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# Developments in Supergravity Unified Models\*

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## Abstract

A review is given of developments in supergravity unified models proposed in 1982 and their implications for current and future experiment are discussed.

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## I. INTRODUCTION

Supersymmetry (SUSY) was initially introduced as a global symmetry [1, 2] on purely theoretical grounds that nature should be symmetric between bosons and fermions. It was soon discovered, however, that models of this type had a number of remarkable properties [3]. Thus the bose-fermi symmetry led to the cancellation of a number of the infinities of conventional field theories, in particular the quadratic divergences in the scalar Higgs sector of the Standard Model (SM). Thus SUSY could resolve the gauge hierarchy problem that plagued the SM. Further, the hierarchy problem associated with grand unified models [4] (GUT), where without SUSY, loop corrections gave all particles GUT size masses [5, 6] was also resolved. In addition, SUSY GUT models with minimal particle spectrum raised the value for the scale of grand unification,  $M_G$ , to  $M_G \cong 2 \times 10^{16}$  GeV, so that the predicted proton decay rate [6, 7] was consistent with existing experimental bounds. Thus in spite of the lack of any direct experimental evidence for the existence of SUSY particles, supersymmetry became a highly active field among particle theorists.

However, by about 1980, it became apparent that global supersymmetry was unsatisfactory in that a phenomenologically acceptable picture of spontaneous breaking of supersymmetry did not exist. Thus the success of the SUSY grand unification program discussed above was in a sense spurious in that the needed SUSY threshold  $M_S$  (below which the SM held) could not be theoretically constructed. In order to get a phenomenologically viable model, one needed “soft breaking” masses [8] (i.e. supersymmetry breaking terms of dimension  $\leq 3$  which maintain the gauge hierarchy) and these had to be introduced in an ad hoc fashion by hand. In the minimal SUSY model, the MSSM [9], where the particle spectrum is just that of the supersymmetrized SM, one could introduce as many as 105 additional parameters (62 new masses and mixing angles and 43 new phases) leaving one with a theory with little predictive power.

A resolution of the problem of how to break supersymmetry spontaneously was achieved by promoting supersymmetry to a local symmetry [10], and specifically supergravity [11]. Here gravity is included into the dynamics. One can then construct supergravity [SUGRA] grand unified models [12, 13] where the spontaneous breaking of supersymmetry occurs in a “hidden” sector via supergravity interactions in a fashion that maintains the gauge hierarchy. In such theories there remains, however, the question of at what scale does supersymmetry

break, and what is the “messenger” that communicates this breaking from the hidden to the physical sector. In this chapter we consider models where supersymmetry breaks at a scale  $Q > M_G$  with gravity being the messenger[12, 14–17]. Such models are economical in that both the messenger field and the agency of supersymmetry breaking are supersymmetrized versions of fields and interactions that already exist in nature (i.e. gravity). Alternately within gravity mediation, supersymmetry could be broken by gaugino condensation[18]. This mechanism is a likely possibility within string theory.

The strongest direct evidence supporting supergravity GUT models is the apparent experimental grand unification of the three gauge coupling constants [19]. This result is non-trivial not only because three lines do not ordinarily intersect at one point, but also because there is only a narrow acceptable window for  $M_G$ . Thus one requires  $M_G \gtrsim 5 \times 10^{15} \text{ GeV}$  so as not to violate current experimental bounds on proton decay for the  $p \rightarrow e^+ + \pi^0$  channel (which occurs in almost all GUT models) and one requires  $M_G \lesssim 5 \times 10^{17} \text{ GeV} \cong M_{string}$  (the string scale) so that gravitational effects do not become large invalidating the analysis. Further, assuming an MSSM type of particle spectrum between the electroweak scale  $M_Z$  and  $M_G$ , acceptable grand unification occurs only with one pair of Higgs doublets and at most four generations. Finally, naturalness requires that SUSY thresholds be at  $M_S \lesssim 1 \text{ TeV}$  which turns out to be the case. Thus the possibility of grand unification is tightly constrained.

As discussed in Sec.(V) below, the grand unified models with R parity invariance produce a natural candidate for the dark matter observed astronomically. Further, the amount of dark matter produced in the early universe can be calculated, and remarkably the theory naturally predicts a relic density of dark matter today of size seen by WMAP and other observations. Thus SUGRA GUTS allows the construction of models valid from mass  $M_G$  down to the electroweak scale, and backwards in time to  $\sim 10^{-8}$  sec after the Big Bang (when the dark matter was created), a unification of particle physics and early universe cosmology. At present, grand unification in SUGRA GUTs can be obtained to within about 2-3 std. [20, 21] However, the closeness of  $M_G$  to the Planck scale,  $M_{Pl} = (\hbar c/8\pi G_N)^{1/2} \cong 2.4 \times 10^{18} \text{ GeV}$ , suggests the possibility that there are  $O(M_G/M_{Pl})$  corrections to these models. One might, in fact, expect such structures to arise in string theory as nonrenormalizable operators (NROs) obtained upon integrating out the tower of Planck mass states. Such terms would produce  $\approx 1\%$  corrections at  $M_G$  which might grow to  $\approx 5\%$  corrections at  $M_Z$ . Indeed, as will be seen in Sec.(II), it is just such NRO terms involving the hidden sector fields that give

rise to the soft breaking masses, and so it would not be surprising to find such structures in the physical sector as well. Thus SUGRA GUTs should be viewed as an effective theory and, as will be discussed in Sec.(VIII), with small deviations between theory and experiment perhaps opening a window to Planck scale physics.

