denote Y_{123}, Y_{abc} and Y_{abm} , referring to the three terms in Eq.(62), are non-vanishing.

This reduction determines the classical, visible part of the total effective superpotential W_{eff} . Since we do not want to specify the mechanism that finally breaks supersymmetry we do not restrict the form of \hat{W} or the bilinear μ -term in Eq.(16).

We have argued earlier that we would favor a situation with $\mu_{\alpha\beta} = 0$ in the tree level superpotential. Such a term may eventually be generated by the coefficients $\tilde{\mathcal{Z}}_{\alpha\beta}$ in the Kähler potential however.

3.4 D-terms, Fayet-Iliopolous parameters and axions

The DBI action Eq.(27) also contains terms that only involve the internal components of the gauge fields through $(\delta\mathcal{F}^A)_{ij} = f^{ABC} A_i^B A_j^C$ and the background moduli $\{s, t_m, u_m\}$. These are then part of the scalar potential and, since \mathcal{F} is a $(1, 1)$ form in complex notation, originate from D-terms and FI terms. Since we focus here completely on the breaking of supersymmetry via F_I taking non-vanishing expectation values, we assume that all D-terms vanish in the vacuum. Of course, having D-term breaking in these models sounds like an interesting alternative to the present approach.¹⁰ The ‘‘D-flatness’’ will turn out to be implied by the condition Eq.(11) and by restricting the open string scalars $C_\alpha^A \lambda^A$ to take values only in the Cartan subalgebra of the gauge group.

This can actually easily be demonstrated to leading order in the field strength \mathcal{F}_a . Since it involves the non-abelian DBI action, which is unknown beyond $\mathcal{O}(\mathcal{F}^6)$, and reliably tested so far only up to $\mathcal{O}(\mathcal{F}^4)$, we cannot easily find the full condition Eq.(11). The leading term in $\tilde{\mathcal{F}}_a = \mathcal{F}_a + \delta\mathcal{F}_a$ is simply the YM Lagrangian

$$-\frac{1}{4g_{10}^2} \text{Tr} \int d^{10}x \sqrt{-g_4} \sqrt{\det(\mathbf{n}_a \cdot G)} e^{-\Phi} \text{tr} (G^{-1}(\mathcal{F}_a + \delta\mathcal{F}_a))^2 \quad (63)$$

because $\text{tr}(G^{-1}\tilde{\mathcal{F}}) = 0$ in Eq.(30). After rescaling to Einstein frame, and using $\tilde{\mathcal{F}}_{m\bar{m}} = (u_m + \bar{u}_{\bar{m}})\tilde{\mathcal{F}}_{12}^{(m)}$, one then gets

$$\sum_a N_a \sqrt{\frac{\det(\mathbf{n}_a \cdot G)}{\det(G)^2}} e^{3\Phi} \text{tr} (G^{-1}(\mathcal{F}_a + \delta\mathcal{F}_a))^2 = \quad (64)$$

$$\frac{16}{s + \bar{s}} \sum_a N_a \prod_{m=1}^3 \mathbf{n}_a^{(m)} \left(\sum_{m=1}^3 \frac{(\mathcal{F}_a)_{m\bar{m}} + (\delta\mathcal{F}_a)_{m\bar{m}}}{(t_m + \bar{t}_{\bar{m}})(u_m + \bar{u}_{\bar{m}})} \right)^2 - 4g_{10}^2 \mathcal{T}_{\text{O5}} .$$

The extra term \mathcal{T}_{O5} is opposite equal to the sum of the tensions of all O5_m -planes and arises after applying the tadpole constraint Eq.(9). It therefore cancels out, as does the leading constant term in Eq.(30) with the tension of the O9 -planes. By going to the Cartan basis one can further put the terms involving $(\delta\mathcal{F}_a)_{m\bar{m}}$ into the form $\text{Tr}|C_m|^2$, $C_m = u_m A_{2m-1} + iA_{2m}$, a sum of absolute squares of charged complex scalars weighted by their charges. These C_m are the surviving scalars of

¹⁰This is actually the way supersymmetry gets broken if D-branes in the hidden obey different or no calibration condition compared to the visible branes [36, 14].

strings between the two stacks, after turning on the gauge flux to split the stack a into two. Since

$$2\Re(f_a) = (s + \bar{s}) + o(\mathcal{F}) , \quad \partial_a \mathcal{K} = \mathcal{K}_\alpha = \frac{\delta_{\alpha\bar{\beta}} \bar{C}_{\bar{\beta}}}{(t_m + \bar{t}_{\bar{m}})(u_m + \bar{u}_{\bar{m}})} + o(\mathcal{F}) , \quad (65)$$

this is just of the expected form

$$(D_a + \xi_a)^2 \sim \frac{1}{\Re(f_a)} (q_\alpha^a \mathcal{K}_\alpha C_\alpha + \xi_a)^2 , \quad (66)$$

to the given leading order in \mathcal{F}_a . The q_α^a are the charges of C_α under the $U(1)$ labelled by a , and we read off the FI-term

$$\xi_a = \sum_{m=1}^3 \frac{(\mathcal{F}_a)_{m\bar{m}}}{(t_m + \bar{t}_{\bar{m}})(u_m + \bar{u}_{\bar{m}})} . \quad (67)$$

The supersymmetry condition is

$$\xi_a = \sum_{m=1}^3 \frac{\mathbf{F}_a^{(m)}}{t_m + \bar{t}_{\bar{m}}} = 0 + o(\mathcal{F}_a) , \quad (68)$$

which is just Eq.(12) to leading order in \mathcal{F}_a . This condition is in fact the Donaldson-Uhlenbeck-Yau condition $G^{m\bar{m}} \mathcal{F}_{m\bar{m}} = 0$, well known for the heterotic string [93]. The full derivation of Eq.(12) from the D-term would evidently involve terms of order \mathcal{F}^4 and \mathcal{F}^6 . In terms of pure Yang-Mills theory this scenario has been studied as early as in [94, 95], in the context of identifying these scalars with Higgs fields in four dimensions. In the YM approximation, some components always get a negative squared mass, but in the full string spectrum, this does not have to happen, see e.g. [96].

The pure FI parameter ξ_a , however, can already be derived from the abelian DBI, and thus Eq.(12) can be recovered from an equation $\xi_a = 0$, and only the fluctuations of charged scalars require the non-abelian action. This was also discussed in [97] and is based on the exact conditions for κ -symmetry of the abelian DBI action, derived in [98]. The relevant constraints on $\tilde{\mathcal{F}}_a$ and the Kähler form

$$J = i \sum_{m=1}^3 \frac{\Re(T_m)}{\Re(U_m)} dz_m \wedge d\bar{z}_{\bar{m}} = \sum_{m=1}^3 \Re(T_m) dx_{2m-1} \wedge dx_{2m} \quad (69)$$

are

$$\frac{1}{2} J \wedge J \wedge \tilde{\mathcal{F}}_a - \frac{1}{6} \tilde{\mathcal{F}}_a \wedge \tilde{\mathcal{F}}_a \wedge \tilde{\mathcal{F}}_a = 0 , \quad (70)$$

for all a , and $\tilde{\mathcal{F}}_a$ has to be of type (1, 1). For the constant background, setting fluctuations to zero,

$$\mathcal{F}_a = \sum_{m=1}^3 \mathbf{F}_a^{(m)} dx_{2m-1} \wedge dx_{2m} , \quad (71)$$

and this reproduces Eq.(12).

It is also interesting to determine the fate of the imaginary parts of the complex scalar moduli fields s and t_m , which are components of the RR 2- and 6-form. The Chern-Simons part Eq.(35) of the brane action contains couplings $C_2 \wedge \tilde{\mathcal{F}}^4$ and $C_6 \wedge \tilde{\mathcal{F}}^2$ that reduce to linear axionic couplings of a_0 and a_m , defined above Eq.(15), to $\tilde{\mathcal{F}}$ in four dimensions [36]. Together with the kinetic terms $(da)^2$ of the axions, these couplings give Stückelberg masses to the respective gauge bosons, i.e. the kinetic terms after a duality transformation trading $\{C_2, C_2^{(m)}\}$ for $\{a_0, a_m\}$ turn into $(da + A)^2$ and the axion can be gauged away through $A \rightarrow A - da$, leaving a mass term for the gauge field A . Precisely this mechanism was used in [99] to propose a solution for the strong CP problem by promoting the QCD θ -parameter to a dynamical axion field that has additional Stückelberg couplings and can be gauged away then. Therefore, the axions as well decouple as longitudinal components of massive vectors from the effective action, together with their moduli partners, when gauge fluxes are turned on. An important point noticed in [29] is that the abelian gauge bosons may even decouple if the gauge symmetry is not anomalous. For an anomalous $U(1)_a$ the Green-Schwarz mechanism actually requires a second axionic coupling of the a to $\tilde{\mathcal{F}}_a^2$, which then allows the cancellation of triangle anomalies by axionic contributions. To see that the mechanism really works out properly, by which we mean, moduli and axions decouple in a one to one fashion, one has to verify that the axionic couplings come exactly in the same patterns that the D-terms arise for the $\Re(T_m)$. We define coupling constants [36]

$$\begin{aligned} c_a^{(0)} C_2 \wedge \delta \mathcal{F}_a &= C_2 \wedge \delta \mathcal{F}_a \int_{\mathcal{W}_a} \frac{1}{6} \mathcal{F}_a \wedge \mathcal{F}_a \wedge \mathcal{F}_a , \\ c_a^{(m)} C_2^{(m)} \wedge \delta \mathcal{F}_a &= C_2^{(m)} \wedge \delta \mathcal{F}_a \int_{\mathcal{W}_a} C_4^{(m)} \wedge \mathcal{F}_a^{(m)} d\xi_{2m-1} \wedge d\xi_{2m} . \end{aligned} \quad (72)$$

