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# Remanufacturing Control in Multistage Systems with Stochastic Recovery Rates

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## ABSTRACT

Remanufacturing operations involved with highly uncertain recovery rate of used products, subassemblies and parts that complicate the planning and control of the process. In this paper, we develop a comprehensive procedure that determines the optimal input quantities at each stage of the remanufacturing operations in which recovery rates at each stage of the process are stochastic. We model the remanufacturing system as an open queueing network and use the decomposition principle and expansion methodology to analyze it. Each station in the system is subject to breakdown and has a finite buffer capacity. Repair times, breakdown times and service times follow exponential distributions. Optimization is done on the system's expected total cost using a dynamic programming (DP) algorithm. A numerical example is presented to show the applicability of the model.

**Keywords:** Remanufacturing, recycling, disassembly, production planning and inventory control, reusable rate, disposal, open queueing network, expansion methodology, throughput, throughput approximation.

## 1. INTRODUCTION

A growing number of manufacturing companies have begun to consider recovering of their used products rather than to dispose them after they are discarded by the customers. This trend is a direct result of the implementation of extended manufacturer responsibility together with the new more rigid environmental legislation and the public awareness. In addition, the economic attractiveness of reusing products, subassemblies or parts instead of disposing them has further promoted this phenomenon.

Remanufacturing is an industrial process in which worn-out products are restored to "like-new" conditions. Thus, remanufacturing provides quality standards of new products with used parts. Remanufacturing is not only a direct and preferable way to reduce the amount of waste generated, it also reduces the consumption of natural resources.

One of the challenges of remanufacturing companies are facing is the supply-side of used products. The difficulty is not only quantity and timing uncertainty of return flow but also the quality and the associated recovery (re-usability) rate of the parts that are inducted from returned products in a series of remanufacturing operations. Thus, remanufacturing operations involved with highly variable reusability rate of the returned products that complicate the planning and control of the process. Thierry *et al.*<sup>20</sup> summarized the type of the return flow and identified their characteristics.

- Manufacturers could be required by law or by contract to take back used products.
- Off-lease and off-rent products.
- Products with technical failure.

Although these three major return forms provide an approximate information about return time and quantity of used products, recovery rate of the certain subassemblies or parts consist of a prominent variability rate through the remanufacturing operations. This situation is particularly acute for electronic components that typically tend to follow a random failure pattern.

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Remanufacturing operations are labor intensive that lead to significant variability in the processing times at various shop floor operations. The uncertainties surrounding the returned products further complicate the modeling and analysis of product recovery problems. As such, forecasting the quantity and the quality level of used products is difficult. There are two different types of uncertainties that affect the remanufacturing process: internal uncertainty and external uncertainty. Internal uncertainty comprises of the variations within the remanufacturing process such as the quality level of the product, the remanufacturing lead time, the yield rate of the process and the possibility of system failure. External uncertainty comprises of the variations originating from factors outside the remanufacturing process which include the timing, quantity and quality (reusable rate) of the returned products, the timing and the level of demand, and the procurement lead times of new parts/products. The results of the aforementioned uncertainties include undersupply or obsolescence of inventory, improper remanufacturing plan and loss of competitive edge in the market.

The objective of this paper is to find the optimal remanufacturing policies for multi stage system with stochastic re-usability rate of the returned products through the system. We develop a model to determine optimal input rates at each station of a serial remanufacturing system that process output at each stage of stations is stochastic. This paper considers a single item remanufacturable product that follow a fixed routing through the remanufacturing system. We model the remanufacturing system using an open queueing network with finite buffers and unreliable servers<sup>1, 2</sup>. In order to analyze the queueing network, we use the decomposition principle and expansion methodology<sup>7, 8, 10, 11, 12</sup>. We analyze the system behavior with different parameter settings and conduct the sensitivity analysis.

## 2. LITERATURE REVIEW

Optimal production control policies with yield uncertainty have been studied extensively in recent years. Yano and Lee<sup>21</sup> compiled the research in this area in the survey paper. Lee and Yano<sup>14</sup> analyze a single-period, single product, serial production system. In their model it's considered that the demand is known and the yield at each production stage is a probabilistic multiple of the input batch size. The optimal production policy for each stage is characterized by a single critical number representing the target input quantity. Ciarallo *et al.*<sup>4</sup> extend the similar model considering a single-stage production system with uncertain capacity and demand. They show that a critical level which provides the optimal policy in a multi-stage period.