One of the fundamental aspects of the SM, not explained by that theory, is the origin of the spontaneous breaking of  $SU(2) \times U(1)$ . SUGRA GUTS offers an explanation of this due to the existence of soft SUSY breaking masses at  $M_G$ . Thus as long as at least one of the soft breaking terms are present at  $M_G$ , breaking of  $SU(2) \times U(1)$  can occur at a lower energy [12, 22], providing a natural Higgs mechanism. Further, radiative breaking occurs at the electroweak scale provided the top quark is heavy ie.  $100 \text{ GeV} \lesssim m_t \lesssim 200 \text{ GeV}$ . The minimal SUGRA model[12, 15, 16](mSUGRA), which assumes universal soft breaking terms, requires only four additional parameters and one sign to describe all the interactions and masses of the 32 SUSY particles. Thus the mSUGRA is predictive model producing many sum rules among the sparticle masses [23], and for that reason the model is used in much of the phenomenological analysis of the past decades. However, we will see in Sec.(II) that there are reasons to consider non-universal extensions of the mSUGRA, and inclusion of the nonuniversalities can produce significant modifications of the sparticle masses and their signatures.

## II. SOFT BREAKING MASSES

Supergravity interactions with chiral matter fields,  $\{\chi_i(x), \phi_i(x)\}$  (where  $\chi_i(x)$  are left (L) Weyl spinors and  $\phi_i(x)$  are complex scalar fields) depend upon three functions of the scalar fields: the superpotential  $W(\phi_i)$ , the gauge kinetic function  $f_{\alpha\beta}(\phi_i, \phi_i^\dagger)$  (which enters in the Lagrangian as  $f_{\alpha\beta} F_{\mu\nu}^\alpha F^{\mu\nu\beta}$  with  $\alpha, \beta =$  gauge indices) and the Kahler potential  $K(\phi_i, \phi_i^\dagger)$  (which appears in the scalar kinetic energy as  $K_j^i \partial_\mu \phi_i \partial^\mu \phi_j^\dagger$ ,  $K_j^i \equiv \partial^2 K^2 / \partial \phi_i \partial \phi_j^\dagger$  and elsewhere).  $W$  and  $K$  enter only in the combination

$$G(\phi_i, \phi_i^\dagger) = \kappa^2 K(\phi_i, \phi_i^\dagger) + \ell n[\kappa^6 |W(\phi_i)|^2] \quad (1)$$

where  $\kappa = 1/M_{Pl}$ . Writing  $\{\phi_i\} = \{\phi_a, z\}$  where  $\phi_a$  are physical sector fields (squarks, sleptons, higgs) and  $z$  are the hidden sector fields whose VEVs  $\langle z \rangle = \mathcal{O}(M_{Pl})$  break supersymmetry, one assumes that the superpotential decomposes into a physical and a hidden

part,

$$W(\phi_i) = W_{phy}(\phi_a, \kappa z) + W_{hid}(z) \quad (2)$$

Supersymmetry breaking is scaled by requiring  $\kappa^2 W_{hid} = \mathcal{O}(M_S) \tilde{W}_{hid}(\kappa z)$  and the gauge hierarchy is then guaranteed by the additive nature of the terms in Eq.(2). Thus only gravitational interactions remain to transmit SUSY breaking from the hidden sector to the physical sector.

A priori, the functions W, K and  $f_{\alpha\beta}$  are arbitrary. However, they are greatly constrained by the conditions that the model correctly reduce to the SM at low energies, and that non-renormalizable corrections be scaled by  $\kappa$  (as would be expected if they were the low energy residue of string physics of the Planck scale). Thus one can expand these functions in polynomials of the physical fields  $\phi_a$

$$\begin{aligned} f_{\alpha\beta}(\phi_i) &= c_{\alpha\beta} + \kappa d_{\alpha\beta}^a(x, y) \phi_a + \dots, \\ W_{phys}(\phi_i) &= \frac{1}{6} \lambda^{abc}(x) \phi_a \phi_b \phi_c + \frac{1}{24} \kappa \lambda^{abcd}(x) \phi_a \phi_b \phi_c \phi_d + \dots, \\ K(\phi_i, \phi_i^\dagger) &= \kappa^{-2} c(x, y) + c_b^a(x, y) \phi_a \phi_b^\dagger \\ &+ (c^{ab}(x, y) \phi_a \phi_b + h.c.) + \kappa (c_{bc}^a \phi_a \phi_b^\dagger \phi_c^\dagger + h.c.) + \dots. \end{aligned} \quad (3)$$

Here  $x = \kappa z$  and  $y = \kappa z^\dagger$ , so that  $\langle x \rangle, \langle y \rangle = \mathcal{O}(1)$ . The scaling hypothesis for the NRO's imply then that the VEVs of the coefficients  $c_{\alpha\beta}$ ,  $c_{\alpha\beta}^a$ ,  $\lambda^{abc}$ ,  $c$ ,  $c_b^a$ ,  $c^{ab}$  etc. are all  $\mathcal{O}(1)$ . The holomorphic terms in K labeled by  $c^{ab}$  can be transformed from K to W by a Kahler transformation,  $K \rightarrow K - (c^{ab} \phi_a \phi_b + h.c.)$  and

$$W \rightarrow W \exp[\kappa^2 c^{ab} \phi_a \phi_b] = W + \tilde{\mu}^{ab} \phi_a \phi_b + \dots \quad (4)$$

where  $\tilde{\mu}^{ab}(x, y) = \kappa^2 W c^{ab}$ . Hence  $\langle \tilde{\mu}^{ab} \rangle = \mathcal{O}(M_S)$ , and one obtains a  $\mu$ -term with the right order of magnitude after SUSY breaking provided only that  $c^{ab}$  is not zero [24]. The cubic terms in W are just the Yukawa couplings with  $\langle \lambda^{abc}(x) \rangle$  being the Yukawa coupling constants. Also  $\langle c_{\alpha\beta} \rangle = \delta_{\alpha\beta}$ ,  $\langle c_b^a \rangle = \delta_b^a$  and  $\langle c_{xy} \rangle = 1$  ( $c_x \equiv \partial c / \partial x$  etc.) so that the field kinetic energies have canonical normalization.