In other words, $c_a^{(0)}$ is the coupling of the universal axion, the partner of $s + \bar{s}$ and $c_a^{(m)}$ the coupling of the partner of $t_m + \bar{t}_m$. Since the number of stacks will be at least four or larger, the generic situation seems to be that all four axions have non-vanishing couplings to one linear combination of $U(1)_a$ gauge bosons. However, the calibration condition Eq.(12) implies that one can find a linear combinations of the four axions that decouples from the gauge fields. Setting the internal components of C_6 to 1, and defining $C'_2 = (s + \bar{s})C_2$, $C_2^{(m)'} = -(t_m + \bar{t}_m)C_2^{(m)}$, this “diagonal” axion is given by the direction $C'_2 = C_2^{(1)'} = C_2^{(2)'} = C_2^{(3)'}$, since then its couplings vanish,

$$\left(\frac{c_a^{(0)}}{\prod_{m=1}^3 \Re(T_m)} C'_2 - \sum_{m=1}^3 \frac{c_a^{(m)}}{\Re(T_m)} C_2^{(m)'} \right) \wedge \delta \mathcal{F}_a \Big|_{C'_2=C_2^{(m)'}} = 0 , \quad (73)$$

by Eq.(12) and for any a . Given the D-flatness, we thus have only three axions participating and the diagonal axion in Eq.(73) survives. The linear combination

of axions that decouples corresponds to the surviving modulus, due to rewriting Eq.(12) as

$$\sum_{m=1}^3 \frac{s + \bar{s}}{t_m + \bar{t}_m} \mathbf{F}_a^{(m)} = \prod_{m=1}^3 \mathbf{F}_a^{(m)}. \quad (74)$$

A variation along the complex direction $s = t_1 = t_2 = t_3$ leaves the D-flatness Eq.(74) intact as well as Eq.(73), identifying $\Im(s) = C_2$ and $\Im(t_m) = C_2^{(m)}$. It is evident that less generic cases, in which the D-flatness Eq.(12) leaves more moduli massless, one can also construct more linear combination of axions that survive together with the abelian gauge bosons.

The above is interesting for several reasons. First it implies that starting from a generic four stack model, we precisely expect one massless abelian gauge factor to survive, which is then the unique candidate for the hypercharge $U(1)_Y$ of the supersymmetric Standard Model. The other three $U(1)$ gauge symmetries decouple from the massless theory and survive as global symmetries only. It also follows that three among the seven phases we started with in parametrizing the values of the auxiliary fields F^I for $\{s, t_m, u_m\}$, are frozen. Technically, this unfortunately does not appear to be too helpful, since it does not imply, that any of the phases of $\{s, t_m\}$ actually vanish, but instead only that they are parametrized through a single phase along Eq.(73), but all four being non-vanishing. Only in the situation when all phases are aligned, the soft-breaking parameters and the patterns for CP violation will simplify considerably. This situation may arise if the scale of supersymmetry breaking is well below the scale where the relative moduli among s, t_m get fixed. Then the four complex scalars and auxiliary fields would effectively already be aligned and the phases of the respective auxiliary fields in the chiral multiplets would be equal.

4 Soft supersymmetry breaking

We now straightforwardly apply the formulas for computing the soft parameters using the expressions given in the previous section. First we address the vacuum energy, which we have to assume to take a very small or even vanishing value, of course. Note that this in $\mathbb{Z}_2 \times \mathbb{Z}_2$ orbifolds this is actually satisfied for any model, where the moduli part of the superpotential depends only on either complex structure moduli u_m or Kähler moduli t_m . Then the vacuum energy is non-negative, $V_0 \geq 0$, since the potential is of the classical no-scale type [100, 101]. In the first case, which would be realized, as long as the effects that drive the breaking are perturbative in the string coupling, the contribution of the t_m moduli to the

potential cancel the negative term through

$$\sum_{m=1}^3 \hat{\mathcal{K}}^{t_m \bar{t}_m} D_{t_m} \hat{W}(u_m) \bar{D}_{\bar{t}_m} \bar{\hat{W}}(u_m) = 3|\hat{W}|^2 . \quad (75)$$

The (flat Minkowski) vacuum is then supersymmetric whenever all other contributions to the potential exactly vanish. In models with less than three Kähler moduli, the situation would possibly look different. The case of gaugino condensation in a hidden sector, as considered in [46], is of the opposite type. In the absence of perturbative contributions, the effective superpotential is depending on the moduli only through the gauge kinetic functions $f_a(s, t_m)$. In that case, the contribution of the generic three u_m moduli can cancel the negative term, just as in Eq.(75) upon substituting $t_m \leftrightarrow u_m$.

The models we are considering have at most seven complex scalar moduli fields above the scale where the constraint Eq.(11) fixes up to three among $\{s, t_m\}$, so that at least one combination of these and three u_m survive. At low energies there will then be at least four free real parameters and phases for the values of the respective auxiliary fields, determining the patterns of supersymmetry breaking. In the formulas we now always use the maximal number of seven moduli. To proceed further we parametrize the auxiliary fields in a standard fashion (see e.g. [67]),

$$\begin{aligned} F^s &= c(s + \bar{s}) \sin(\theta) e^{-i\gamma_s} , \\ F^{t_m} &= c(t_m + \bar{t}_m) \cos(\theta) \Theta_{t_m} e^{-i\gamma_{t_m}} , \quad F^{u_m} = c(u_m + \bar{u}_m) \cos(\theta) \Theta_{u_m} e^{-i\gamma_{u_m}} \end{aligned} \quad (76)$$

with

$$c = C\sqrt{3}M_{3/2} , \quad C^2 = 1 + \frac{V_0}{3M_{3/2}^2} , \quad \sum_{m=1}^3 (|\Theta_{t_m}|^2 + |\Theta_{u_m}|^2) = 1 . \quad (77)$$

We further define [78]

$$\begin{aligned} D &= -\ln(s + \bar{s}) , \quad e^{-i\rho} = \frac{\langle \hat{W} \rangle}{|\langle \hat{W} \rangle|} , \\ f &= \prod_{m=1}^3 (t_m + \bar{t}_m) \prod_{m=1}^3 (u_m + \bar{u}_m) . \end{aligned} \quad (78)$$

4.1 The soft breaking parameters

For the gaugino masses Eq.(20) we now obtain

$$M_a = \frac{c}{2\Re(f_a)} \left(\sin(\theta) e^{-i\gamma_s} (s + \bar{s}) \prod_{m=1}^3 \mathbf{n}_a^{(m)} \right) \quad (79)$$

$$-\frac{1}{2} \sum_{m,n,p=1}^3 \gamma_{mnp} \cos(\theta) \Theta_{t_m} e^{-i\gamma_{t_m}} (t_m + \bar{t}_{\bar{m}}) \mathbf{n}_a^{(m)} \mathbf{m}_a^{(n)} \mathbf{m}_a^{(p)} .$$

Whenever just a single term in the bracket survives, i.e. for D9- or D5-branes, the dependence of M_a on the moduli drops out, but in all other cases the gaugino masses will depend on (the real parts of) s and the Kähler parameters t_m . For the scalar mass parameters of extra matter fields in symmetric and anti-symmetric representations we get

$$\begin{aligned} M_{m\bar{m}}^{[aa]2} &= M_{3/2}^2 + V_0 - F^I \bar{F}^{\bar{J}} \partial_I \partial_{\bar{J}} \ln \left(\tilde{\mathcal{K}}_{m\bar{m}}^{[aa]} \right) \\ &= M_{3/2}^2 + V_0 + c^2 \cos^2(\theta) |\Theta_{u_m}|^2 + \Gamma_{m\bar{m}}^{[aa]} , \end{aligned} \quad (80)$$

with

$$\begin{aligned} \Gamma_{m\bar{m}}^{[aa]} &= 4|M_a|^2 - \frac{c^2}{(1 + \Delta_a^{(m)})^2} \times \\ &\times \left(\left| \sin(\theta) e^{-i\gamma_s} + \cos(\theta) \Theta_{t_m} e^{-i\gamma_{t_m}} + \frac{1}{2} \cos(\theta) \sum_{n,p} \Delta_a^{(m|np)} (\Theta_{t_n} e^{-i\gamma_{t_n}} + \Theta_{t_p} e^{-i\gamma_{t_p}}) \right|^2 \right. \\ &\quad \left. - 2(1 + \Delta_a^{(m)}) \left(\sin(\theta) \cos(\theta) \Theta_{t_m} \cos(\gamma_s - \gamma_{t_m}) \right. \right. \\ &\quad \left. \left. + \cos^2(\theta) \sum_{n,p} \Delta_a^{(m|np)} \Theta_{t_n} \Theta_{t_p} \cos(\gamma_{t_n} - \gamma_{t_p}) \right) \right) . \end{aligned} \quad (81)$$

