

January 01, 2001

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### Recommended Citation

Goldberg, Haim and Weiler, Thomas J., "Clustering in highest energy cosmic rays: physics or statistics?" (2001). *Physics Faculty Publications*. Paper 133. <http://hdl.handle.net/2047/d20000744>

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# Clustering in Highest Energy Cosmic Rays: Physics or Statistics?

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## Abstract

Directional clustering can be expected in cosmic ray observations due to purely statistical fluctuations for sources distributed randomly in the sky. We develop an analytic approach to estimate the probability of random cluster configurations, and use these results to study the strong potential of the HiRes, Auger, Telescope Array and EUSO/OWL/AirWatch facilities for deciding whether any observed clustering is most likely due to non-random sources.

## Introduction

An unsolved astrophysical mystery is the origin and nature of the extreme energy cosmic ray primaries (EECRs) responsible for the observed events at highest energies,  $\sim 10^{20}$  eV [1]. About twenty events above  $E_{\text{GZK}}$  have been observed by five different experiments [2]. The longitudinal profile for one of these events (the highest energy Fly’s Eye event at  $3 \times 10^{20}$  eV) is available; it favors a nucleon or nuclear primary over a photon primary [3]. The origin of these events is a mystery, for there are no visible source candidates within 50 Mpc except possibly M87, a radio-loud AGN at  $\sim 20$  Mpc. Since the observed events display a large-scale isotropy, many sources rather than one source seems to be required. The nature of the primary is a

mystery, because interactions with the 2.73K cosmic microwave background (CMB) renders the Universe opaque to nucleons at these energies, and double pair production on the cosmic radio background (CRB) renders the Universe opaque to photons at these energies. The theoretical prediction of the end of transparency for nucleons at  $E_{\text{GZK}} \sim 5 \times 10^{20}$  eV is the famous ‘‘GZK cutoff’’ [4].

Models for the origin and nature of the primaries may be put into two broad categories. In the first category are postulated sources of protons and photons ‘‘locally’’ distributed within 50 to 100 Mpc. For these models, the propagation problem is mitigated. However, the source problem is aggravated. In traversing a distance  $D$ , a charged particle interacting with magnetic domains having coherence length  $\lambda$  will bend through an energy-dependent angle<sup>1</sup>

$$\delta\theta \sim 0.5^\circ \times \frac{Z B_{nG}}{E_{20}} \sqrt{D_{\text{Mpc}} \lambda_{\text{Mpc}}}. \quad (1)$$

Here  $B_{nG}$  is the magnetic field in units of nanogauss,  $E_{20}$  and  $Z$  are the particle energy in units of  $10^{20}$  eV and charge, and the lengths  $D$  and  $\lambda$  are given in units of Mpc. It is thought likely that coherent extragalactic fields are nanogauss in magnitude [5], in which case super-GZK primaries from  $\lesssim 50$  Mpc will bend only a few degrees, typically (but note that protons at  $10^{19}$  eV will bend through  $\sim 30^\circ$ ). Thus, local models either postulate many invisible sources isotropically-distributed with respect to the Galaxy to provide the roughly isotropic flux observed above  $E_{\text{GZK}}$ , or postulate a large extragalactic magnetic field to isotropize over our Northern Hemisphere the highest-energy particles<sup>2</sup> from a small number of sources [6]. A common prediction for local models is little or no directional pairing on small scales, especially when events with energy  $\sim E_{\text{GZK}}$  are included with the  $10^{20}$  eV events<sup>3</sup>. In particular, models invoking randomly distributed, decaying super-massive par-

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<sup>1</sup>On average, half of the interactions of a super-GZK nucleon with the CMB change the isospin. At energies for which  $c\tau$  of the neutron is small compared to the interaction mfp of  $\sim 6$  Mpc, the neutron decays back to a proton with negligible energy loss and the bending-angle formula is unchanged. However, at the energy  $6 \times 10^{20}$ ,  $c\tau$  for the neutron is comparable to the interaction mfp, so at higher energies the nucleon bending-angle is reduced by  $\lesssim 2$ .

<sup>2</sup>Some models postulate helium or iron nuclei as the primaries, to increase the bending by the charge factors 2 and 26, respectively.

<sup>3</sup>There is the possibility of pairing due to magnetic focussing, if the projection of large-scale extragalactic

ticles (SMPs) as sources [8], and models invoking a large magnetic field with considerable incoherent component, predict a chance distribution of observed events on the sky.