The breaking of SUSY in the hidden sector leads to the generation of a series of soft breaking terms [12, 14–17]. We consider here the case where  $\langle x \rangle = \langle y \rangle$  (i.e. the hidden sector SUSY breaking does not generate any CP violation) and state the leading terms. Gauginos gain a soft breaking mass term at  $M_G$  of  $\tilde{m}_{\alpha\beta} \lambda^\alpha \gamma^0 \lambda^\beta$  ( $\lambda^\alpha =$  gaugino Majorana field) where

$$\tilde{m}_{\alpha\beta} = \kappa^{-2} \langle G^i (K^{-1})_j^i \text{Re} f_{\alpha\beta j}^\dagger \rangle m_{3/2} \quad (5)$$

Here  $G^i \equiv \partial G / \partial \phi_i$ ,  $(K^{-1})^i_j$  is the matrix inverse of  $K^i_j$ ,  $f_{\alpha\beta j} = \partial f_{\alpha\beta} / \partial \phi_j^\dagger$  and  $m_{3/2}$  is the gravitino mass:  $m_{3/2} = \kappa^{-1} \langle \exp[G/2] \rangle$ . In terms of the expansion of Eq.(3) one finds

$$\tilde{m}_{\alpha\beta} = [c + \ell n(W_{hid})]_x \text{Re } c_{\alpha\beta y}^* m_{3/2}, \quad (6)$$

and  $m_{3/2} = (\exp \frac{1}{2} c) \kappa^2 W_{hid}$  where it is understood from now on that  $x$  is to be replaced by its VEV in all functions (e.g.  $c(x) \rightarrow c(\langle x \rangle) = \mathcal{O}(1)$ ) so that  $m_{3/2} = \mathcal{O}(M_S)$ . One notes the following about Eq.(6): (i) For a simple GUT group, gauge invariance implies that  $c_{\alpha\beta} \sim \delta_{\alpha\beta}$  and so gaugino masses are universal (labeled by  $m_{1/2}$ ) at mass scales above  $M_G$ . (ii) While  $m_{1/2}$  is scaled by  $m_{3/2} = \mathcal{O}(M_S)$ , it can differ from it by a significant amount. (iii) From Eq.(6) one sees that it is the NRO such as  $\kappa z m_{3/2} \lambda^\alpha \gamma^0 \lambda^\alpha$  that gives rise to  $m_{1/2}$ . Below  $M_G$ , where the GUT group is broken, the second term in the  $f_{\alpha\beta}$  of Eq.(3) would also contribute yielding a NRO of size  $\kappa d_{\alpha\beta}^a \phi_a m_{3/2} \lambda^\alpha \gamma^0 \lambda^\beta \sim (M_G/M_{Pl}) m_{3/2} \lambda \gamma^0 \lambda$  [25] for fields with VEV  $\langle \phi_a \rangle = \mathcal{O}(M_G)$  which break the GUT group. Such terms give small corrections to the universality of the gaugino masses and affect grand unification. They are discussed in Sec.(VIII).

The effective potential for the scalar components of chiral multiplets is given by [12, 26]

$$\begin{aligned} V &= e^{\kappa^2 K} [(K^{-1})^j_i (W^i + \kappa^2 K^i W) (W^j + \kappa^2 K^j W)^\dagger - 3\kappa^2 |W|^2] + V_D, \\ V_D &= \frac{1}{2} g_\alpha g_\beta (\text{Re } f^{-1})_{\alpha\beta} (K^i (T^\alpha)_{ij} \phi_j) (K^k (T^\beta)_{kl} \phi_l), \end{aligned} \quad (7)$$

where  $W^i = \partial W / \partial \phi_i$  etc., and where  $g_\alpha$  are the gauge coupling constants. Eqs.(2-4) then lead to the following soft breaking terms at  $M_G$ :

$$V_{soft} = (m_0^2)_b^a \phi_a \phi_b^\dagger + \left[ \frac{1}{3} \tilde{A}^{abc} \phi_a \phi_b \phi_c + \frac{1}{2} \tilde{B}^{ab} \phi_a \phi_b + h.c. \right] \quad (8)$$

In the following, we impose for simplicity the condition that the cosmological constant vanish after SUSY breaking, i.e.  $\langle V \rangle = 0$ . [One of course could accommodate the tiny cosmological constant suggested by the supernova observation.] This is a fine tuning of  $\mathcal{O}(M_S^2 M_{Pl}^2)$ . From Eq.(7) one notes that the soft breaking terms are in general not universal unless one assumes that the fields  $z$  couple universally to the physical sector.