We note that one hallmark of the case of general background world volume flux is the appearance of cross terms with $\Theta_{t_n} \Theta_{t_p}$, as they are absent for only D9- and D5-branes. For completeness, we state the trilinear coupling parameters for the $C_m^{[aa]}$ fields,

$$\begin{aligned} A_{123}^0 - \bar{c} F^I \partial_I \ln(Y_{123}) &= \\ &= -c \frac{e^{-i\rho + \frac{D}{2}}}{\sqrt{f}} \left(\sum_{m=1}^3 \frac{\Delta_a^{(m)}}{1 + \Delta_a^{(m)}} (\sin(\theta) e^{-i\gamma_s} + \cos(\theta) \Theta_{t_m} e^{-i\gamma_{t_m}}) \right. \\ &\quad \left. + \frac{6}{c} M_a - 2 \sin(\theta) e^{-i\gamma_s} - \cos(\theta) \sum_{m,n,p=1}^3 \frac{\Delta_a^{(m|np)}}{1 + \Delta_a^{(m)}} (\Theta_{t_n} e^{-i\gamma_{t_n}} + \Theta_{t_p} e^{-i\gamma_{t_p}}) \right) . \end{aligned} \quad (82)$$

The most interesting matter sector is the one which involves the bifundamentals, the squarks and sleptons. Here for $M^{[ab]2}$ we find

$$\begin{aligned} M^{[ab]2} &= M_{3/2}^2 + V_0 - c^2 \sin^2(\theta) \left(1 - \frac{\nu_{ab}}{2} \right) \\ &\quad - c^2 \cos^2(\theta) \sum_{m=1}^3 \nu_{ab}^{(m)} \left(\frac{1}{2} - |\Theta_{t_m}|^2 - |\Theta_{u_m}|^2 \right) \end{aligned} \quad (83)$$

From this expression one can derive simple sum-rules. In a rather isotropic case, when all $\nu_{ab}^{(m)} \sim \nu_{ab}/3$ one finds e.g.

$$M^{[ab]2} \sim M_{3/2}^2 + V_0 - c^2 \left(1 - \frac{\nu_{ab}}{2} - \cos^2(\theta) \left(1 + \frac{\nu_{ab}}{3} \right) \right). \quad (84)$$

While the general expression for the scalar masses is already independent of the phases of the F^I , this formula does only involve the single angle parameter θ which distinguishes between the popular scenarios of dilaton or moduli domination. For the trilinear couplings involving three bifundamentals we get

$$A_{abc}^0 - \bar{c} F^I \partial_I \ln(Y_{abc}) = c \frac{e^{-i\rho + \frac{D}{2}}}{\sqrt{f}} \left(\left(2 - \frac{\nu_{ab} + \nu_{bc} + \nu_{ca}}{2} \right) \sin(\theta) e^{-i\gamma_s} \right. \\ \left. - \cos(\theta) \sum_{m=1}^3 \left(1 + \nu_{ab}^{(m)} + \nu_{bc}^{(m)} + \nu_{ca}^{(m)} - \frac{\nu_{ab} + \nu_{bc} + \nu_{ca}}{2} \right) (\Theta_{t_m} e^{-i\gamma_{t_m}} + \Theta_{u_m} e^{-i\gamma_{u_m}}) \right). \quad (85)$$

When again the $\nu_{ab}^{(m)}$ etc. have all equal elements $\nu_{ab}/3 = \nu_{bc}/3 = \nu_{ca}/3$ the relation of Eq.(85) simplifies so that

$$A_{abc}^0 - \bar{c} F^I \partial_I \ln(Y_{abc}) \sim c \frac{e^{-i\rho + \frac{D}{2}}}{\sqrt{f}} \left(\left(2 - \frac{3\nu_{ab}}{2} \right) \sin(\theta) e^{-i\gamma_s} \right. \\ \left. - \cos(\theta) \left(1 - \frac{\nu_{ab}}{2} \right) \sum_{m=1}^3 (\Theta_{t_m} e^{-i\gamma_{t_m}} + \Theta_{u_m} e^{-i\gamma_{u_m}}) \right) \quad (86)$$

We also record here the trilinear couplings involving only two bifundamentals,

$$A_{abm}^0 - \bar{c} F^I \partial_I \ln(Y_{abc}) = c \frac{e^{-i\rho + \frac{D}{2}}}{\sqrt{f}} \left(-\frac{1}{c} M_a + \sin \theta e^{-i\gamma_s} (2 - \nu_{ab}) \right. \\ \left. + \cos(\theta) \left(\Theta_{t_m} e^{-i\gamma_{t_m}} + \Theta_{u_m} e^{-i\gamma_{u_m}} + \sum_{n=1}^3 (-1 - 2\nu_{ab}^{(n)} + \nu_{ab}) (\Theta_{t_n} e^{-i\gamma_{t_n}} + \Theta_{u_n} e^{-i\gamma_{u_n}}) \right) \right. \\ \left. - \frac{\Delta_a^{(m)}}{1 + \Delta_a^{(m)}} (\sin(\theta) e^{-i\gamma_s} + \cos(\theta) \Theta_{t_m} e^{-i\gamma_{t_m}}) \right. \\ \left. + \frac{\cos(\theta)}{2} \sum_{n,p} \frac{\Delta_a^{(m|np)}}{1 + \Delta_a^{(m)}} (\Theta_{t_m} e^{-i\gamma_{t_m}} + \Theta_{u_m} e^{-i\gamma_{u_m}}) \right) \quad (87)$$

Finally, we compute the parameter relevant for the Higgs bilinear term $B_{ab}^0 = B_{[ab][ba]}^0$, which refers to a sector of open strings between two given branes $[ab]$.

$$B_{ab}^0 = c \frac{e^{-i\rho + \frac{D}{2}}}{\sqrt{f}} \left(\sin(\theta) e^{-i\gamma_s} (1 - \nu_{ab} + (s + \bar{s}) \partial_s \ln(\mu_{ab})) \right) \quad (88)$$

$$\begin{aligned}
& -\cos(\theta) \sum_{m=1}^3 \Theta_{t_m} e^{-i\gamma t_m} \left(1 + 2\nu_{ab}^{(m)} - \nu_{ab} - (t_m + \bar{t}_{\bar{m}}) \partial_{t_m} \ln(\mu_{ab}) \right) \\
& -\cos(\theta) \sum_{m=1}^3 \Theta_{u_m} e^{-i\gamma u_m} \left(1 + 2\nu_{ab}^{(m)} - \nu_{ab} - (u_m + \bar{u}_{\bar{m}}) \partial_{u_m} \ln(\mu_{ab}) \right) \Big) .
\end{aligned}$$

When μ_{ab} is independent of the moduli, as we have argued above may be natural, the terms proportional to derivatives drop out. An application of these general formulae to a specific model is given in appendix A.

4.2 Simplifications for $s = t_1 = t_2 = t_3$

Since it appears possible that the supersymmetry constraints Eq.(11) fix the four mentioned complex moduli fields to be effectively aligned, and also the auxiliary fields in their respective chiral multiplets, we briefly investigate the simplifications that arise when replacing the four fields by a single one. Of course, we do not mean that the vacuum expectation values of s and t_m are equal, but just that the surviving modulus points into the diagonal direction, which we call λ . This is perhaps a very naive way to implement the integrating out of the relative coordinates among the s, t_m by just substituting λ , but the analysis here may give a rough idea of what can happen in this situation. The auxiliary field is $F^\lambda = c(\lambda + \bar{\lambda})\Theta_\lambda e^{-i\gamma\lambda}$. One can then observe that

$$\Delta_a^{(m)} \propto (\mathbf{F}_a^{(m)})^2, \quad (89)$$

independent of the remaining moduli. It also follows immediately that the gaugino masses are completely determined by the value of F^λ ,

$$M_a = \frac{1}{2\Re(f_a)} F^\lambda \partial_\lambda f_a = c\Theta_\lambda e^{-i\gamma\lambda} \quad (90)$$

and all explicit dependence on the world volume gauge fields drops out. This follows already from $f_a = f_a(\lambda)$ only depending linearly and holomorphically on λ , but no u_m . Thus the gaugino masses are approximately universal. The simplifications for the other parameters can in fact also be quite dramatic. The scalar masses in the $[aa]$ sectors simplify so that

$$M_{m\bar{m}}^{[aa]2} = M_{3/2}^2 + V_0 + c^2 \cos^2(\theta) |\Theta_{u_m}|^2 + 2c^2 |\Theta_\lambda|^2, \quad (91)$$

still depending on the F^{u_m} , but universal in a , and therefore independent of \mathcal{F}_a . An interesting point is that the cross terms in Eq.(81) drop out in this simpler case. In the same way their trilinear couplings A_{123}^0 become independent of the stack of branes, and one would predict a set of extra matter at some universal mass in anti-symmetric and symmetric representations of the gauge group for this

simple scenario. It is very easy to get the specialized formulas for the squarks and sleptons as well. These, fortunately, do not lose their dependence on the type $[ab]$ of intersection, where the fields are localized, which enters through the $\nu_{ab}^{(m)}$, and they still depend on F^{um} , such that no qualitative reduction of the complexity of possible solutions takes place. The number of independent parameters is of course reduced by three.

5 Gauge unification, FCNC, EDM, Dark matter

In this section we discuss a number of phenomena within intersecting brane models which are of interest in model building in particle theory. Specifically we discuss issues regarding gauge coupling unification, flavor changing neutral currents, CP violation, the electric dipole moments, and dark matter. Another interesting application of the intersecting brane model concerns proton decay via dimension six operators [90]. However, this topic will not be discussed here. A more systematic analysis and a deeper investigation of these topics is left to a future work.