In the second category of models, the cosmic ray sources are put at cosmic distances ( $\gtrsim 100$  Mpc, i.e.  $z \gtrsim 0.02$ ). These large- $z$  models mitigate the source issue. However, the propagation problem is exacerbated due to the increased distance. Most proposals of this type postulate some stable, charge-neutral primary having limited interaction with the radiation background. Examples include the neutrino (which may regenerate a local nucleon/photon flux via “Z-bursts” [9], or may develop a strong interaction at high energies [10], magnetic monopoles [11], and the lightest SUSY baryon [12] (if the gluino mass is  $\sim 1$  GeV). Other large- $z$  models employ broken Lorentz invariance or broken CPT-symmetry with effects generally suppressed by  $M_P^{-1}$  factors but still large enough at  $E \gtrsim E_{\text{GZK}}$  to suppress the nucleon-CMB interaction [13]. The nucleon-magnetic field interaction may also be suppressed, in which case these nucleon primaries, like the charge-neutral primaries, are not significantly bent by the intervening extragalactic magnetic fields. A common prediction, then, of large- $z$  models is direct pointing of the primary’s arrival direction back to the source<sup>4</sup>. If a source is of sufficient intensity and duration, then the observation of multiple events from the direction of the source may be expected, beyond what is expected from chance coincidence.

So it is seen that a major discriminator between the local and the large- $z$  models is the occurrence of directional pairing compared to that expected from chance coincidence alone. The AGASA experiment has already presented data strongly suggesting that directional pairing is occurring at higher than chance coincidence [16]. Comparisons of event directions in a combined data sample of four experiments further supports non-chance coincidences [17]. Furthermore, the energy-time correlation within pairs seems to disfavor models with charged primaries originating from a common source of relatively short duration. This is because magnetic diffusion should lead to a later arrival time (more bending) for the lower energy charged-primary in the pair, contrary to what is observed in some pairs [18].

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magnetic fields on our sky contains caustics [7], and if the incoherent magnetic fields are sufficiently small.

<sup>4</sup>There is some evidence that the highest energy events may indeed point to distant compact radio-loud quasars [14]; however, more recent data do not seem to support the earlier result [15].

It is the purpose of this paper to estimate in a straightforward manner the significance of multiple events in future data. We do so by providing an analytic calculation of the chance occurrence of directional multiples. It is relative to these chance probabilities that observed rates will determine the rise or fall of models. We present a formula for all possible multiples based on statistics alone, as a function of the total number of observed events, and the number of angular bins. We show that our analytic formula reproduces the published AGASA probabilities (obtained by Monte Carlo simulation) fairly well, even without inputting detailed knowledge of the experiment. We present the chance occurrence of directional multiples for the HiRes experiment, and the next-generation Auger and Telescope Array experiments, and finally the proposed EUSO/OWL/AirWatch experiment. We show that for 20 events at HiRes, the observation of a triplet or two doublets at resolution of  $2^\circ$  or less is unlikely at the  $3\sigma$  level, as is the observation of two triplets or a quadruplet for 100 events at Auger. For the EUSO/OWL/AirWatch facility, the anticipated very large number of events can be binned into a sizeable number of subsets. As will be shown, this process can allow an explicit quantitative test of the probabilities predicted for certain cluster groupings on the basis of purely chance coincidence.

## Formula for Chance Coincidence

We now present the combinatoric formula for the probability of various event distributions in angle. Our formula is exact in the limit where the experimental efficiency for observing events is effectively uniform over the coverage of the celestial sky. We imagine that the sky coverage consists of a solid angle  $\Omega$  divided into  $N$  equal angular bins, each with solid angle  $\omega \simeq \pi\theta^2$  steradian; the number of bins with cone half-angle  $\theta$  is

$$N \simeq \frac{\Omega}{\pi\theta^2} = 1045 \frac{(\Omega/1.0 \text{ sr})}{(\theta/1.0^\circ)^2} \quad (2)$$

where  $\Omega$  is the solid angle (sidereal or galactic) on the celestial sphere covered by the experiment. We toss  $n$  events at random into these bins. As mentioned in the introduction, such a chance distribution of events is just what is expected in some models for the EECRs, e.g. randomly situated decaying SMPs, or charged-particle or monopole primaries traversing incoherent magnetic fields.