Product recovery management puts the responsibility of collection of all used and discarded products, components, materials and their reuse opportunities on the manufacturing company. The objective of product recovery management as stated by Thierry *et al.* is "to recover as much of the economic (and ecological) value as reasonably possible, thereby reducing the ultimate quantities of waste"<sup>20</sup>. The product recovery options include repairing, refurbishing, remanufacturing, cannibalizing and recycling. The problem of determining optimal recovery/disposal strategy is discussed by a number of researchers. Simpson<sup>17</sup> developed a dynamic programming based algorithm to determine the optimal values of the several decision variables, viz., the quantity of products that is disposed of in planning period  $t$  ( $Q_d(t)$ ), the quantity of products that is procured outside or internally produced in planning period  $t$  ( $Q_p(t)$ ), the quantity of products that is remanufactured in planning period  $t$  ( $Q_r(t)$ ). Krikke *et al.*<sup>13</sup> proposed a comprehensive model that determines an optimal product recovery and disposal strategy for one product type. The objective function takes into account technical, commercial and ecological as well as uncertainty on these criteria due to lack of information about the condition of the returned products. Particularly, authors model the quality classes of the products and disassembly transitions in terms of conditional probabilities. Procedure starts with a disassembly tree, for each assembly all the recovery options and the cost associated with them are identified. Then, using a dynamic programming (DP) algorithm optimal recovery/disposal strategy is determined. In a later paper, Teunter<sup>19</sup>, generalized the solution strategy by maximizing the net profit for each product, module, part etc. Furthermore, the author discussed how these optimized net profits can be used to find optimal inventories.

The first crucial step of product recovery is disassembly. Disassembly is a methodical extraction of valuable parts/subassemblies and/or materials from post-used products through a series of operations. After disassembly, reusable parts/subassemblies are cleaned, refurbished, tested and directed to the part/subassembly inventory for remanufacturing operations. The recyclable materials can be sold to raw-material suppliers and the residuals are disposed of. The problems associated with disassembly and scheduling have been investigated by Brennan *et al.*<sup>3</sup>. Many

authors proposed different disassembly algorithms explicit optimization of disassembly sequences for full as well as partial disassembly (Penev and De Ron<sup>15</sup>, Johnson and Wang<sup>9</sup> and Taleb *et. al.*<sup>18</sup>). Gungor and Gupta<sup>6</sup> review the literature in the area of environmentally conscious manufacturing and product recovery. The problems associated with remanufacturing have been addressed by Guide and Srivastava<sup>5</sup>.

### 3. PROBLEM DESCRIPTION

In this paper we consider one of the strategic problems of hybrid systems where finished good demand is satisfied by means of manufacturing and remanufacturing operations together. For a hybrid system uncertainties belonging to remanufacturing stream is much higher than that the manufacturing stream. Due to the high uncertainty in the reusability of returned products (recovery rate, quality of parts etc.), coordinating the serviceable inventory for remanufactured products is a challenge. The model under study here is a continuous flow process with a serial remanufacturing operation shops (Figure 1). The inventory system is produce-to-stock type that customer orders are satisfied from the finished good inventory (serviceable inventory). The remanufacturing system reclaims a single part product and that item has a fixed or predetermined routing through the system. The goal of the remanufacturing process is to bring the product at the original quality level as manufactured first time. In the model, each remanufacturing operation station is subject to breakdown and has a finite buffer capacity. The objective of the paper is not only controlling the return flow to the system but also implementing optimal used product recovery and disposal strategies in each stage of the recovery operations to run the system in an economically optimal level.