5.1 Gauge coupling unification

The unification of gauge couplings using standard renormalization group running is one of the facts about the minimal extension of the Standard Model, that is usually considered among its most attractive features. Starting from LEP data and using the particle spectrum of the MSSM the couplings meet at $M_{\text{GUT}} = 2 \times 10^{16} \text{GeV}$. A D-brane model derived from string theory, that tries to construct the MSSM already at a high scale, without any extra matter or gauge group, will have to be able to reproduce this unification pattern, since otherwise it would lead to the wrong couplings at low energies, just turning the evolution of couplings around. So even if we do not expect a unification of the gauge group, e.g. an enhancement towards $SU(5)$ or $SO(10)$ at the GUT scale, since we are still dealing with separate stacks of D-branes then, we have to worry about unification as an accidental property of our models. This argument does, however, not apply in practice so far, since all examples of models known involve extra sectors with additional matter and gauge factors. Their dynamics at intermediate scales would substantially affect the running of the couplings, and thus the apparent unification of couplings may be an illusion and spoiled by these effects.

In any case, we now investigate the minimal requirements that would arise from imposing gauge unification (see [42, 47, 48]). What is meant as an attempt to further constrain the models and increase their predictive power will turn out to lead to a system of overconstraining conditions in a sufficiently generic case. Using

$SU(5)$ conventions, the unification actually reads

$$\frac{1}{g_3^2(M_{\text{GUT}})} = \frac{1}{g_2^2(M_{\text{GUT}})} = \frac{5}{3} \frac{1}{g_Y^2(M_{\text{GUT}})}, \quad (92)$$

where the g_i denote the couplings of $SU(3)$, $SU(2)$ and $U(1)_Y$. The latter will actually be some linear combinations $\sum_a c_a U(1)_a$ of the $U(1)_a$ that are present in the total model, one per stack of branes. The starting point of [42] is that Eq.(92) is compatible with the condition that arises by embedding the hypercharge $U(1)$ into the original abelian gauge symmetries that live on the $U(3) \times U(2) \times U(1)^n$ stacks to start with.

One may now distinguish three cases. *i)* This is the apparently most generic case. There is at least one stack of branes such that all three $\mathbf{F}_a^{(m)}$ are non-vanishing, which does not even have to be a stack in the visible sector of the model. Then Eq.(12) will fix the overall volume of the internal space and, if not very special accidents appear, also all ratios $\Re(T_1) : \Re(T_2) : \Re(T_3)$, i.e. fix $\{s + \bar{s}, t_m + \bar{t}_m\}$ except for a simultaneous rescaling of all four. Now Eq.(92) means

$$\Re(f_3(s, t_m)) = \Re(f_2(s, t_m)) = \frac{5}{3} \sum_a c_a \Re(f_a(s, t_m)). \quad (93)$$

Since the $f_a(s, t_m)$ are linear functions in $\{s, t_m\}$ an overall rescaling does not affect this relation. Therefore, the D-flatness Eq.(12) already fixes all freedom in the relevant parameters that could be used to unify the gauge couplings, and Eq.(92) would be very accidental. *ii)* All stacks have at least one m for which $\mathbf{F}_a^{(m)} = 0$. Though this sounds less generic, it is actually the case in all the examples to be found in the literature. The reason is probably just that the practical search for models is very much simplified, if Eq.(12) can be turned into a linear relation. But we do not see any reason why this situation should be favored on general grounds. Now s drops out of Eq.(12) and the overall volume remains undetermined, while the ratios $\Re(T_1) : \Re(T_2) : \Re(T_3)$ are fixed as long as no further relations among the conditions exist. This situation provides one single free parameter in Eq.(92), not enough to satisfy two relations. Unification would again be unnatural, and is not achieved in the cases considered in [13]. *iii)* If in case *ii)* there are further relations among the conditions Eq.(12) then two parameters may survive, and only one ratio of $\Re(T_m)$ is fixed. This is actually the case considered in [42], and applies to some of the simpler examples given in the literature. Imposing the extra constraint Eq.(92) is then just enough to fix the remaining two Kähler parameters. Thus, only in the most symmetric case, unification appears possible generically.

If a class of examples of the type *i)* were found that produced precisely the MSSM spectrum, imposing the extra constraint Ref.(92) could then maybe point

toward the “true” solution. Otherwise, one should look for more compelling reasons to consider special examples where some flux quantum numbers vanish and simplifying symmetries exist. As a final remark, note that the difficulty in obtaining a unification of gauge couplings in the present class of D-brane models is about the opposite of the situation in the heterotic string, where grand unification is somehow automatic.

5.2 Flavor changing neutral currents

In a general situation of soft supersymmetry breaking the parameters in the soft breaking terms of the effective Lagrangian can be arbitrary for different generations of matter multiplets. This kind of anisotropy leads to phenomenological problems, since then interferences of these fields no longer cancel out. A prominent example of such effects are flavor changing neutral currents that for instance lead to unacceptable rates for transitions which contribute to the same process as the $\Delta S = 2$ box diagram that allows for CP violation in the K-system (see [102, 103, 44]¹¹). The absence of such transitions puts bounds on the differences of the masses of squark and slepton doublets, but in principle not the anti-squark singlets. In the present setting of brane models however, all three generations are generically on a very symmetric footing. They arise from multiple intersections of the same two stacks of branes, and therefore the relevant Kähler metrics are equal. The only difference could arise in their Yukawa couplings $Y_{\alpha\beta\gamma}$, which would only lead to mass differences comparable to those of the quarks and leptons of the Standard Model itself. The only exception to this is indeed the interesting case of the quark doublets, where in a large variety of models the three generations split into two from one set of intersections and a third from an extra intersection with potentially different relative angles and Kähler metric. The reason for this to happen is the fact that this allows to circumvent a problem related to the anomaly cancellation within the $U(2)$, which demands the number of positively and negatively charged doublets to be equal. This can be arranged if the two $(\mathbf{3}, \mathbf{2})$ carry charge plus, the one extra $(\mathbf{3}, \mathbf{2})$ charge minus, and the three lepton doublets $(\mathbf{1}, \mathbf{2})$ also minus [29].

For the FCNC constraints to be satisfied for the first two generations, the squarks have to be essentially degenerate with mass differences of the order of the charm quark mass. The constraints on the third generation squark masses on the Higgs masses consistent with FCNC constraints are far less stringent. Thus if we label the first two generation squark masses by $M_q^{[ab]}$ and the third generation squark masses by $M_q^{[cd]}$ then the condition $M_q^{[cd]2} = (1 + \delta)M_q^{[ab]2}$ is consistent with FCNC constraints with $|\delta| \leq 1$ [105, 106]. The above translates to the following

¹¹A completely different source for FCNC was discussed in [104] in the context of brane models with supersymmetry breaking at a low string scale, where the massive fields can contribute as well.

conditions on the intersection angles

$$\delta \left(\frac{1}{3} + \frac{1}{2} \sin^2(\theta)(\nu_{cd} - 2) - \cos^2(\theta) \sum_{m=1}^3 \nu_{cd}^{(m)} \left(\frac{1}{2} - |\Theta_{t_m}|^2 - |\Theta_{u_m}|^2 \right) \right) = \quad (94)$$

$$\frac{1}{2} \sin^2(\theta)(\nu_{cd} - \nu_{ab}) + \cos^2(\theta) \sum_{m=1}^3 (\nu_{cd}^{(m)} - \nu_{ab}^{(m)}) \left(\frac{1}{2} - |\Theta_{t_m}|^2 - |\Theta_{u_m}|^2 \right) ,$$

where we have set $V_0 = 0$ in writing the above constraint. Now Eq.(94) imposes only mild constraints on the intersection angles for the third generation compared to the the first generation. Depending on the breaking scenario, they can become sharpened. As an example, complete dilaton domination with $\cos(\theta) = 0$ would put rather stringent bounds on ν_{ab} and ν_{cd} . One also has similar FCNC constraints for the Higgs doublet masses at the string scale. Thus replacing $M_q^{[cd]}$ with $M_H^{[ef]}$, where $M_H^{[ef]}$ is the Higgs mass for the generations localized at the intersection of $[ef]$, then Eq.(94) holds again with δ replaced by δ_H , where FCNC constraints are consistent with $|\delta_H| \leq 1$. Again the constraints on the intersection angles in this case are rather mild. The anisotropy between the first two and the third generation could only become dangerous if experimental bounds were sharpened, comparable to the first two generations, which are completely degenerate in the brane models at hand.

5.3 CP-violation

The soft breaking sector of the intersecting brane models contains seven phases, i.e. $\gamma_s, \gamma_{t_m}, \gamma_{u_m}$ ($m = 1, 2, 3$). The typical size of these phases is $\text{o}(1)$ and they produce large effects on the electric dipole moments (EDMs) of the electron and of the neutron (complete expressions for these can be found in [107, 108]). Thus the moduli phase generated EDMs may in general exceed the very sensitive experimental bounds on the electric dipole moments of the electron, the neutron and of the Hg-atom [109, 110, 111]. These models can be made compatible with experiment either via mass suppression [112] or via the cancellation mechanism [113, 114]. The phases can affect a large number of phenomena accessible to experiment (for a recent review see [115]). These include sparticle decays, $g_\mu - 2$, proton decay, $B_{(s,d)}^0 \rightarrow l + l^-$, and baryogenesis [116], to name a few. There is one case in which drastic simplification occurs on the dependence on the phases. If all phases are equal,

$$\gamma_s = \gamma_{t_1} = \gamma_{t_2} = \gamma_{t_3} \equiv \gamma \quad (95)$$

one finds that the common phase factors out of the gaugino masses and in view of the discussion of sec 4.2 one may write

$$M_a = M_{1/2} e^{-i\gamma} . \quad (96)$$

Here the gaugino masses are universal independent of the brane stack. Further, the common phase can be rotated away from the gaugino sector by redefinition of fields although it will appear in other sectors of the theory. Thus the only phases that remain in this case are γ and γ_{u_m} . An important phase that enters in physical processes is the μ -phase θ_μ , the phase of the term $\mu_{\alpha\beta}$ in the superpotential Eq.(16). In principle it is determined in terms of the fundamental moduli phases, but we have not specified the μ -term here, in case it is non-vanishing. If one considers the subset of intersecting brane models where θ_μ vanishes or is very small (i.e. of order 10^{-2} , which may be natural, since we have argued that it may be vanishing to leading order), then the only avenue for the phases to enter the physical processes is via the trilinear parameter. Further, an arrangement that the trilinear parameters for the first two generations are relatively real while the third generation A^0 is complex will satisfy the experimental EDM limits for a significantly large A^0 phase[117]. This scenario has some interesting features. Thus, for example, there will be no CP-phase dependent contribution from the dominant chargino-sneutrino contribution to $g_\mu - 2$, because the chargino mass is independent of the phases in this case and so is the sneutrino mass. However, the phase of the third generation A^0 can still produce large effects. Such effects will be visible in the decay of the stops once they are produced at the Fermilab Tevatron and at the Large Hadron Collider at CERN.