### III. RADIATIVE BREAKING AND THE LOW ENERGY THEORY

In Sec.(II), the SUGRA GUT model above the GUT scale i.e. at  $Q > M_G$  was discussed. Below  $M_G$  the GUT group is spontaneously broken, and we will assume here that the SM

group,  $SU(3) \times SU(2) \times U(1)$ , holds for  $Q < M_G$ . Contact with accelerator physics at low energy can then be achieved using the renormalization group equations (RGE)[27] running from  $M_G$  to the electroweak scale  $M_Z$ . As one proceeds downward from  $M_G$ , the coupling constants and masses evolve, and provided at least one soft breaking parameter and also the  $\mu$  parameter at  $M_G$  is not zero, the large top quark Yukawa can turn the  $H_2$  running  $(\text{mass})^2$ ,  $m_{H_2}^2(Q)$ , negative at the electroweak scale [22]. Thus the spontaneous breaking of supersymmetry at  $M_G$  triggers the spontaneous breaking of  $SU(2) \times U(1)$  at the electroweak scale. In this fashion all the masses and coupling constants at the electroweak scale can be determined in terms of the fundamental parameters (Yukawa coupling constants and soft breaking parameters) at the GUT scale, and the theory can be subjected to experimental tests.

The conditions for electroweak symmetry breaking arise from minimizing the effective potential  $V$  at the electroweak scale with respect to the Higgs VEVs  $v_{1,2} = \langle H_{1,2} \rangle$ . This leads to the equations [22]

$$\mu^2 = \frac{\mu_1^2 - \mu_2^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{1}{2} M_Z^2; \quad \sin^2 \beta = \frac{-2B\mu}{2\mu^2 + \mu_1^2 + \mu_2^2} \quad (9)$$

where  $\tan \beta = v_2/v_1$ ,  $B$  is the quadratic soft breaking parameter ( $V_{soft}^B = B\mu H_1 H_2$ ),  $\mu_i = m_{H_i}^2 + \Sigma_i$ , and  $\Sigma_i$  are loop corrections [28]. All parameters are running parameters at the electroweak scale which one takes for convenience to be  $Q \simeq \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$  to minimize loop corrections. Eq.(9) then determines the  $\mu$  parameter and allows the elimination of  $B$  in terms of  $\tan \beta$ . This determination of  $\mu$  greatly enhances the predictive power of the model.

In general there are two broad regions of electroweak symmetry breaking implied by the soft parameters appearing in Eq.(9). One region is where the soft parameters can be arranged to lie on the surface of an ellipsoid with their radii fixed by the value of  $\mu$ . In this case for fixed  $\mu$ ,  $m_0$  and  $m_{\frac{1}{2}}$  cannot get too large since the surface of an ellipsoid is a closed surface. However, it turns out that when loop corrections[28] to the effective potential are included the nature of electroweak symmetry breaking can change rather drastically. Then the soft parameters instead of lying on the surface of an ellipsoid, lie on the surface of a hyperboloid and this branch may appropriately be called the hyperbolic branch (HB)(see the first paper of [29]). Since the surface of a hyperboloid is open, the soft parameters can get large with  $\mu$  fixed. Specifically, the HB allows TeV size scalars with small values of  $\mu$  and thus small fine tunings. The region of TeV size scalars is also known as the Focus Point

(FP) region (see the second paper of [29]).

The renormalization group equations evolve the universal gaugino mass  $m_{1/2}$  at  $M_G$  to separate masses for SU(3), SU(2) and U(1) at  $M_Z$

$$\tilde{m}_i = (\alpha_i(M_Z)/\alpha_G)m_{1/2}; \quad i = 1, 2, 3 \quad (10)$$

where at 1-loop, the gluino mass  $m_{\tilde{g}} = \tilde{m}_3$  [30]. The simplest model is the one with universal soft breaking masses. This model depends on only the four SUSY parameters and one sign at the GUT scale[12, 15, 16]

$$m_0, \quad m_{1/2}, \quad A_0, \quad B_0, \quad \text{sign}(\mu_0) \quad (11)$$

Alternately, at the electroweak scale one may choose  $m_0$ ,  $m_{\frac{1}{2}}$ ,  $A_t$ ,  $\tan\beta$ , and  $\text{sign}(\mu)$  as the independent parameters. Universality can be derived in a variety of ways. From a string view point it could arise, for example, from dilaton dominance, or when modular weights are all equal, and in both GUTS and in strings it could arise from a family symmetry at the GUT/string scale. This case has been extensively discussed in the literature [31–35]. The deviations from universality can be significant, however, and affect  $\mu^2$  and sparticle masses, and these parameters play a crucial role in predictions of the theory. Several analyses exist where the SUGRA models have been extended to include non-universalities [36–39]. These extensions include non-universalities in the gaugino sector, in the Higgs sector and in the third generation sector consistent with flavor changing neutral current constraints.

#### IV. SUPERSYMMETRIC CORRECTIONS TO ELECTROWEAK PHENOMENA

SUGRA models make contributions to all electroweak processes at the loop level through the exchange of sparticles. We discuss here some of the more prominent ones which include the muon anomalous magnetic moment  $g_\mu - 2$ , and the the flavor changing neutral current processes  $b \rightarrow s\gamma$  and  $B_s^0 \rightarrow \mu^+\mu^-$ . These processes are all probes of new physics. Thus in  $g_\mu - 2$  the sparticle loops at one loop make contributions (see Fig.1)[40] which are comparable to the electroweak contributions from the Standard Model[40]. The most recent evaluations of the difference between experiment and theory give for  $\delta a_\mu = (g_\mu^{exp} - g_\mu^{SM})/2$ , the result[41]

$$\delta a_\mu = (24.6 \pm 8.0) \times 10^{-10} \quad (12)$$

If the above result holds up it would imply upper limits on sparticle masses within the range of the LHC energies <sup>1</sup>. These conclusions were already drawn earlier[40, 43–45].