Define each event distribution by specifying the partition of the  $n$  total events into a number  $m_0$  of empty bins, a number  $m_1$  of single hits, a number  $m_2$  double hits, etc., among the  $N$  angular bins which constitute the total sky exposure. The probability to obtain a given event topology is:

$$P(\{m_i\}, n, N) = \frac{1}{N^n} \frac{N!}{m_0! m_1! m_2! m_3! \dots} \frac{n!}{(0!)^{m_0} (1!)^{m_1} (2!)^{m_2} (3!)^{m_3} \dots} . \quad (3)$$

The  $N!$  and  $n!$  factors in the numerator count the permutations of the bins and the events, respectively. The  $m_j!$  and  $j!$  factors in the denominator remove the overcounting of those bins containing  $j$  events, and the events within those bins, respectively. The normalization factor  $N^n$  is total number of ways to partition  $n$  events among  $N$  bins.

The variables in the probability are not all independent. The partitioning of events is related to the total number of events by

$$\sum_{j=1} j \times m_j = n , \quad (4)$$

and to the total number of bins by

$$\sum_{j=0} m_j = N . \quad (5)$$

Because of the constraints in eqs. (4) and (5), we infer that the process is not described by a simple multinomial or Poisson probability distribution.

It is useful to use eqs. (4) and (5) to rewrite our exact probability (3) as

$$P(\{m_i\}, n, N) = \frac{N!}{N^N} \frac{n!}{n^n} \prod_{j=0} \frac{(\overline{m}_j)^{m_j}}{m_j!} , \quad (6)$$

where we have defined

$$\overline{m}_j \equiv N \left( \frac{n}{N} \right)^j \frac{1}{j!} . \quad (7)$$

In the  $n \ll N$  limit,  $\overline{m}_j$  is expected to approximate the mean number of  $j$ -plets, and eq. (6) becomes roughly Poissonian. As an approximate mean,  $\overline{m}_j$  defined in eq. (7) provides a simple estimate of cluster probabilities due to chance for the  $n \ll N$  case.

Scaling laws relating mean cluster numbers to total event numbers and binning angles may be derived by inserting eq. (2) into eq. (7). Results are

$$\overline{m}_j = \left( \frac{\pi n \theta^2}{\Omega} \right)^{j-1} \left( \frac{n}{j!} \right) , \quad (8)$$

and

$$\frac{\overline{m}_j}{\overline{m}_{j-1}} = \frac{n}{jN} \sim \frac{\pi n \theta^2}{j\Omega}. \quad (9)$$

These scaling laws may be used as a further test of the randomness of clustering in a data sample. For example, the angular binning may be artificially expanded from the experimental resolution to see if the cluster numbers follow the  $\theta^{2(j-1)}$  scaling law. Of course, signal-to-chance is optimised by choosing the binning angle close to the natural angle of the source on the sky.

In the next section we examine the simplification of the exact formulas (3) and (6) that results in the large  $N, n$  limit. Even in the  $N \gg n \gg 1$  limit, the resulting approximate formula will be seen to be not quite Poissonian, since the variables in the set  $\{m_i, n, N\}$  are not independent.

## Large $N, n$ Limit

Two large-number limits of interest are  $N \gg n \gg 1$ , and  $n > N \gg 1$ . With bin numbers typically  $\sim 10^3$ , the first limit applies to the AGASA, HiRes, Auger and Telescope Array experiments; the second limit becomes relevant for the EUSO/OWL/AW experiment after a year or more of running.

### Approximation for $N \gg n \gg 1$

When  $N \gg n$ , the number  $m_0$  of empty bins is of order  $N$ , and the number of bins  $m_1$  with single events (singlets) is order  $n$ ; the number of clusters (doublets, triplets, etc.) is small. It is sensible to explicitly evaluate the not-so-interesting  $j = 0$  and 1 terms in eqs. (6) and (7). With the use of Stirling's approximation for the factorials, one arrives at a simple form for the probability, valid when  $N \gg n \gg 1$ :

$$P(\{m_i\}, n, N) \approx \mathcal{P} \left[ \prod_{j=2} \frac{(\overline{m}_j)^{m_j}}{m_j!} e^{-\overline{m}_j} r^j (j-2)! \right], \quad (10)$$

where  $r \equiv (N - m_0)/n \approx 1$ , and the prefactor  $\mathcal{P}$  is

$$\mathcal{P} = e^{-(n-m_1)} \left( \frac{n}{m_1} \right)^{m_1 + \frac{1}{2}}. \quad (11)$$