5.4 Dark matter

The recent data from WMAP [118] indicates that there is a significant cold dark matter component to the dark matter- dark energy in the universe. Like SUGRA models, the intersecting brane models with R-parity constraint have the possibility that the lowest mass neutralino could be the lowest mass supersymmetric particle (LSP) and a candidate for cold dark matter. Recent analyses of the WMAP data indicate that the accurate WMAP results produce a strong correlation between the sfermion and gaugino masses and the allowed mass range can extend to even tens of TeV [119] (for a review see [120]). Previous analyses indicate that brane models can indeed allow for the desired amount of dark matter [121]. Since the pattern of soft breaking in the intersecting brane models is essentially a modification of that for the parallel brane case, one expects that the intersecting brane model will sustain dark matter in sufficient amounts to be compatible with the WMAP data. As in the case of heterotic string [78] the constraints of radiative breaking of the electroweak symmetry in the intersecting brane model will also determine $\tan(\beta)$. In this case the analysis of dark matter would be much more predictive compared to the SUGRA models. It would in fact be interesting to investigate this possibility in further detail for the case of intersecting brane models.

6 Conclusions and prospects

In this work we have investigated the effective action and soft breaking in a generic class of intersecting brane models with $\mathcal{N} = 1$ supersymmetry. There are in general two equivalent approaches to such constructions, i.e. models based on intersecting D6-branes in type IIA or alternatively to use constructions with D9-branes with magnetic flux on their world volume in type IIB. The type IIB framework is found more convenient to construct the Kähler metric, the gauge kinetic function and the superpotential, and we have followed this approach in the present work. Thus one of the main results of this paper are Eq.(47) and Eq.(57) which give the Kähler metric for chiral matter in the intersecting brane case and reduce correctly in certain limits to the appropriate expressions for the parallel brane case. The effective potential constructed from these Kähler metrics is then used to derive soft breaking under the standard assumptions of a hidden sector breaking used in supergravity models. The soft breaking results obtained are in general valid for a wide class of models since they are given in terms of the general attributes that would characterize the chiral matter. Remarkably the entire soft breaking in the sector that involves the bifundamental fields can be characterised in terms of indices which are the intersection angles measured in units of π and are the analogue of the twist vectors in the heterotic string case. These results reduce correctly in certain limits to the soft breaking for the parallel brane case. A weakness of the methods used clearly is the lack of a direct derivation of Eq.(57) from first principles, as well as the complete neglect of twisted moduli, that may actually involve important physics.

We have also analyzed in this paper the question of gauge coupling unification and its interconnection with the constraints that preserve $\mathcal{N} = 1$ supersymmetry. The analysis indicates that while gauge coupling unification is not generic in intersecting brane models, there are regions of the moduli space where it is possible. Other phenomenological implications of these models were also explored. Specifically we discussed the constraints on the brane intersection angles from flavor changing neutral currents and the implications of the CP violating phases that arise quite naturally in the soft breaking sector. It is argued that such models have the potential to satisfy the EDM constraints and at the same time allow phases which are sufficiently large to generate visible effects in phenomena at colliders and also provide sufficient new sources of CP violation in baryogenesis. Similarly, the pattern of soft breaking indicates that such models with R-parity will allow for dark matter in sufficient amounts to satisfy the current astrophysical constraints on cold dark matter. These and other issues deserve further study.

Acknowledgements

B. K. would like to thank Marcus Berg, Dieter Lüst, Stephan Stieberger, and

Jan Troost for helpful advice and stimulating discussions. The work of B. K. was supported by the German Science Foundation (DFG) and in part by funds provided by the U.S. Department of Energy (D.O.E.) under cooperative research agreement #DF-FC02-94ER40818. Part of this work was done when P. N. was visiting the Max Planck Institute fur Kernphysik, Heidelberg, Germany, and he thanks the Max Planck Institute for hospitality and the Alexander von Humboldt Foundation for support during this visit. The work of P. N. was also supported in part by the U.S. National Science Foundation under the grant NSF-PHY-0139967

A An application of the soft breaking analysis

We have included this appendix to demonstrate how to compute some of the relevant parameters in practice and show how the procedure of converting relative angles into shift vectors works. The example also shows that many problems can arise in the details of any given model. We just choose to look at one of the first supersymmetric models constructed in the literature, a four generation Standard-like Model given in [13]. The set of brane stacks is defined by the following table.

Sector $[ab]$	N_a	$(\mathbf{n}_a^{(m)}, \mathbf{m}_a^{(m)})$
A_1	6+2	(1,1) (1,-2)(1,0)
A_2	2	(-1,0) (-1,0)(1,0)
B_1	4	(1,0) (1,2)(1,-1)
B_2	8	(1,0) (0,1)(0,-1)
C_1	2	(1,2) (1,0)(1,-2)
C_2	8	(0,1) (1,0)(0,-1)

Table 1

The constraints of supersymmetry require in this case the relation $\Re(T_1) : \Re(T_2) : \Re(T_3) = 1 : 2 : 1$. Compared to [13] we have flipped two of the orientations of the stack denoted A_2 to achieve that all angles $\varphi_a^{(m)}$ add up to 2π for any stack a , according to our conventions. All other stacks produce angles of the kind $(\alpha, 2\pi - \alpha, 0)$, or permutations thereof, for some values of α . The spectrum, gauge group and ideas on the physical relevance of this model can be found in the original works. The model, as all examples in the literature, does not fix all three Kähler parameters $\Re(T_m)$, since in Eq.(12) the product of fluxes always vanishes for all brane stacks, as one of the winding numbers is always zero. Therefore, we can only parametrize the angle variables in terms of the one remaining modulus, which we call $\chi = 1/\Re(T_1)$. This is somehow against our general philosophy, since we neglect the implicit dependence of the shift vectors on the moduli in the soft parameters, but cannot be avoided in the absence of more general models, where Eq.(12) fixes all moduli. Now we apply Eq.(53) to compute the shift vectors, which gives Table 2.

Sector $[ab]$	$\nu_{ab}^{(m)}$	ν_{ab}	Fields
$\nu_{A_1 B_1}^{(m)}$	$(\alpha, 2\alpha, \alpha)$	4α	Q_L, L
$\nu_{A_1 B_2}^{(m)}$	$(\alpha, \frac{1}{2} + \alpha, \frac{1}{2})$	$1 + 2\alpha$	$\bar{U}, \bar{D}, \bar{\nu}, \bar{E}$
$\nu_{A_1 C_2}^{(m)}$	$(\frac{1}{2} - \alpha, \alpha, \frac{1}{2})$	1	U, D, ν, E
$\nu_{B_1 C_2}^{(m)}$	$(\frac{1}{2}, \alpha, \frac{1}{2} - \alpha)$	1	H_U, H_D
$\nu_{B_2 C_1}^{(m)}$	$(\alpha', \frac{1}{2}, \frac{1}{2} - \alpha')$	1	S_U, S_D

Table 2

where S_U, S_D are $SU(2)$ singlets and where α, α' are defined by

$$\begin{aligned}\alpha &= \frac{1}{\pi} \arccos \left(\frac{1}{\sqrt{1 + \chi^2}} \right), \quad 0 \leq \alpha \leq \frac{1}{2}, \\ \alpha' &= \frac{1}{\pi} \arccos \left(\frac{1}{\sqrt{1 + 4\chi^2}} \right), \quad 0 \leq \alpha' \leq \frac{1}{2}.\end{aligned}\tag{97}$$

In table 2 we see some non-perturbative sectors ($\nu_{ab} = 1$) and two ‘‘interpolating’’ sectors. We notice that this model contains precisely the D9-D5₁-D5₂ system discussed in [61] and given in Eq.(50)¹² in the set of branes A_2, B_2, C_2 . Again blindly applying Eq.(53) and get,

Sector $[ab]$	$\nu_{ab}^{(m)}$	ν_{ab}
$\nu_{A_2 B_2}^{(m)}$	(1, 1/2, 1/2)	2
$\nu_{A_2 C_2}^{(m)}$	(1/2, 1, 1/2)	2
$\nu_{B_2 C_2}^{(m)}$	(1/2, 1/2, 0)	1