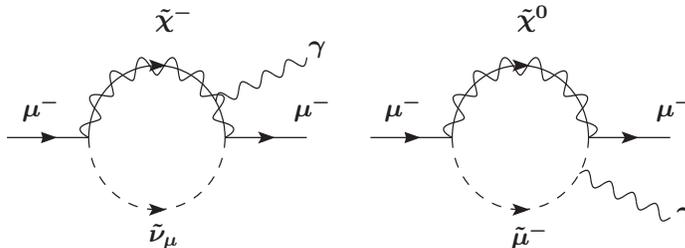


FIG. 1: Supersymmetric electroweak contributions to  $g_\mu - 2$

Flavor changing neutral current processes also provide an important constraint on supergravity unified models. A process of great interest here is the decay  $b \rightarrow s + \gamma$ . Further, it is well known from the early days that  $b \rightarrow s + \gamma$  experiment imposes an important constraint on the parameter space of supergravity models[46] and specifically on the analysis of dark matter. The current experimental value for this branching ratio from the the Heavy Flavor Averaging Group (HFAG) [47] along with the BABAR, Belle and CLEO experimental results gives  $\mathcal{B}r(B \rightarrow X_s \gamma) = (352 \pm 23 \pm 9) \times 10^{-6}$ . In the SM this decay proceeds at the loop level with the exchange of W and Z bosons and the most recent evaluation including the next to next leading order (NNLO) QCD corrections is given by [48]  $\mathcal{B}r(b \rightarrow s \gamma) = (3.15 \pm 0.23) \times 10^{-4}$ . In supersymmetry there are additional diagrams which contribute to this process [49]. Thus in SUGRA unification one has contributions from the exchange of the charged Higgs, the charginos, the neutralinos and from the gluino. It is well known that the contribution from the charged Higgs exchange is always positive [50] while the contribution from the exchange of the other SUSY particles can be either positive or negative with the contribution of the charginos being usually the dominant one [51].

A comparison of the experimental and theoretical evaluations in the SM point to the

<sup>1</sup> In most extra dimension models the corrections to  $g_\mu - 2$  are rather small [42] and it is difficult to accommodate a deviation of size Eq.(12).

possibility that a positive correction to the SM value is needed. As noted above such a positive correction can arise from supersymmetry specifically from the exchange of the charged higgs which implies the possibility of a relatively light charged Higgs. Further, over most of the parameter space the chargino exchange contributions are often negative pointing to a cancellation between the charged Higgs and the chargino exchange contributions and also hint at the possibility of a relatively light chargino and possibly of a relatively light stop.

The rare process  $B_s \rightarrow \mu^+ \mu^-$  is of interest as it is a probe of physics beyond the standard model[52, 53]. The branching ratio for this process in the SM is  $\mathcal{B}r(B_s \rightarrow \mu^+ \mu^-) = (3.1 \pm 1.4) \times 10^{-9}$  (for  $V_{ts} = 0.04 \pm 0.002$ ). In supersymmetric models it can get large for large  $\tan \beta$  since decay branching ratio increases as  $\tan^6 \beta$ . The current experimental limit at 95% (90%) C.L. reported by CDF is  $\mathcal{B}r(B_s \rightarrow \mu^+ \mu^-) = 5.8 \times 10^{-8}$  ( $4.7 \times 10^{-8}$ ) [54]. Since in supersymmetric theories this branching ratio can increase as  $\tan^6 \beta$  the experimental data does constrain the analysis at least for large  $\tan \beta$  and the implications of this constraint have been analyzed in several works[55, 56]. Additionally, this specific decay is very sensitive to CP phases and thus the experiment also constrains the CP phases in SUGRA models in certain regions of the parameter space[57].

## V. DARK MATTER IN SUGRA UNIFICATION

As mentioned earlier one of the remarkable results of supergravity grand unification with R parity invariance is the prediction that the lightest neutralino  $\chi_1^0$  is the LSP over most of the parameter space [58]. In this part of the parameter space the  $\chi_1^0$  is a candidate for cold dark matter (CDM). We discuss now the relic density of  $\chi_1$  within the framework of the Big Bang Cosmology. The quantity that is computed theoretically is  $\Omega_{\chi_1} h^2$  where  $\Omega_{\chi_1} = \rho_{\chi_1} / \rho_c$ ,  $\rho_{\chi_1}$  is the neutralino relic density and  $\rho_c$  is the critical relic density needed to close the universe,  $\rho_c = 3H^2 / 8\pi G_N$ , and  $H = h100km/sMpc$  is the Hubble constant. One of the important elements in the computation of the relic density concerns the correct thermal averaging of the quantity  $(\sigma v)$  where  $\sigma$  is the neutralino annihilation cross section in the early universe and  $v$  is the relative neutralino velocity. Normally the thermal average is calculated by first making the approximation  $\sigma v = a + bv^2$  and then evaluating its thermal average [59]. However, it is known that such an approximation breaks down in the vicinity

of thresholds and poles [60]. Precisely such a situation exists for the case of the annihilation of the neutralino through the Z and Higgs poles. An accurate analysis of the neutralino relic density in the presence of Z and Higgs poles was given in ref. [61]<sup>2</sup> and similar analyses have also been carried out since by other authors [63]. There are a number of possibilities for the detection of dark matter both direct and indirect [64]. We discuss first the direct method which involves the scattering of incident neutralino dark matter in the Milky Way from nuclei in terrestrial targets. The event rates consist of two parts [65]: one involves an axial interaction and the other a scalar interaction. The axial (spin dependent) part  $R_{SD}$  falls off as  $R_{SD} \sim 1/M_N$  for large  $M_N$  where  $M_N$  is the mass of the target nucleus, while the scalar (spin independent) part behaves as  $R_{SI} \sim M_N$  and increases with  $M_N$ . Thus for heavy target nuclei the spin independent part  $R_{SI}$  dominates over most of the parameter space of the model.