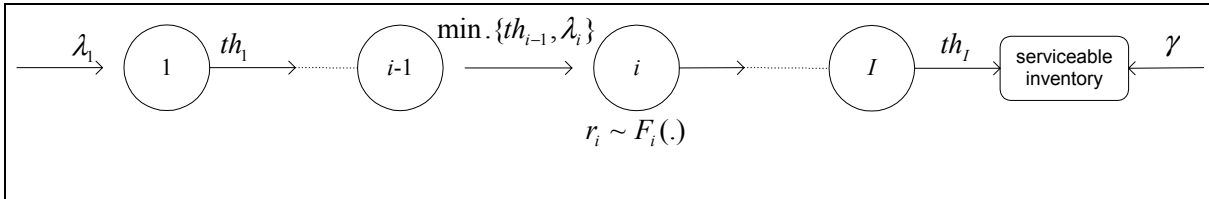


Figure 1. Serial remanufacturing system

We assume that there is one server and finite buffer capacity at each station represented by  $B_i$ . The service rate  $\mu_i$  for each operation is exponentially distributed and the service discipline is First Come First Serve (FCFS). The breakdown rate  $\alpha_i$  and the repair time rate  $\beta_i$  for broken machine at station  $i$  are also exponentially distributed. The service rate, breakdown rate and repair rates of servers are exponential random variables distributed according to the following distribution functions.

$$\left. \begin{aligned} f_{serv.,i}(t) &= \mu_i e^{-\mu_i t}, \quad t \geq 0, \\ f_{break.,i}(t) &= \alpha_i e^{-\alpha_i t}, \quad t \geq 0 \\ f_{rep.,i}(t) &= \beta_i e^{-\beta_i t}, \quad t \geq 0 \end{aligned} \right\} \quad (1)$$

Therefore, the availability of station  $i$ ,  $\varphi_i$ , the average fraction of the time that station  $i$  is operational when it's operated isolation, i.e., never starved or blocked. For a reliable station  $\varphi_i = 1$  and for an unreliable station  $\varphi_i < 1$ .

$$\varphi_i = \frac{\beta_i}{\alpha_i + \beta_i}, \quad \forall i, \quad i = 1, \dots, I. \quad (2)$$

Also, actual process rate of station  $i$  under isolation is denoted by  $\pi_i$ .

$$\pi_i = \frac{\mu_i \beta_i}{(\alpha_i + \beta_i)}, \quad \forall i, \quad i = 1, \dots, I \quad (3)$$

In addition, there are two more states for each server, viz. idle and blocking period. Idle period is the time interval during which a server is idle because of the lack of work piece. Blocking period is the time interval during which a

station cannot provide service on an item, due to the last item serviced at this station cannot move to the downstream station because of down stream station buffer is full or the downstream server is failed. The blocking mechanism in the remanufacturing system is ‘block after service’, (BAS). When an item is ready to join a station, either the buffer at that station is not full, in which case the item joins the queue at that station, or the buffer is full and the item cannot join the queue, in which case it stays where it is originated from and blocks that server.

Throughout the paper, for recovery potential we use the term re-usable rate ( $r_i$ ) to refer to the acceptable fraction of used parts that satisfies the quality specifications for remanufacturing. We assume that the returned products re-usable rate probability distribution functions are mutually independent among the stations. Also, it’s assumed that re-usable rate distributions are independent with the magnitude of the returned rate flow. The p.d.f. and c.d.f. of the re-usable rate at stage  $i$  is denoted as  $f_i(r_i)$  and  $F_i(r_i)$  respectively. Re-usable rate doesn’t need to be estimated from a fitted probability distribution, as it’s valid for most of the real-world applications, historical recovery data are available and this information can be used empirically to estimate the re-usable rate of the parts from returned products. In each station there is a scrap rate of  $(1 - r_i)$  for the returned products and disposition (excess) quantity of  $(\lambda_i - th_{i-1})^+$ , where  $(.)^+$  points to the positive part. Disposition of a part occur when marginal cost of remanufacturing is surpass the disposition cost and unavailability of end markets for the particular part or material. There are special cases exist at the final stage of the system, lost sales costs ( $c_l$ ) incur when the system output is not sufficient for the demand, i.e.  $(\gamma - th_l)^+$  and inventory holding cost ( $c_h$ ) incur when there is an excess inventory exists, i.e.  $(th_l - \gamma)^+$ .

At any stage of the remanufacturing operations, the system’s productive capacity may be uncertain due to a number of factors such as machine failures, service time variations because of the unknown condition of the returned product, repair rate of the machine and the buffer size of the station. The production capacity of the station  $i$  can be determined by these parameters  $(\mu_i, \alpha_i, \beta_i, B_i)$ .