Table 3

exactly reproducing what was given in Eq.(50), via Eq.(57). However, one may note that the procedure is only unique once the winding quantum numbers ($\mathbf{n}_a, \mathbf{m}_a$) have been specified, and switching the orientations differently on the A_2 brane would have produced slightly different results. With the above analysis at hand we can implement our soft breaking formulae for the 4 generation model. Using the tables above and the general relations derived in section 4.1 we get

$$\begin{aligned}M_{Q_L}^2 &= c^2 \left(\frac{1}{3} - \sin^2(\theta) - 2\alpha \cos(2\theta) + \alpha \cos^2(\theta)[1 + F_2] \right), \\ M_{\bar{U}, \bar{D}}^2 &= c^2 \left(-\frac{1}{6} - \alpha \cos(2\theta) + \cos^2(\theta) \left[\frac{1}{2} + \left(\alpha - \frac{1}{2} \right) F_1 + \alpha F_2 \right] \right), \\ M_{U, D}^2 &= c^2 \left(-\frac{1}{6} + \frac{1}{2} \cos^2(\theta) + \cos^2(\theta) \left[-\alpha F_1 + \left(\alpha - \frac{1}{2} \right) F_2 \right] \right), \\ M_{H_U, H_D}^2 &= c^2 \left(-\frac{1}{6} + \left(\frac{1}{2} - \alpha \right) \cos^2(\theta) + \cos^2(\theta) \left[\alpha F_1 + \left(-\frac{1}{2} + 2\alpha \right) F_2 \right] \right), \\ M_{S_U, S_D}^2 &= c^2 \left(-\frac{1}{6} + \left(\frac{1}{2} - \alpha' \right) \cos^2(\theta) + \cos^2(\theta) \left[\left(-\frac{1}{2} + 2\alpha' \right) F_1 + \alpha' F_2 \right] \right),\end{aligned}\tag{98}$$

where

$$\sum_{i=1}^3 F_i = 1, \quad F_i = |\Theta_{t_i}|^2 + |\Theta_{u_i}|^2, \quad i = 1, 2, 3$$

¹²The D5₃ brane would need to be defined with ((0, 1), (0, 1), (-1, 0)).

and

$$M_{Q_L}^2 = M_L^2, \quad M_{U,D}^2 = M_{\nu,E}^2, \quad M_{\bar{U},\bar{D}}^2 = M_{\bar{\nu},\bar{E}}^2.$$

Next we focus on the trilinear couplings. We notice that the sectors that “connect” and have non-vanishing tree level Yukawa couplings are $[A_1 B_1, B_1 C_2, C_2 A_1]$. Thus only the two generations of right handed quarks (leptons) in the $A_1 C_2$ sector couple and the other two generations of right handed quarks (leptons) in the sector $A_1 B_2$ do not couple with the left handed quarks (leptons) and the Higgs. Consequently only two generations of quarks (leptons) can gain mass by the Higgs phenomenon in this model. In the sector where the quark (lepton) mass growth can occur the trilinear couplings are

$$A_{[A_1 B_1 C_2]}^0 = A_{[q_L H_U U]}^0 = A_{[q_L H_D D]}^0 = A_{[L H_D E]}^0 = A^0,$$

and using the analysis of section 4.1 one has

$$A^0 = -c \frac{e^{-\rho + \frac{D}{2}}}{\sqrt{f}} \cos(\theta) (\Theta_{t_2} e^{-i\gamma_{t_2}} + \Theta_{u_2} e^{-i\gamma_{u_2}}).$$

So for instance, a purely dilaton dominated scenario with $\theta = \pi/2$ would not have any soft trilinear couplings in this model.

References

- [1] For a sample of heterotic string model building see, A. Font, L. E. Ibanez, H. P. Nilles and F. Quevedo, *Degenerate Orbifolds*, Nucl. Phys. B **307** (1988) 109 [Erratum-ibid. B **310** (1988) 764];
I. Antoniadis, J. R. Ellis, J. S. Hagelin and D. V. Nanopoulos, *Supersymmetric Flipped $SU(5)$ Revitalized*, Phys. Lett. B **194** (1987) 231;
B. R. Greene, K. H. Kirklin, P. J. Miron and G. G. Ross, *A Three Generation Superstring Model. 2. Symmetry Breaking and the Low-Energy Theory*, Nucl. Phys. B **292** (1987) 606;
P. Nath and R. Arnowitt, *Symmetry Breaking in Three Generation Calabi-Yau Manifolds*, Phys. Rev. D **39** (1989) 2006;
A. E. Faraggi, *A New standard - like model in the Four-dimensional Free Fermionic String Formulation*, Phys. Lett. B **278** (1992) 131;
S. Chaudhuri, S. W. Chung, G. Hockney and J. Lykken, *String Consistency for Unified Model Building*, Nucl. Phys. B **456** (1995) 89 [hep-ph/9501361].
- [2] C. Angelantonj and A. Sagnotti, *Open Strings*, Phys. Rept. **371** (2002) 1 [Erratum-ibid. **376** (2003) 339] [hep-th/0204089].
- [3] R. Blumenhagen, L. Görlich and B. Körs, *Supersymmetric Orientifolds in 6D with D-branes at Angles*, Nucl. Phys. B **569** (2000) 209 [hep-th/9908130].
- [4] R. Blumenhagen, L. Görlich and B. Körs, *Supersymmetric 4D Orientifolds of Type IIA with D6-branes at Angles*, JHEP **0001** (2000) 040 [hep-th/9912204].
- [5] G. Pradisi, *Type I Vacua from Diagonal $Z(3)$ -orbifolds*, Nucl. Phys. B **575** (2000) 134 [hep-th/9912218].
- [6] R. Blumenhagen, L. Görlich and B. Körs, *A New Class of Supersymmetric Orientifolds with D-branes at Angles*, hep-th/0002146.
- [7] C. Angelantonj, I. Antoniadis, E. Dudas and A. Sagnotti, *Type-I Strings on Magnetised Orbifolds and Brane Transmutation*, Phys. Lett. B **489** (2000) 223 [hep-th/0007090].
- [8] S. Förste, G. Honecker and R. Schreyer, *Supersymmetric $Z(N) \times Z(M)$ Orientifolds in 4D with D-branes at Angles*, Nucl. Phys. B **593** (2001) 127 [hep-th/0008250].
- [9] C. Angelantonj and A. Sagnotti, *Type-I Vacua and Brane Transmutation*, hep-th/0010279.
- [10] S. Förste, G. Honecker and R. Schreyer, *Orientifolds with Branes at Angles*, JHEP **0106** (2001) 004 [hep-th/0105208].

- [11] R. Blumenhagen, B. Körs, D. Lüst and T. Ott, *The Standard Model from Stable Intersecting Brane World Orbifolds*, Nucl. Phys. B **616** (2001) 3 [hep-th/0107138].
- [12] M. Cvetič, G. Shiu and A. M. Uranga, *Three-family Supersymmetric Standard like Models from Intersecting Brane Worlds*, Phys. Rev. Lett. **87** (2001) 201801 [hep-th/0107143].
- [13] M. Cvetič, G. Shiu and A. M. Uranga, *Chiral Four-dimensional $N = 1$ Supersymmetric Type IIA Orientifolds from Intersecting D6-branes*, Nucl. Phys. B **615** (2001) 3 [hep-th/0107166].
- [14] R. Blumenhagen, V. Braun, B. Körs and D. Lüst, *Orientifolds of $K3$ and Calabi-Yau Manifolds with Intersecting D-branes*, JHEP **0207** (2002) 026 [hep-th/0206038].
- [15] R. Blumenhagen, L. Görlich and T. Ott, *Supersymmetric Intersecting Branes on the type IIA $T^{**6}/Z(4)$ Orientifold*, JHEP **0301** (2003) 021 [hep-th/0211059].
- [16] G. Honecker, *Chiral Supersymmetric Models on an Orientifold of $Z(4) \times Z(2)$ with Intersecting D6-branes*, Nucl. Phys. B **666** (2003) 175 [hep-th/0303015].
- [17] T. J. Li and T. Liu, *Supersymmetric G^{**3} Unification from Intersecting D6-branes on Type IIA Orientifolds*, hep-th/0304258.
- [18] M. Larosa and G. Pradisi, *Magnetized Four-dimensional $Z(2) \times Z(2)$ Orientifolds*, Nucl. Phys. B **667** (2003) 261 [hep-th/0305224].
- [19] T. Ott, *Aspects of Stability and Phenomenology in Type IIA Orientifolds with Intersecting D6-branes*, hep-th/0309107.
- [20] A. H. Chamseddine, R. Arnowitt and P. Nath, *Locally Supersymmetric Grand Unification*, Phys. Rev. Lett. **49** (1982) 970.
- [21] R. Barbieri, S. Ferrara and C. A. Savoy, *Gauge Models with Spontaneously Broken Local Supersymmetry*, Phys. Lett. B **119** (1982) 343.
- [22] L. J. Hall, J. Lykken and S. Weinberg, *Supergravity as the Messenger of Supersymmetry Breaking*, Phys. Rev. D **27** (1983) 2359.
- [23] P. Nath, R. Arnowitt and A. H. Chamseddine, *Gauge Hierarchy in Supergravity Guts*, Nucl. Phys. B **227** (1983) 121.
- [24] R. Blumenhagen, L. Görlich, B. Körs and D. Lüst, *Noncommutative Compactifications of Type I Strings on Tori with Magnetic Background Flux*, JHEP **0010** (2000) 006 [hep-th/0007024].