In recent years the direct detection dark matter experiments have begun to provide significant bounds on the spin independent neutralino -proton cross section  $\sigma_{\tilde{\chi}^0 p}$  and thus theoretical computations of this quantity can be directly compared with the data. The predictions of the SUGRA models lie over a wide range. With typical assumptions of naturalness on sparticle masses below 1 TeV,  $\sigma_{\tilde{\chi}^0 p}$  can lie in the range  $10^{-43}$  cm<sup>2</sup> to  $10^{-48}$  cm<sup>2</sup>. The current experiments such as CDMS[66] and XENON[67] have already begun to constrain a part of the SUGRA parameter space and improved experiments[68] are expected to probe the parameter space further<sup>3</sup>. Inclusion of non-universalities is seen to produce definite signatures in the event rate analysis [37, 38]. The satisfaction of the relic density in SUGRA models can occur via coannihilations. One of the most studied coannihilation is with the coannihilation of the neutralino with the stau. However, with the inclusion of non-universalities of soft parameters many other coannihilations become possible such as coannihilations of the LSP with charginos, stops, gluinos, and heavier neutralinos etc. The coannihilation with the gluino which occurs most dominantly when the gluino is the NLSP and exhibits the interesting phenomenon that the sparticle production cross sections are dominated by the gluino production making the observation of other sparticles challenging[70].

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<sup>2</sup> The analysis of [61] has been used to show that annihilation near a Breit-Wigner pole generates a significant enhancement of  $\langle \sigma v \rangle_H$  in the halo of the galaxy relative to  $\langle \sigma v \rangle_{X_f}$  at the freezeout[62].

<sup>3</sup> Recently the CDMS experiment has observed two events fitting the behavior of WIMPS with a background of  $0.6 \pm 0.1$  events[69]. Further, data is needed to confirm this.

In addition to the direct detection dark matter experiments, there are also indirect signatures for dark matter. Thus, e.g., neutrinos arising from the annihilation of neutralinos in the center of the Earth and Sun can produce detectable signals. Further, it was suggested quite sometime ago [71] that the annihilation process  $\tilde{\chi}^0\tilde{\chi}^0 \rightarrow W^+W^-$  with the subsequent decays of the W's, e.g.,  $W^+ \rightarrow e^+\nu$  could generate a detectable positron excess in anti-matter probes. One of the typical problems encountered in most theoretical analyses of the positron excess is the following: in order to have the appropriate positron signal in PAMELA [72] one needs to have a  $\chi^0\chi^0$  annihilation cross section to  $WW$  with  $\langle\sigma v\rangle_{WW} \simeq 10^{-24} \text{cm}^3/\text{s}$ . However, the relic density has an inverse proportionality to the annihilation cross section at the freeze out, i.e.,  $\Omega_{\tilde{\chi}^0} h^2 \propto [\int_{x_f}^{\infty} \langle\sigma_{eff} v\rangle \frac{dx}{x^2}]^{-1}$  which leads to too low a relic density. To overcome this problem most works typically resort to large so called boost factors. Effectively, what this implies is that the annihilation cross section of dark matter is taken to satisfy the relic density and then to get the right strength positron signal a boost factor is assumed. It is argued that such boost factors can arise from clumping of dark matter in the galaxy. However, while a boost factor of O(2-10) could arise from clumping of dark matter in the galaxy, it appears unreasonable to assume large clumping factors (sometimes as large as  $10^3$  or even larger) to fit the data. In the context of the minimal supergravity model, one simple solution arises due to the automatic suppression of the relic density from coannihilation effects with hidden sector matter in SUGRA models with an extended  $U(1)^n$  sector [73]. In this case with  $n=3$ , one finds good fits to the positron excess from PAMELA while maintaining the neutralino relic density in the WMAP [74] error corridor. At the same time one can maintain compatibility with the anti-proton flux [72] and the photon flux from the FERMI-LAT experiment [75].

## VI. SIGNATURES AT COLLIDERS

Sparticle decays produce missing energy signals since at least one of the carriers of missing energy will be the neutralino. Signals of this type were studied early on after the advent of supergravity models in the supersymmetric decays of the W and Z bosons [76] and such analyses have since been extended to the decays of all of the supersymmetric particles (For a review of sparticle decays see [13]). Using these decay patterns one finds a variety of supersymmetric signals for SUSY particles at colliders where SUSY particles are expected