The demand rate represented by  $\gamma$  with c.d.f  $Q(\gamma)$  and p.d.f.  $q(\gamma)$ . The market environment for the remanufactured products is followed the “make-for-stock” policy. In the make-for-stock environment all customer demand is planned to be met from finished serviceable inventory. The detailed mechanism of the model is depicted in Figure 2.

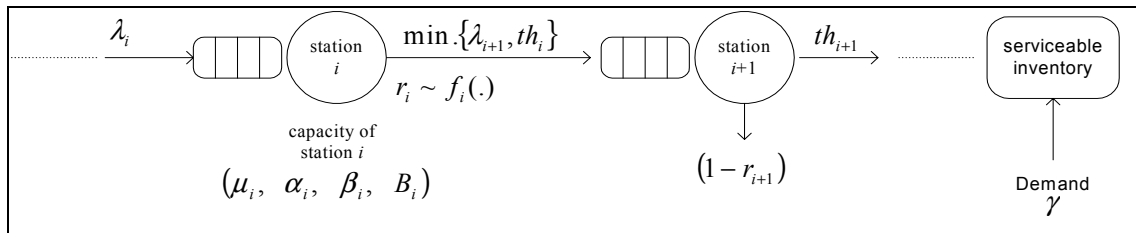


Figure 2. Serial remanufacturing system with recovery losses and uncertain capacities

The objective is to determine a process input rate  $\lambda_i^*$  ( $i=1, \dots, I$ ) at each stage of the remanufacturing system that minimized system expected total cost. The input rate to station  $i$  is constrained by the output of the upstream station. The output rate of station  $i$ ,  $th_i$ , which is a function includes the uncertain production capacity of the server and random recovery rate ( $r_i$ ) of the returned product at station  $i$ . The output of the remanufacturing system is represented by  $th_l$ .

#### 4. MODEL FORMULATION

For a typical remanufacturing system the long-run average total cost per unit of time is define as follows,

$$E(TC) = c_p E(RP) + c_d E(D) + c_l E(T) + c_{dis} E(Dis) + c_h E(Inv) + c_i E(Ls) + \sum_{j=1}^J c_{rj} E(R_j) \quad (4)$$

$E(RP)$ : expected number of returned products per unit of time.

- $E(D)$  : expected rate of disposed items per unit of time.  
 $E(T)$  : expected number of tested products per unit of time.  
 $E(Dis)$  : expected number of disassembled products per unit of time.  
 $E(Inv)$  : expected on hand inventory level per unit of time.  
 $E(Ls)$  : expected lost sales per unit of time.  
 $E(R_j)$  : expected number of remanufactured products in remanufacturing station  $j$  per unit of time.  
 $c_p$  : variable purchase cost of returned product (\$/product).  
 $c_d$  : variable disposition cost per product (\$/product).  
 $c_t$  : variable testing cost per returned product (\$/product)  
 $c_{dis}$  : variable disassembly cost per returned product (\$/product)  
 $c_h$  : variable inventory holding cost per returned product (\$/product/time)  
 $c_l$  : variable lost sales cost (\$/product/time)  
 $c_{rj}$  : variable remanufacturing operations cost at station  $j$  (\$/product)

Let  $\lambda_i$ ,  $i = 1, \dots, I$  (where  $\lambda_i = \lambda_{ar}$ ) denotes the input rate of the returned parts into each station in the remanufacturing system. Then the general formulation of the problem for remanufacturing a used product with associated setup and variable costs in each stage can be formulated as follows;

$$\min_{\lambda_i} E \left\{ \sum_{i=1}^I [(A_i + c_i \lambda_i) + cd_n (th_{i-1} - \lambda_i)^+] + c_h (th_i - \gamma)^+ + c_l (\gamma - th_i)^+ \right\} \quad (5)$$

subject to  $0 \leq \lambda_i \leq th_{i-1}$ ,  $i = 1, \dots, I$ .

and  $th_i$  is a function of  $(r_i, \mu_i, \alpha_i, \beta_i, i = 1, \dots, I)$

Optimal part recovery strategy is represented by  $\Omega = \{\lambda_i^*, i = 1, \dots, I\}$  and  $\Omega_i$  is function of  $(r_i, \mu_i, \alpha_i, \beta_i, B_i)$ .