In the “sparse events” case here, where  $N \gg n$ , one expects the number of singlets  $m_1$  to approximate the number of events  $n$ . In this case the prefactor is near unity. The non-Poisson nature of Eq. (10) is reflected in the factorials and powers of  $r$  in the exponents, and the deviation of the prefactor from unity. In our numerical work, we will provide curves for the exact result, and for the Poisson approximation (obtained from our expressions (10) and (7) by setting  $\mathcal{P}$  to unity and omitting  $r^j (j - 2)!$  in the exponent, and by neglecting the constraints of eqs. (4) and (5). Note that in this Poisson approximation,  $\overline{m}_j$  as defined in Eq. (7) is truly the mean number of  $j$ -plets.

### Approximation for $n > N \gg 1$

In the case where  $n > N \gg 1$ , higher  $j$ -plets are common and the distribution of clusters can be rather broad in  $j$ , since according to Eq. (9).

Already at  $j = 1$  (2), Stirling’s approximation to  $j!$  is good to 8% (4%), and so we may write  $\overline{m}_j$  in the approximate form for  $j \geq 1$ :

$$\overline{m}_j \approx \sqrt{\frac{N^3}{2\pi en}} \left( \frac{en}{jN} \right)^{j+\frac{1}{2}}. \quad (12)$$

Extremizing this expression with respect to  $j$ , one learns that the most populated  $j$ -plet occurs near  $j \sim n/N$ . Combining this result with the broad distribution expected for large  $n/N$ , one expects clusters with  $j$  up to several  $\times \frac{n}{N}$  to be common in the EUSO/OWL/AW experiment.

## Comparison with Experiments

There are two ongoing EECR experiments, AGASA [19] and HiRes [20]. There are two larger area experiments under development, Auger [21] and Telescope Array (TA) [22]. Finally, there are experiments proposed with still larger areas, EUSO [23], OWL [24] and AirWatch [25]. A non-hostile merger of these latter experiments appears likely, so we refer to them collectively as EUSO/OWL/AW. In Table 1 we list the relevant parameters defining for our purposes each of these experiments.

Table 1: Typical values of effective area  $A$ , celestial solid angle  $\Omega$  [26], and angular resolution  $\theta_{\min}$  for the existing and proposed EECR experiments. The incident flux  $F(\geq E_{\text{GZK}}) = 10^{-19} \text{cm}^{-2} \text{s}^{-1} \text{sr}^{-1}$  has been used to estimate the number of events above  $E_{\text{GZK}} = 5 \times 10^{19} \text{ eV}$ .

Experiment	AGASA	HiRes	Auger/TA	EUSO/OWL/AW
$A$ (km <sup>2</sup> sr)	150	800	$6 \times 10^3$	$3 \times 10^5$
$n/\text{yr} = A \times F(\geq E_{\text{GZK}})$	5	30	200	$10^4$
$\Omega$ (sr)	4.8	7.3	4.8	$4\pi$
$\theta_{\min}$	$3.0^\circ$	$0.5^\circ$	$1.0^\circ$	$1.0^\circ$

We proceed to normalize the analytic approach described above against the Monte Carlo method used by the AGASA Collaboration [16], and then to provide some concrete examples for future observation. In searching for cluster probabilities using Monte Carlo, a fixed number  $n$  of events is tossed into phase space, and clusters are defined by choosing a correlation angle  $\theta'$  (in principle independent from the resolution angle discussed earlier). For example, a doublet is registered when an event falls within an angle  $\theta'$  of a preceding event. In such a manner, the AGASA Collaboration [16] found that 92 events yielded 12 or more doublets in 1.5% of their trials. Their simulations utilized a correlation angle  $\theta' = 3^\circ$  and declination angles (roughly) between  $10^\circ$  and  $70^\circ$ . This gives a solid angle  $\Omega \simeq 4.8$  steradian. If for the moment we identify the correlation angle  $\theta'$  with the resolution angle  $\theta$ , then from Eq. (2), with  $\theta = 3^\circ$ , we find  $N = 557$ . Eq.(3) then gives

$$P(m_2 \geq 12, m_3 \geq 0, m_4 \geq 0, 92, 557) = 1.4\% \quad . \quad (13)$$

Considering the crudeness of the approximations, this result agrees with the 1.5% simulation result.<sup>5</sup>

For Auger, we adopt the same solid angle ( $\Omega = 4.8$  sr). For HiRes, we estimate on geometric grounds a solid angle of 10.9 sr [26]. The same geometric estimate for AGASA gives a solid angle of 6.9 sr, which the acceptance profile reduced to the above-mentioned 4.8 sr. For the sake of our modeling, we apply the same reduction factor ( $4.8/6.9$ ) to the

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<sup>5</sup>Note that if our estimate of the solid angle were to change somewhat, agreement with the Monte Carlo results could still be achieved by introducing a slight deviation from the assumed equality  $\theta' = \theta$ .