- [25] R. Blumenhagen, L. Görlich, B. Körs and D. Lüst, *Magnetic Flux in Toroidal Type I Compactification*, Fortsch. Phys. **49** (2001) 591 [hep-th/0010198].
- [26] G. Aldazabal, S. Franco, L. E. Ibanez, R. Rabadan and A. M. Uranga, *Intersecting Brane Worlds*, JHEP **0102** (2001) 047 [hep-ph/0011132].
- [27] G. Aldazabal, S. Franco, L. E. Ibanez, R. Rabadan and A. M. Uranga, *$D = 4$ Chiral String Compactifications from Intersecting Branes*, J. Math. Phys. **42** (2001) 3103 [hep-th/0011073].
- [28] R. Blumenhagen, B. Körs and D. Lüst, *Type I Strings with F- and B-flux*, JHEP **0102** (2001) 030 [hep-th/0012156].
- [29] L. E. Ibanez, F. Marchesano and R. Rabadan, *Getting Just the Standard Model at Intersecting Branes*, JHEP **0111** (2001) 002 [hep-th/0105155].
- [30] C. Kokorelis, *GUT Model Hierarchies from Intersecting Branes*, JHEP **0208** (2002) 018 [hep-th/0203187].
- [31] C. Kokorelis, *New Standard Model Vacua from Intersecting Branes*, JHEP **0209** (2002) 029 [hep-th/0205147].
- [32] C. Kokorelis, *Exact Standard Model Compactifications from Intersecting Branes*, JHEP **0208** (2002) 036 [hep-th/0206108].
- [33] N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, *The Hierarchy Problem and New Dimensions at a Millimeter*, Phys. Lett. B **429**, 263 (1998) [hep-ph/9803315].
- [34] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, *New Dimensions at a Millimeter to a Fermi and Superstrings at a TeV*, Phys. Lett. B **436** (1998) 257 [hep-ph/9804398].
- [35] M. Berkooz, M. R. Douglas and R. G. Leigh, *Branes Intersecting at Angles*, Nucl. Phys. B **480** (1996) 265 [hep-th/9606139].
- [36] D. Cremades, L. E. Ibanez and F. Marchesano, *SUSY Quivers, Intersecting Branes and the Modest Hierarchy Problem*, JHEP **0207** (2002) 009 [hep-th/0201205].
- [37] M. Cvetič, P. Langacker and G. Shiu, *Phenomenology of a Three-family Standard-like String Model*, Phys. Rev. D **66** (2002) 066004 [hep-ph/0205252].
- [38] M. Cvetič, P. Langacker and G. Shiu, *A Three-family Standard-like Orientifold Model: Yukawa Couplings and Hierarchy*, Nucl. Phys. B **642** (2002) 139 [hep-th/0206115].

- [39] M. Cvetič, I. Papadimitriou and G. Shiu, *Supersymmetric Three Family $SU(5)$ Grand Unified Models from Type IIA Orientifolds with Intersecting D6-branes*, hep-th/0212177.
- [40] D. Cremades, L. E. Ibanez and F. Marchesano, *Yukawa Couplings in Intersecting D-brane Models*, JHEP **0307** (2003) 038 [hep-th/0302105].
- [41] M. Cvetič and I. Papadimitriou, *More Supersymmetric Standard-like Models from Intersecting D6-branes on Type IIA Orientifolds*, Phys. Rev. D **67** (2003) 126006 [hep-th/0303197].
- [42] R. Blumenhagen, D. Lüst and S. Stieberger, *Gauge Unification in Supersymmetric Intersecting Brane Worlds*, JHEP **0307** (2003) 036 [hep-th/0305146].
- [43] P. Nath, R. Arnowitt and A. H. Chamseddine, *Applied $N=1$ Supergravity*, NUB-2613 *Lectures given at Summer Workshop on Particle Physics, Trieste, Italy, Jun 20 - Jul 29, 1983*, Vol I, World Scientific, Singapore 1983.
- [44] H. P. Nilles, *Supersymmetry, Supergravity and Particle Physics*, Phys. Rept. **110** (1984) 1.
- [45] V. S. Kaplunovsky and J. Louis, *Model Independent Analysis of Soft Terms in Effective Supergravity and in String Theory*, Phys. Lett. B **306** (1993) 269 [hep-th/9303040].
- [46] M. Cvetič, P. Langacker and J. Wang, *Dynamical Supersymmetry Breaking in Standard-like Models with Intersecting D6-branes*, Phys. Rev. D **68** (2003) 046002 [hep-th/0303208].
- [47] R. Blumenhagen, *String Unification of Gauge Couplings with Intersecting D-branes*, hep-th/0309146.
- [48] N. Chamoun, S. Khalil and E. Lashin, *Fermion Masses and Mixing in Intersecting Branes Scenarios*, hep-ph/0309169.
- [49] R. Blumenhagen, L. Görlich, B. Körs and D. Lüst, *Asymmetric Orbifolds, Noncommutative Geometry and Type I String Vacua*, Nucl. Phys. B **582** (2000) 44 [hep-th/0003024].
- [50] N. Kaloper and R. C. Myers, *The $O(dd)$ Story of Massive Supergravity*, JHEP **9905** (1999) 010 [hep-th/9901045].
- [51] C. Kokorelis, *$N=1$ Locally Supersymmetric Standard Models from Intersecting Branes*, hep-th/0309070.
- [52] L. J. Dixon, J. A. Harvey, C. Vafa and E. Witten, *Strings on Orbifolds*, Nucl. Phys. B **261** (1985) 678.

- [53] L. J. Dixon, J. A. Harvey, C. Vafa and E. Witten, *Strings on Orbifolds. 2*, Nucl. Phys. B **274** (1986) 285.
- [54] G. Lopes Cardoso, D. Lüüst and T. Mohaupt, *Moduli Spaces and Target Space Duality Symmetries in $(0,2)$ $Z(N)$ Orbifold Theories with Continuous Wilson Lines*, Nucl. Phys. B **432** (1994) 68 [hep-th/9405002].
- [55] L. E. Ibanez and D. Lüüst, *Duality Anomaly Cancellation, Minimal String Unification and the Effective Low-energy Lagrangian of 4-D Strings*, Nucl. Phys. B **382** (1992) 305 [hep-th/9202046].
- [56] C. Angelantonj, *Comments on Open-string Orbifolds with a Non-vanishing $B(ab)$* , Nucl. Phys. B **566** (2000) 126 [hep-th/9908064].
- [57] M. R. Douglas, *D-branes and Discrete Torsion*, hep-th/9807235.
- [58] N. Ohta and P. K. Townsend, *Supersymmetry of M-branes at Angles*, Phys. Lett. B **418** (1998) 77 [hep-th/9710129].
- [59] A. Hashimoto and W. I. Taylor, *Fluctuation Spectra of Tilted and Intersecting D-branes from the Born-Infeld Action*, Nucl. Phys. B **503** (1997) 193 [hep-th/9703217].
- [60] I. Antoniadis, C. Bachas, C. Fabre, H. Partouche and T. R. Taylor, *Aspects of Type I - Type II - Heterotic Triality in Four Dimensions*, Nucl. Phys. B **489** (1997) 160 [hep-th/9608012].
- [61] L. E. Ibanez, C. Munoz and S. Rigolin, *Aspects of Type I String Phenomenology*, Nucl. Phys. B **553** (1999) 43 [hep-ph/9812397].
- [62] M. Dine, N. Seiberg, X. G. Wen and E. Witten, *Nonperturbative Effects on the String World Sheet*, Nucl. Phys. B **278** (1986) 769.
- [63] M. Dine, N. Seiberg, X. G. Wen and E. Witten, *Nonperturbative Effects on the String World Sheet. 2*, Nucl. Phys. B **289** (1987) 319.
- [64] S. Kachru, S. Katz, A. E. Lawrence and J. McGreevy, *Open String Instantons and Superpotentials*, Phys. Rev. D **62** (2000) 026001 [hep-th/9912151].
- [65] S. Kachru, S. Katz, A. E. Lawrence and J. McGreevy, *Mirror Symmetry for Open Strings*, Phys. Rev. D **62** (2000) 126005 [hep-th/0006047].
- [66] E. Cremmer, S. Ferrara, L. Girardello and A. van Proeyen, *Coupling Supersymmetric Yang-Mills Theories to Supergravity*, Phys. Lett. B **116** (1982) 231; *Yang-Mills Theories with Local Supersymmetry: Lagrangian, Transformation Laws and Superhiggs Effect*, Nucl. Phys. B **212** (1983) 413.

- [67] A. Brignole, L. E. Ibanez and C. Munoz, *Soft Supersymmetry-breaking Terms from Supergravity and Superstring Models*, In *Kane, G.L. (ed.): Perspectives on supersymmetry* 125-148. hep-ph/9707209.
- [68] A. H. Chamseddine, *Interacting Supergravity in Ten-Dimensions: The Role of the Six - Index Gauge Field*, Phys. Rev. D **24** (1981) 3065.
- [69] E. Witten, *Dimensional Reduction of Superstring Models*, Phys. Lett. B **155** (1985) 151.
- [70] C. P. Burgess, A. Font and F. Quevedo, *Low-Energy Effective Action for the Superstring*, Nucl. Phys. B **272** (1986) 661.
- [71] S. Ferrara, C. Kounnas and M. Porrati, *General Dimensional Reduction of Ten-Dimensional Supergravity and Superstring*, Phys. Lett. B **181** (1986) 263.
- [72] D. Lüst and S. Stieberger, *Gauge Threshold Corrections in Intersecting Brane World Models*, hep-th/0302221.
- [73] N. Seiberg and E. Witten, *String Theory and Noncommutative Geometry*, JHEP **9909** (1999) 032 [hep-th/9908142].
- [74] P. van Baal, *$SU(N)$ Yang-Mills Solutions with Constant Field Strength on $T^{**}4$* , Commun. Math. Phys. **94** (1984) 397.
- [75] J. Troost, *Constant Field Strengths on $T(2n)$* , Nucl. Phys. B **568** (2000) 180 [hep-th/9909187].
- [76] F. Denef, A. Sevrin and J. Troost, *Non-abelian Born-Infeld versus String Theory*, Nucl. Phys. B **581** (2000) 135 [hep-th/0002180].
- [77] R. C. Myers, *Dielectric-branes*, JHEP **9912** (1999) 022 [hep-th/9910053].
- [78] P. Nath and T. R. Taylor, *Modular Invariance, Soft Breaking, μ and $\tan(\beta)$ in Superstring Models*, Phys. Lett. B **548** (2002) 77 [hep-ph/0209282].
- [79] J. Polchinski and E. Witten, *Evidence for Heterotic - Type I String Duality*, Nucl. Phys. B **460** (1996) 525 [hep-th/9510169].
- [80] M. Cvetič, J. Louis and B. A. Ovrut, *A String Calculation of The Kähler Potentials for Moduli of $Z(N)$ Orbifolds*, Phys. Lett. B **206** (1988) 227.
- [81] L. J. Dixon, V. Kaplunovsky and J. Louis, *On Effective Field Theories Describing $(2,2)$ Vacua of the Heterotic String*, Nucl. Phys. B **329** (1990) 27.
- [82] A. A. Tseytlin, *On Non-abelian Generalisation of the Born-Infeld Action in String Theory*, Nucl. Phys. B **501** (1997) 41 [hep-th/9701125].