- $A_i$  : setup cost of remanufacturing operations at stage  $i$ .  
 $cd_i$  : cost of disposing a part retrieved at stage  $i-1$  but not used in at stage  $i$ .  
 $c_i$  : variable costs at stage  $i$ .  
 $c_h$  : inventory holding cost.  
 $c_l$  : lost sales cost for each unit of unsatisfied demand.  
 $c_p$  : production cost of a new part or cost of the item procuring from outside.  
 $\gamma$  : demand rate for the remanufactured parts.  
 $\lambda_i$  : planned remanufacturing quantity at stage  $i$ .  
 $\mu_i$  : service rate at station  $i$ .  
 $\alpha_i$  : breakdown rate of station  $i$ .  
 $\beta_i$  : repair rate of station  $i$ .  
 $B_i$  : buffer capacity of station  $i$ .  
 $r_i$  : recovery (reusability) rate of the part at station  $i$ .  
 $th_i$  : throughput of station  $i$ .

Note that  $th_i$  is a function of the productive capacity of the station that determined by  $(\mu_i, \alpha_i, \beta_i, B_i)$  and  $r_i$ ,  $i = 1, \dots, I$ .

In this cost formulation the following conditions are necessary to verify that remanufacturing is economically feasible.

**Condition 1:** Remanufacturing a part is cheaper than manufacturing in house or procuring a new one from outside.

$$c_p \gamma \geq \sum_{i=1}^I (A_i + c_i \lambda_i) + cd_i (th_{i-1} - \lambda_i)^+$$

**Condition 2:** It's more expensive to dispose a part at one stage than to process it at the downstream station.

$$c_i + cd_i (r_i \lambda_i) > cd_{i+1}$$

**Condition 3:** Lost sales cost is greater than inventory holding cost.

$$c_l > c_h$$

The dynamic nature of the problem can be captured by defining the cost function as follows. Let  $TC_i(th_{i-1})$  is the expected cost of operating the system optimally from station 1 through the final remanufacturing station, given that the throughput rate from stage  $i$  is  $th_i$ . Then  $TC_i(th_{i-1})$  represents the minimum total cost to operate the remanufacturing system. Clearly, at each station we must have  $\lambda_i \leq th_{i-1}$  (viz. the input cannot exceed the output of the upstream station). Hence, the dynamic programming recursion relationships are

$$TC_i(th_{i-1}) = \min_{0 \leq \lambda_i \leq th_{i-1}} \{[(A_i + c_i \lambda_i) + cd_i (th_{i-1} - \lambda_i)] + E[TC_{i+1}(th_i)]\} \quad i = 1, \dots, I-1 \quad (6)$$

$$TC_I(th_{I-1}) = \min \{[(A_I + c_I \lambda_I) + cd_I (th_{I-1} - \lambda_I)^+ + c_h (th_I - \gamma)^+ + c_l (\gamma - th_I)^+]\} \quad (7)$$

To obtain the throughput rate and other performance measures of the stations we use the robust approximation technique, viz. the Expansion Method. By means of this technique we empirically showed the convexity of cost function. Figure 3 shows the behavior of the  $E(TC)$  respecting to return rate of the used products for three different parameter settings and different re-usability rates in each station.

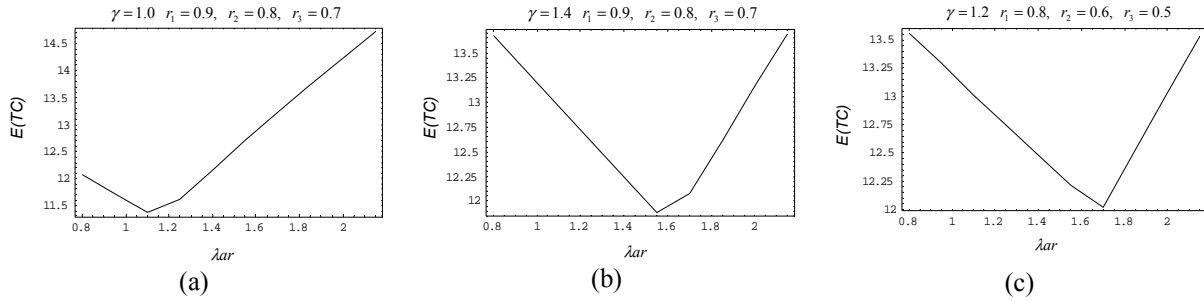


Figure 3.  $E(TC)$  respect to return rate  $\lambda_{ar} = \lambda_i$  and parameters are,