HiRes geometry, to arrive at an effective solid angle of 7.3 sr (Table I). The purely statistical probabilities for various cluster topologies can now be calculated as a function of the angular resolution and the accumulated number  $n$  of events.

## Discussion of Results

### HiRes

For the HiRes experiment, about 20 events at  $10^{20}$  eV are expected when the first full year's data is analyzed. We calculate the inclusive probabilities for one or more, two or more, and three or more doublets; and one or more triplets, over a range of angular binning using Eqs. (3) and (2). Note that by “inclusive probability” we mean the stated number of  $j$ -plets *plus any other clusters*; e.g. a topology with two doublets and one triplet counts as one doublet, as two doublets, and as one triplet in the inclusive probabilities. The results are displayed in Figure 1.

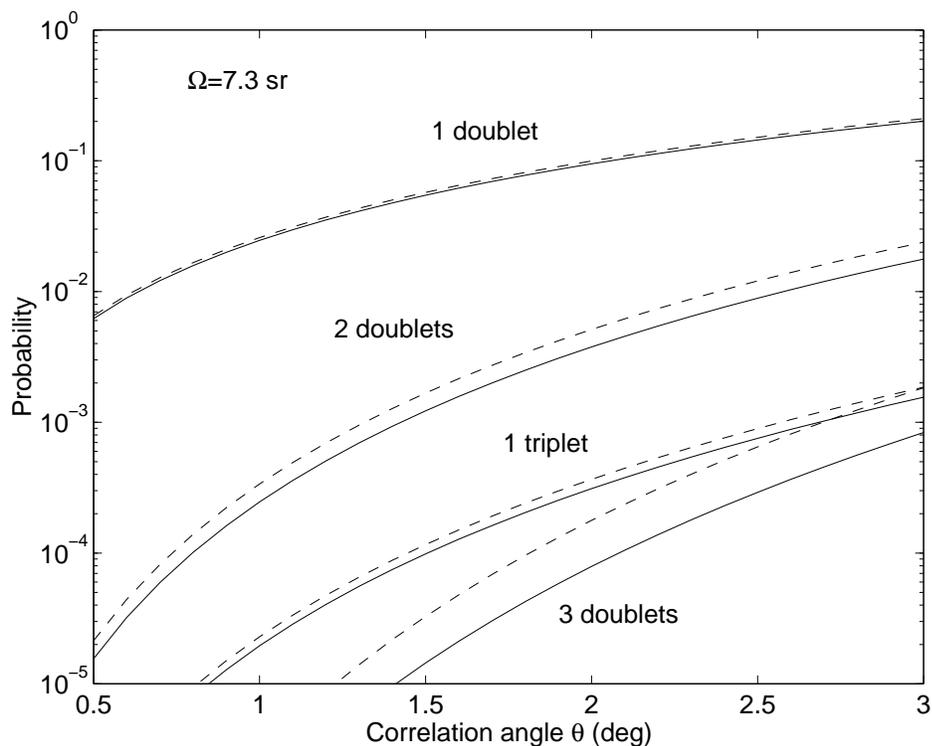


Figure 1: Inclusive probabilities for various clusters, given 20 events at HiRes. The solid line is the exact result, the dashed line is the Poisson approximation.

Several comments may be made with reference to this figure:

- (a) For all except the 3 doublet configuration, the Poisson approximation using the mean values in Eq.(7) provides an estimate good to within 50% of the non-approximate form; for the (much suppressed) 3 doublet configuration, it overestimates the probability by about a factor of 3 in much of the angular region.
- (b) For angular binning tighter than  $2^\circ$ , an observation of two doublets among the first 20 events has a chance probability of less than 0.5%. Thus the observation of this topology could be construed as evidence (at the  $3\sigma$  level) for clustering beyond statistical. The observation of a triplet within  $\leq 3^\circ$  has a random probability of less than  $10^{-3}$ , and hence observation of such a triplet would most likely signify clustered or repeating sources, or magnetic focussing effects.
- (c) With the accumulation of 40 events (not shown in the figure), the appearance of two doublets has a probability of less than 0.5% for a correlation angle of  $1^\circ$  or less. This illustrates how the good angular resolution of HiRes may be used to detect non-statistical clustering with only a few observed clusters.