- [83] I. Antoniadis, H. Partouche and T. R. Taylor, *Lectures on Heterotic-Type I Duality*, Nucl. Phys. Proc. Suppl. **61A** (1998) 58 [Nucl. Phys. Proc. Suppl. **67** (1998) 3] [hep-th/9706211].
- [84] Z. Kakushadze, *Aspects of $N = 1$ Type I-Heterotic Duality in Four Dimensions*, Nucl. Phys. B **512** (1998) 221 [hep-th/9704059].
- [85] Z. Kakushadze, G. Shiu and S. H. Tye, *Type IIB Orientifolds, F-theory, Type I Strings on Orbifolds and Type I Heterotic Duality*, Nucl. Phys. B **533** (1998) 25 [hep-th/9804092].
- [86] Z. Kakushadze, *Non-perturbative Orientifolds*, Phys. Lett. B **455** (1999) 120 [hep-th/9904007].
- [87] A. Gregori, *String-String Triality for $d = 4$, Z_2 Orbifolds*, JHEP **0206** (2002) 041 [hep-th/0110201].
- [88] M. Cvetič and I. Papadimitriou, *Conformal Field Theory Couplings for Intersecting D-branes on Orientifolds*, Phys. Rev. D **68** (2003) 046001 [hep-th/0303083].
- [89] S. A. Abel and A. W. Owen, *Interactions in Intersecting Brane Models*, Nucl. Phys. B **663** (2003) 197 [hep-th/0303124].
- [90] I. R. Klebanov and E. Witten, *Proton Decay in Intersecting D-brane Models*, Nucl. Phys. B **664** (2003) 3 [hep-th/0304079].
- [91] D. Lüst, P. Mayr, R. Richter and S. Stieberger, *Scattering of Gauge, Matter, and Moduli Fields from Intersecting Branes*, hep-th/0404134.
- [92] M. Berkooz and R. G. Leigh, *A $D = 4$ $N = 1$ Orbifold of Type I Strings*, Nucl. Phys. B **483** (1997) 187 [hep-th/9605049].
- [93] E. Witten, *New Issues in Manifolds of $SU(3)$ Holonomy*, Nucl. Phys. B **268** (1986) 79.
- [94] N. K. Nielsen and P. Olesen, *An Unstable Yang-Mills Field Mode*, Nucl. Phys. B **144** (1978) 376.
- [95] S. Randjbar-Daemi, A. Salam and J. Strathdee, *Spontaneous Compactification in Six-Dimensional Einstein-Maxwell Theory*, Nucl. Phys. B **214** (1983) 491; *Instability of Higher Dimensional Yang-Mills Systems*, Phys. Lett. B **124** (1983) 345 [Erratum-ibid. B **144** (1984) 455].
- [96] R. Rabadan, *Branes at Angles, Torons, Stability and Supersymmetry*, Nucl. Phys. B **620** (2002) 152 [hep-th/0107036].

- [97] R. Blumenhagen, D. Lüst and T. R. Taylor, *Moduli Stabilization in Chiral Type IIB Orientifold Models with Fluxes*, Nucl. Phys. B **663** (2003) 319 [hep-th/0303016].
- [98] M. Marino, R. Minasian, G. W. Moore and A. Strominger, *Nonlinear Instantons from Supersymmetric p-branes*, JHEP **0001** (2000) 005 [hep-th/9911206].
- [99] G. Aldazabal, L. E. Ibanez and A. M. Uranga, *Gauging Away the Strong CP Problem*, hep-ph/0205250.
- [100] E. Cremmer, S. Ferrara, C. Kounnas and D. V. Nanopoulos, *Naturally Vanishing Cosmological Constant in $N = 1$ Supergravity*, Phys. Lett. B **133** (1983) 61.
- [101] J. R. Ellis, A. B. Lahanas, D. V. Nanopoulos and K. Tamvakis, *No - Scale Supersymmetric Standard Model*, Phys. Lett. B **134** (1984) 429.
- [102] J. R. Ellis and D. V. Nanopoulos, *Flavor Changing Neutral Interactions in Broken Supersymmetric Theories*, Phys. Lett. B **110** (1982) 44.
- [103] J. F. Donoghue, H. P. Nilles and D. Wyler, *Flavor Changes in Locally Supersymmetric Theories*, Phys. Lett. B **128** (1983) 55.
- [104] S. A. Abel, M. Masip and J. Santiago, *Flavor Changing Neutral Currents in Intersecting Brane Models*, JHEP **0304** (2003) 057 [hep-ph/0303087].
- [105] D. Matalliotakis and H. P. Nilles, *Implications of Nonuniversality of Soft Terms in Supersymmetric Grand Unified Theories*, Nucl. Phys. B **435** (1995) 115 [hep-ph/9407251].
- [106] P. Nath and R. Arnowitt, *Non-universal Soft SUSY Breaking and Dark Matter*, Phys. Rev. D **56** (1997) 2820 [hep-ph/9701301].
- [107] T. Ibrahim and P. Nath, *The Chromoelectric and Purely Gluonic Operator Contributions to the Neutron Electric Dipole Moment in $N = 1$ Supergravity*, Phys. Lett. B **418** (1998) 98 [hep-ph/9707409].
- [108] T. Ibrahim and P. Nath, *The Neutron and the Lepton EDMs in MSSM, Large CP Violating Phases, and the Cancellation Mechanism*, Phys. Rev. D **58** (1998) 111301 [hep-ph/9807501].
- [109] E. D. Commins, S. B. Ross, D. DeMille and B. C. Regan, *Improved Experimental Limit on the Electric Dipole Moment of the Electron*, Phys. Rev. A **50** (1994) 2960.
- [110] P. G. Harris *et al.*, *New Experimental Limit on the Electric Dipole Moment of the Neutron*, Phys. Rev. Lett. **82** (1999) 904.

- [111] S. K. Lamoreaux, J. P. Jacobs, B. R. Heckel, F. J. Raab and E. N. Fortson, *New Limits on Spatial Anisotropy from Optically Pumped He-201 and Hg-199*, Phys. Rev. Lett. **57**, 3125 (1986).
- [112] P. Nath, *CP Violation via Electroweak Gauginos and the Electric Dipole Moment of the Electron*, Phys. Rev. Lett. **66** (1991) 2565.
- [113] T. Ibrahim and P. Nath, *Large CP Phases and the Cancellation Mechanism in EDMs in SUSY, String and Brane Models*, Phys. Rev. D **61** (2000) 093004 [hep-ph/9910553].
- [114] M. Brhlik, L. L. Everett, G. L. Kane and J. Lykken, *A Resolution to the Supersymmetric CP Problem with Large Soft Phases via D-branes*, Phys. Rev. Lett. **83** (1999) 2124 [hep-ph/9905215]; M. Brhlik, L. L. Everett, G. L. Kane and J. Lykken, *Superstring theory and CP violating phases: Can they be related?*, Phys. Rev. D **62** (2000) 035005 [hep-ph/9908326].
- [115] T. Ibrahim and P. Nath, *CP Violation in SUSY, Strings and Branes*, hep-ph/0207213.
- [116] M. Carena, M. Quiros, M. Seco and C. E. Wagner, *Improved Results in Supersymmetric Electroweak Baryogenesis*, Nucl. Phys. B **650** (2003) 24 [hep-ph/0208043].
- [117] D. Chang, W. Y. Keung and A. Pilaftsis, *New two-loop contribution to electric dipole moment in supersymmetric theories*, Phys. Rev. Lett. **82** (1999) 900 [Erratum-ibid. **83** (1999) 3972] [hep-ph/9811202].
- [118] C. L. Bennett *et al.*, *First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Preliminary Maps and Basic Results*, astro-ph/0302207.
- [119] U. Chattopadhyay, A. Corsetti and P. Nath, *WMAP Constraints, SUSY Dark Matter and Implications for the Direct Detection of SUSY*, Phys. Rev. D **68** (2003) 035005 [hep-ph/0303201].
- [120] A. B. Lahanas, N. E. Mavromatos and D. V. Nanopoulos, *WMAPing the Universe: Supersymmetry, Dark Matter, Dark Energy, Proton Decay and Collider Physics*, hep-ph/0308251.
- [121] A. Corsetti and P. Nath, *Gaugino Mass Nonuniversality and Dark Matter in SUGRA, Strings and D-brane Models*, Phys. Rev. D **64** (2001) 125010 [hep-ph/0003186].