(a)  $(\mu_1, \alpha_1, \beta_1, B_1) = (1.5, 0.2, 1, 8)$ ;  $(\mu_2, \alpha_2, \beta_2, B_2) = (1.5, 0.4, 1, 5)$ ;  $(\mu_3, \alpha_3, \beta_3, B_3) = (1.8, 0.4, 1, 5)$

(b)  $(\mu_1, \alpha_1, \beta_1, B_1) = (1.5, 0.6, 1, 8)$ ;  $(\mu_2, \alpha_2, \beta_2, B_2) = (1.8, 0.2, 1, 3)$ ;  $(\mu_3, \alpha_3, \beta_3, B_3) = (2.2, 0.6, 1, 5)$

(c)  $(\mu_1, \alpha_1, \beta_1, B_1) = (1.2, 0.6, 1, 3)$ ;  $(\mu_2, \alpha_2, \beta_2, B_2) = (2.2, 0.6, 1, 8)$ ;  $(\mu_3, \alpha_3, \beta_3, B_3) = (2.5, 0.4, 1, 5)$

Since the relevant cost functions are convex (Figure 3.), using numerical search procedures we can find the optimal input rates  $\lambda_{i-1}^*$ ,  $\lambda_{i-2}^*$ , ...,  $\lambda_1^*$  recursively.

$$\lambda_i^* = \begin{cases} th_{i-1} & \text{if } th_{i-1} < \lambda_i \\ \lambda_i & \text{otherwise} \end{cases} \quad i = 1, \dots, I \quad (8)$$

To obtain the approximate throughput rates of each server and the entire remanufacturing network, we utilize the expansion methodology. In this section we briefly discuss the fundamentals of the expansion methodology. Details of



the method and necessary derivations for unreliable production lines can be obtained from Gupta and Kavusturucu<sup>7, 8</sup> and Kavusturucu and Gupta<sup>10</sup>.

The expansion methodology is an efficient tool for the analysis of nodes with finite buffers. In order to analyze the remanufacturing system, which is presented in Figure 2, we first decompose the network and examine each server separately. After isolating each node we expand the network by adding a node in front of the each server. These extra nodes are modeled as infinite buffer nodes with zero processing times. They act as “holding nodes” for jobs, which cannot enter the destination node because the buffer is full. The blocked jobs stay there until a space becomes available at the full buffer. Next, the parameters that define the expanded network, such as the actual arrival rate to the system, the probability of a job being blocked by the full buffer, etc. are calculated. Finally, using the newly calculated parameters, the throughput of the entire network can be calculated<sup>11</sup>.

### 5. PERFORMANCE EVALUATION AND CONCLUSIONS

In this section we examine the behavior of the model and obtained the important performance measures for the remanufacturing system. Consider a three-stage serial remanufacturing system shown in Figure 2. To this end, we designed several experiments where we changed all system parameters through the model simultaneously. The range for each parameter was picked such that the experiments would cover wide range of values for all parameters (see Table 1). Also, variable cost values of the system are given in Table 2.

To design the experiments, we used orthogonal arrays, instead of full factorial. For the model system shown in Figure 4, we consider the following system parameters, service rate of station  $i$   $\mu_i$ , isolated availability of the station  $i$   $\varphi_i$ , buffer capacity of station  $i$   $B_i$  and reusability rate at station  $i$   $r_i$ . With three levels of each parameter we used the orthogonal array  $L_{27}(3^{13})$  that reduced the number of experiments to 27 as shown in Table 3 (Phadke<sup>16</sup>). The table provides the value of each parameter for every experiment conducted. Note that  $L_{27}(3^{13})$  allows up to 13 variables with 3 levels. Since we only have 12 variables with 3 levels last column is left blank and is not shown in Table1. The advantages of using this methodology include efficiency, ease of use, and the coverage of the experimental region with a considerably small number of experiments.