to be pair produced when sufficient energies in the center of mass system are achieved. One signal of special interest in the search for supersymmetry is the trileptonic signal through off shell  $W^*$  production as well as via other production and decay chains [77]. For example in  $p\bar{p}$  collisions one can have  $p\bar{p} \rightarrow \tilde{\chi}_1^\pm + \tilde{\chi}_2^0 + X \rightarrow (l_1\bar{\nu}_1\tilde{\chi}_1^0) + (l_2\bar{l}_2\tilde{\chi}_1^0) + X$  which gives a signal of three leptons and missing energy. In addition to the trileptonic signal there is a long list of possible signatures for the discovery of supersymmetry and test of SUGRA models. These include multileptons and multijet and missing energy. Thus one can devise a variety of combinations with n number of leptons ( $e$  or  $\mu$ ), m number of  $\tau$ 's, k number of jets ( $m, n = 0, 1, 2, 3, \dots; k \geq 2$ ) leading to a large number of possibilities. Further, one can add to this list tagged b jet signals and kinematical signatures such as missing transverse momentum  $P_T^{miss}$ , effective mass  $P_T^{miss} + \sum_j P_T^j$ , invariant mass of  $e^+e^-$ ,  $\mu^+\mu^-$ ,  $\tau^+\tau^-$ , and invariant mass of all jets which provide important signatures.

An important result concerns the fact that one can utilize measurements at the LHC to predict phenomena related to dark matter, showing the unification of particle physics and cosmology. Within the mSUGRA framework, existing constraints on the parameter space combined with the cold dark matter constraints pick out three regions: (i) the  $\tilde{\chi}_1 - \tilde{\tau}_1$  coannihilation (CA) region where  $m_0$  is small but  $m_{1/2}$  can rise to 1 TeV, (ii) the hyperbolic (HB)/focus (FP) region, where  $m_{1/2}$  is relatively small but  $m_0$  is large, and (iii) the pole region[58] (alternately called the funnel region) where annihilation in the early universe goes via heavy Higgs poles. For the CA region it can be shown[78] that purely from the measurements at the LHC one can predict the dark matter relic density with an uncertainty of 6% with  $30\text{fb}^{-1}$  of data which is comparable to the uncertainty in the determination of the relic density by WMAP. The relevant signal here consists of low energy  $\tau$  leptons from  $\tilde{\chi}_2^0 \rightarrow \tau\tilde{\tau}_1 \rightarrow \tau\tau\tilde{\chi}_1^0$  where the mass difference of the  $\tilde{\tau}_1$  and  $\tilde{\chi}_1^0$  is constrained to lie within in 5-15 GeV by the current experimental bounds[81]. In addition it is possible to test experimentally the universality of the gaugino masses and if not, measure the amount of non-universality as well as obtain precision measurements of the gaugino mass, squark and lighter stau masses. A similar analysis may be carried out in the HB/FP region[79] and very likely can be done for the ILC[80].

We discuss now an approach by which the LHC data can be used to discriminate among a variety of models. This approach utilizes the idea of sparticle mass hierarchies which we now describe. Thus as mentioned already in MSSM there are 32 supersymmetric particles

including the Higgs fields. In general they can generate a large number of mass hierarchies. Assuming the lightest sparticle is the lightest neutralino, there are still in excess of  $10^{25}$  possible mass hierarchies in which the sparticles can arrange themselves. Of these only one will eventually be realized if all the sparticle masses are finally measured at the LHC or in other future collider experiments. The question then is how predictive are SUGRA models in pinning down the mass hierarchical patterns. The above question can be answered within the SUGRA framework including the REWSB constraints, the WMAP and other relevant experimental constraints[82]. An analysis along these lines but limited to four particle mass hierarchies aside from the lightest Higgs boson mass would in general lead to roughly  $O(10^4)$  such mass hierarchical patterns. However, within the mSUGRA framework with the constraints mentioned above one finds that the number of possibilities reduces to just 16 for  $\mu$  positive and 6 more for  $\mu$  negative. These possibilities are labeled as minimal supergravity patterns mSP1-mSP16 for  $\mu > 0$  and mSP17-mSP22 for  $\mu < 0$ . These allowed set of models can be further subdivided into classes according to their next to the lowest mass particle (NLSP). Thus one finds the dominant sub classes of patterns among mSP1-mSP16 to be the Chargino Pattern, Stau Pattern, Stop Pattern and Higgs Pattern. In addition for  $\mu < 0$  one finds additional Stau and Stop Patterns and also a Neutralino Pattern where the second neutralino is the NLSP[82].

A similar analysis can also be carried out for the non-universal SUGRA case. Here allowing for non-universalities in the Higgs sector, gaugino sector and in the third generation sector one finds 22 new sparticle patterns for the first four sparticles (excluding the lightest Higgs boson). These are labeled NUSP1-NUSP22. It is found that no new patterns arise from non-universalities in the Higgs sector and all the new patterns are from non-universalities in the gaugino sector and in the third generation sectors. It is shown in [82] that these new patterns have distinctive signatures and can be discriminated by appropriate choices of events with leptons, jets and missing energy. Specifically using leptons, jets and missing energy events one can discriminate between the stau coannihilation branch and the hyperbolic branch/focus point region. Thus it is also found that one can identify the origin of dark matter using LHC data.