## **Auger**

Coming now to Auger, we present in Figure 2 the probabilities of observing 8 doublets, one triplet, and two triplets, in an event sample of 100 events.

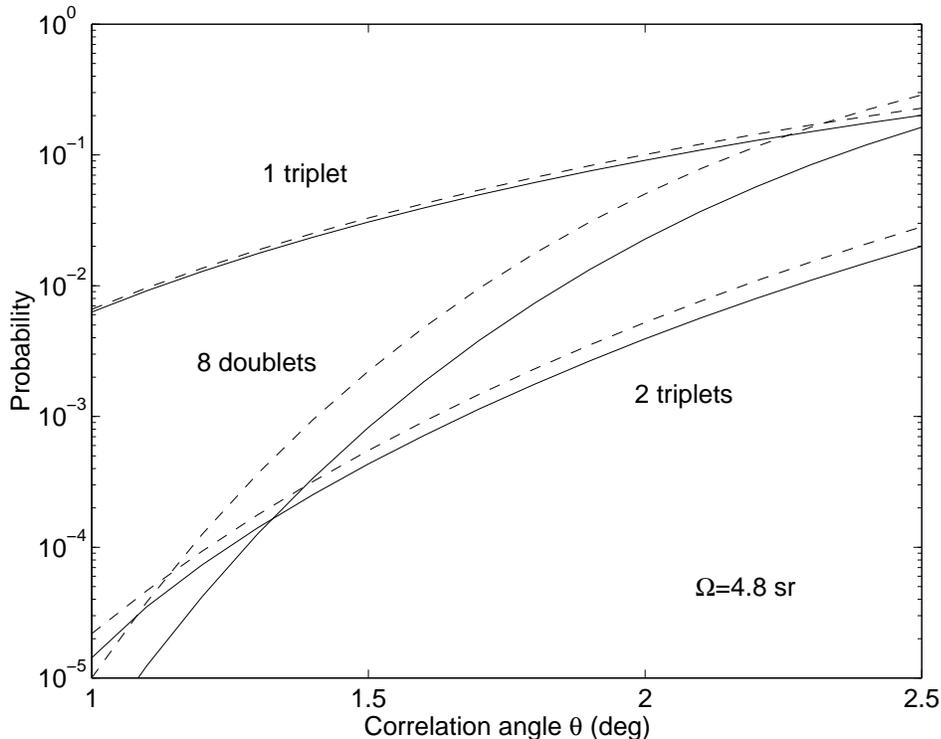


Figure 2: Inclusive probabilities for various clusters in a 100 event sample at Auger; solid (exact), dashed (Poisson).

This graph illustrate several points of interest:

- (a) The 8-doublet probability is extremely sensitive to the angular binning, and thus uncertainties of the order of  $0.5^\circ$  in the region of  $\theta \leq 2^\circ$  would preclude assigning a baseline chance-probability to better than an order of magnitude for this topology. This uncertainty may be avoided by breaking down the 100 events into smaller data sets. On the other hand, it may be possible to use the sensitive dependence displayed here to advantage: observation of a flatter dependence on angular bin-size could signal a non-chance origin for the clustering.
- (b) As in Fig. 1, it is seen that the Poisson approximation is good for some topologies, but an overestimate for others; for 8 doublets, it overestimates the probability by about 3 in much of the angular region.
- (c) The observation of two triplets with angular binning of less than  $2^\circ$  would have a random probability of less than 0.5%, and hence could be construed as 3-sigma evidence for a novel astrophysics, such as clustered or repeating sources or magnetic focussing.

(d) Not shown in the figure is the probability of observing a quadruplet, which turns out to be about 0.5% for a binning angle of  $2.5^\circ$ . Hence the observation of a quadruplet within  $2.5^\circ$  among the first 100 Auger events would be suggestive of clustering due to an astrophysical cause.