## VII. CP VIOLATION

The minimal supergravity model with universal soft breaking has two independent CP violating phases which can be chosen to be the phase of the  $\mu$  parameter ( $\theta_\mu$ ) and the phase of the trilinear coupling  $A_0$  ( $\alpha_{A_0}$ ). In the more general soft breaking of the non-universal supergravity model, one may have many more phases. For instance, with non-universal gaugino masses each of the gaugino masses in the  $U(1)_Y$ ,  $SU(2)_L$  and  $SU(3)_C$  may have a phase, i.e.,  $\tilde{m}_i = |\tilde{m}_i|e^{i\xi_i}$  ( $i=1,2,3$ ) of which two are independent. Similarly, the trilinear couplings may be complex and flavor dependent, so that  $A_a = |A_a|e^{i\alpha_{A_a}}$ . For the most general allowed soft breaking in MSSM, the list of allowed phases is much larger, and even after field redefinitions many CP phases remain. In general the CP phases lead to large supersymmetric contributions to the electric dipole moments (EDMs) of the neutron and of the leptons leading to their EDMs far in excess of experiment. These EDMs can be made compatible by a variety of means, such as through choice of small CP phases [83, 84], large sparticle masses [85], via the cancellation mechanism [86, 87], or via the CP phases arising only in the third generation [88]. If the CP phases are large they will lead to mixings [89] of CP even Higgs and CP odd Higgs states and there would be many phenomenological implications at colliders and elsewhere [90, 91].

## VIII. PLANCK SCALE CORRECTIONS AND FURTHER TESTS OF SUGRA GUT AND POST GUT PHYSICS

Because of the proximity of the GUT scale to the Planck scale one can expect corrections of size  $O(M_G/M_{Pl})$  to grand unification where  $M_{Pl}$  is the Planck mass. For example, Planck scale corrections can modify the gauge kinetic energy function so that one has for the gauge kinetic energy term  $-(1/4)f_{\alpha\beta}F_\alpha^{\mu\nu}F_{\beta\mu\nu}$ . For the minimal  $SU(5)$  theory,  $f_{\alpha\beta}$  in SUGRA models can assume the form  $f_{\alpha\beta} = \delta_{\alpha\beta} + (c/2M_{Pl})d_{\alpha\beta\gamma}\Sigma^\gamma$  where  $\Sigma$  is the scalar field in the 24 plet of  $SU(5)$ . After the spontaneous breaking of  $SU(5)$  and a re-diagonalization of the gauge kinetic energy function, one finds a splitting of the  $SU(3) \times SU(2) \times U(1)$  gauge coupling constants at the GUT scale. These splittings generate a corrections to  $\alpha_i(M_Z)$ , and using the LEP data one can put constraints on  $c$ . One finds that [21]  $-1 \leq c \leq 3$ . The Planck scale correction also helps relax the stringent constraint on  $\tan\beta$  imposed by  $b - \tau$  unification.

Thus in the absence of Planck scale correction one has that  $b - \tau$  unification requires  $\tan\beta$  to lie in two rather sharply defined corridors [92]. One of these corresponds to a small value of  $\tan\beta$ , i.e.,  $\tan\beta \sim 2$  and the second a large value  $\tan\beta \sim 50$ . This stringent constraint is somewhat relaxed by the inclusion of Planck scale corrections [21].

SUSY grand unified models contain many sources of proton instability. Thus in addition to the p decay occurring via the exchange of superheavy vector lepto-quarks, one has the possibility of p decay from dimension (dim) 4 (dim 3 in the superpotential) and dim 5 (dim 4 in the superpotential) operators [93]. The lepto-quarks exchange would produce  $p \rightarrow e^+\pi^0$  as its dominant mode with an expected lifetime [94] of  $\sim 1 \times 10^{35 \pm 1} [M_X/10^{16}]^4 y$  where  $M_X \cong 1.1 \times 10^{16}$  while the current lower limit on this decay mode from Super Kamiokande is  $\sim 2 \times 10^{33}$  yr. Thus the  $e^+\pi^0$  mode may be at the edge of being accessible in proposed experiments[95] such as at DUSEL which will have improved sensitivities for this decay mode. Proton decay from dim 4 operators is much too rapid but is easily forbidden by the imposition of R parity invariance. The p decay from dim 5 operators is more involved. It depends on both the GUT physics as well as on the low energy physics such as the masses of the squarks and of the gauginos. Analysis in supergravity unified models[96] shows that one can make concrete predictions of the p decay modes within these models once the sparticle spectrum is determined.

Precision determination of soft SUSY breaking parameters can be utilized as a vehicle for the test of the predictions of supergravity grand unification. Specifically it has been proposed that precision measurement of the soft breaking parameters can also act as a test of physics at the post GUT and string scales[97]. Thus, for example, if one has a concrete model of the soft breaking parameters at the string scale then these parameters can be evolved down to the grand unification scale leading to a predicted set of non-universalities there. If the SUSY particle spectra and their interactions are known with precision at the electro-weak scale, then this data can be utilized to test a specific model at the post GUT or string scales. Future colliders such as the LHC [98] and the NLC [99] will allow one to make mass measurements with significant accuracy. Thus accuracies of up to a few percent in the mass measurements will be possible at these colliders allowing a test of post GUT and string physics up to an accuracy of  $\sim 10\%$ [97].

## IX. CONCLUSION

Supergravity grand unification provides a framework for the supersymmetric unification of the electro-weak and the strong interactions where supersymmetry is broken spontaneously by a super Higgs effect in the hidden sector and the breaking communicated to the visible sector via gravitational interactions. The minimal version of the model based on a generation independent Kahler potential contains only four additional arbitrary parameters and one sign in terms of which all the SUSY mass spectrum and all the SUSY interaction structure is determined. This model is thus very predictive. A brief summary of the predictions and the phenomenological implications of the model were given. Many of the predictions of the model can be tested at current collider energies and at energies that would be achievable at the LHC. We also discussed here extensions of the minimal supergravity model to include non-universalities in the soft SUSY breaking parameters. Some of the implications of these non-universalities on predictions of the model were discussed. Future experiments should be able to see if the predictions of supergravity unification are indeed verified in nature.

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