## **EUSO/OWL/AW**

The large number of events expected in the EUSO/OWL/AW experiment presents an extraordinary opportunity to test for non-random clustering, but also a numerical problem for both our formula and for a direct Monte Carlo simulation. With  $10^4$  super-GZK events (Table I) distributed among  $10^3$  bins, 10-plets and beyond may be common occurrences (see eq. (12)). One approach to the very large data sample is to envisage the  $10^4$  super-GZK events divided into 500 more manageable subsamples of 20 events each (there are many ways to choose the event partitions, and some subtleties are involved.). Then the probability predictions for clustering in the random model can be straightforwardly assessed. For example, In Figure 3, we show the inclusive probabilities for 1 and 2 doublets in 20 events, for the EUSO/OWL/AW sky aperture.

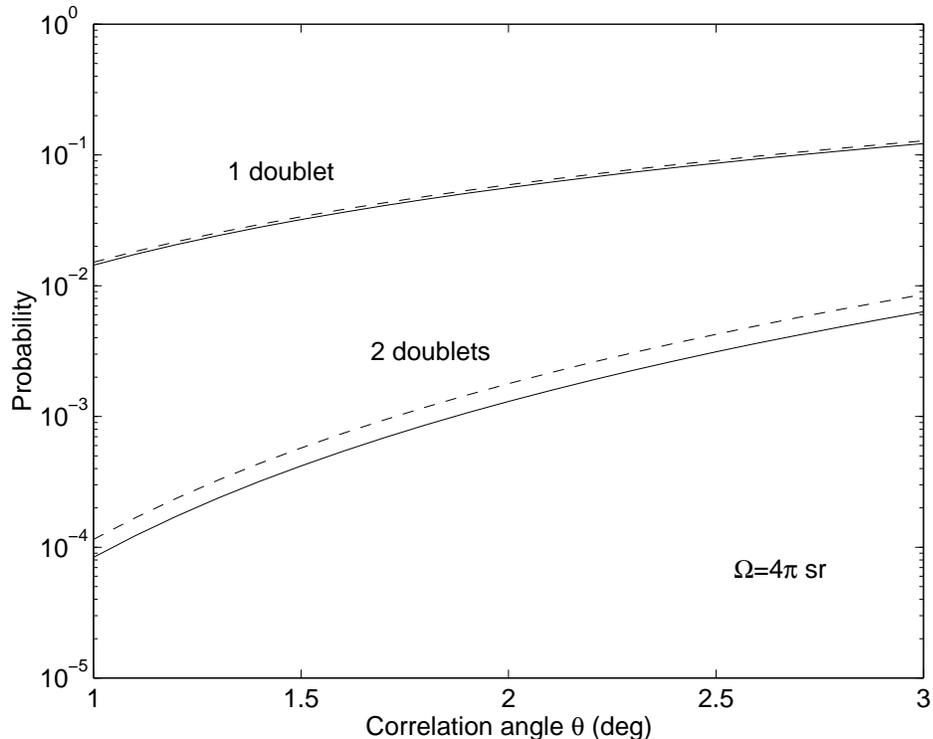


Figure 3: Probabilities for various topologies in a 20 event subsample at EUSO/OWL/AW

We can see that for binning angles of  $2.5^\circ$ , we expect about 10% of the 500 20-event subsamples, or about 50 subsamples to have one doublet, and we expect about 0.3%, or perhaps 1 or 2 samples to have 2 doublets. Comparing subsamples in this way, deviations from random clustering can be *quantitatively* assessed. Of course, it is possible that in the large sample of EUSO/OWL/AW, some non-random high- $j$  clusters will emerge far above background. In such a case, the random probabilities presented here become much less relevant.

The same large data-set approach just described could also be used for Auger after a few years of running time, although with somewhat fewer statistics.

## Summary and Concluding Remarks

We have presented an analytic study of clustering for cosmic rays based on a random angular generation of events in the sky. Our probability formula is based on randomly assigning events into fixed angular bins with uniform a priori probability. In reality, the efficiency of

experiments for observing events is not uniform across the sky coverage. For this reason, the most careful quantitative statements about clustering probabilities of existing data must come from Monte Carlo simulations incorporating experimental efficiencies. Nevertheless, our results are in good agreement with the prior Monte Carlo study by the AGASA Collaboration [16]. For this reason, we believe that our formula makes a significant advance to the field, and is especially useful in predicting event topologies for future experiments and larger data samples.

We found that the use of Poisson distributions, with mean values given by Eq. (7), was approximately valid for some topologies, but yielded overestimates for others. For some of the interesting cases discussed in the preceding section, the Poisson estimates were factors of 2 to 3 larger than the exact probabilities.

Results for the HiRes, Auger and EUSO/OWL/AW experiments were presented. These results reveal which topologies occur with probabilities of 0.5% or less in the various experiments; observation of these topologies would constitute evidence at the  $3\sigma$  level for astrophysical rather than random causes for the clustering. An observation of two or more doublets in the first 20 events at HiRes, each doublet consisting of 2 events within less than  $2^\circ$  of each other, is one example shown in the text.

Topologies with highly suppressed chance probabilities are especially sensitive probes of non-random clustering. This situation is exemplified in our discussion of the 8-doublet topology for the Auger experiment with 100 events. Highly suppressed topologies may be rather difficult to use in practice since they exhibit extreme sensitivity to binning angle, which leads to great ambiguity in their statistical significance. On the other hand, this sensitivity may be useful as a diagnostic to distinguish between random clustering vs. clustering due to astrophysics.

The large number of events previewed in the proposed EUSO/OWL/AW experiment (and to a lesser extent, at the Auger facility) presents an analytical challenge. Partitioning of the total data sample into subsamples, and then comparing these subsamples, would provide a direct test of the purely statistical predictions for clustering.

We have not included any source modeling in our analysis. Our chance probabilities describe arrival directions randomly distributed on the celestial sky. In fact, this distribution

is reality for some models, such as decaying SMPs, and charged primaries with directions randomized by incoherent cosmic magnetic fields. A complementary approach to our work is to consider specific source models generating non-random angular distributions. Steps along this line of inquiry have recently been taken [27]. Future progress in the field will involve comparisons of the random and non-random model predictions with the data.

## Acknowledgements

We acknowledge fruitful discussions with Jim Fry, and thank the Aspen Center for Physics for a beneficial working environment. This work was supported in part by the U.S. Department of Energy grant no. DE-FG05-85ER40226, and by the National Science Foundation grant no. PHY-9722044.

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$$\delta t \sim 300 D_{\text{Mpc}} \left( \frac{Z B_{nG} \lambda}{E_{20}} \right)^2 \text{ yrs} \quad (14)$$

for the time delay. The time separation at earth is obtained by taking differences in eq. (14); to first order in  $\delta E$  it is already large:

$$t_1 - t_2 \sim 600 D_{\text{Mpc}} \left( \frac{Z B_{nG} \lambda}{E_{20}} \right)^2 \left( \frac{\delta E}{E_{20}} \right) \text{ yrs}. \quad (15)$$

The correlation in energy and time becomes even more significant when it is remembered that the higher energy primary has an even higher mean energy in transit, before losses on the 2.7K background. On the other hand, certain magnetic field configurations can lead to a counter-intuitive *earlier* arrival time for the lower energy charged-primary, as shown in D.Harari, S. Mollerach and E. Roulet, J. High Energy Phys. 02, 035 (2000).

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[26] To estimate the celestial sky coverage  $\Omega$  for ground-based experiments, one rotates the fixed-coordinate solid angle about the earth's axis of rotation. For experiments with

vertical acceptance from the zenith to  $\theta_z$ , e.g. AGASA and Auger/TA, the relevant formula is

$$\Omega = 2\pi \begin{cases} 2 \sin \theta_z \cos \alpha, & \text{for } \alpha + \theta_z < 90^\circ \\ 1 + \sin(\theta_z - \alpha), & \text{for } \alpha + \theta_z > 90^\circ \end{cases}$$

where  $\alpha$  is the latitude of the experiment, which is  $36^\circ\text{N}$  for AGASA and  $35^\circ\text{S}$  for Auger.

For experiments with horizontal acceptance from  $\theta_1$  to  $\theta_2$  with respect to the horizon, e.g. HiRes,

$$\Omega = 2\pi \begin{cases} \cos(\alpha + \theta_1) + \cos(\alpha - \theta_2), & \text{for } \theta_2 < \alpha \\ \cos(\alpha + \theta_1) + 1, & \text{for } \theta_2 > \alpha \end{cases}$$

For HiRes,  $\alpha \sim 40^\circ\text{N}$ . Finally, for an orbiting experiment such as EUSO/OWL/AW, the celestial coverage is nearly  $4\pi$  steradians.

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