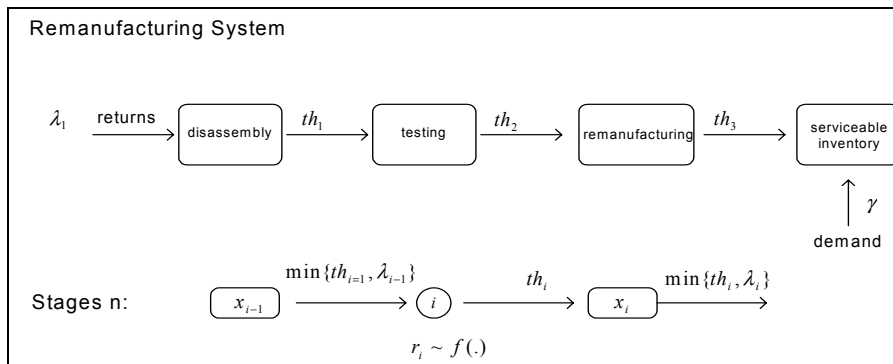


Figure 4. Remanufacturing system with stochastic recovery rates and uncertain capacity servers

Table 1. Experimental frame, modified from  $L_{27}(3^{13})$  orthogonal array.

No.	$\mu_1$	$\varphi_1$	$B_1$	$r_1$	$\mu_2$	$\varphi_2$	$B_2$	$r_2$	$\mu_3$	$\varphi_3$	$B_3$	$r_3$
1	1.5	0.83	3	0.7	1.5	0.83	3	0.7	1.5	0.83	3	0.7
2	1.5	0.83	3	0.7	1.8	0.71	5	0.8	1.8	0.71	5	0.8
3	1.5	0.83	3	0.7	2.2	0.63	8	0.9	2.2	0.63	8	0.9
4	1.5	0.71	5	0.8	1.5	0.83	3	0.8	1.8	0.71	8	0.9
5	1.5	0.71	5	0.8	1.8	0.71	5	0.9	2.2	0.63	3	0.7
6	1.5	0.71	5	0.8	2.2	0.63	8	0.7	1.5	0.83	5	0.8

7	1.5	0.63	8	0.9	1.5	0.83	3	0.9	2.2	0.63	5	0.8
8	1.5	0.63	8	0.9	1.8	0.71	5	0.7	1.5	0.83	8	0.9
9	1.5	0.63	8	0.9	2.2	0.63	8	0.8	1.8	0.71	3	0.7
10	1.8	0.83	5	0.9	1.5	0.71	8	0.7	1.8	0.63	3	0.8
11	1.8	0.83	5	0.9	1.8	0.63	3	0.8	2.2	0.83	5	0.9
12	1.8	0.83	5	0.9	2.2	0.83	5	0.9	1.5	0.71	8	0.7
13	1.8	0.71	8	0.7	1.5	0.71	8	0.8	2.2	0.83	8	0.7
14	1.8	0.71	8	0.7	1.8	0.63	3	0.9	1.5	0.71	3	0.8
15	1.8	0.71	8	0.7	2.2	0.83	5	0.7	1.8	0.63	5	0.9
16	1.8	0.63	3	0.8	1.5	0.71	8	0.9	1.5	0.71	5	0.9
17	1.8	0.63	3	0.8	1.8	0.63	3	0.7	1.8	0.63	8	0.7
18	1.8	0.63	3	0.8	2.2	0.83	5	0.8	2.2	0.83	3	0.8
19	2.2	0.83	8	0.8	1.5	0.63	5	0.7	2.2	0.71	3	0.9
20	2.2	0.83	8	0.8	1.8	0.83	8	0.8	1.5	0.63	5	0.7
21	2.2	0.83	8	0.8	2.2	0.71	3	0.9	1.8	0.83	8	0.8
22	2.2	0.71	3	0.9	1.5	0.63	5	0.8	1.5	0.63	8	0.8
23	2.2	0.71	3	0.9	1.8	0.83	8	0.9	1.8	0.83	3	0.9
24	2.2	0.71	3	0.9	2.2	0.71	3	0.7	2.2	0.71	5	0.7
25	2.2	0.63	5	0.7	1.5	0.63	5	0.9	1.8	0.83	5	0.7
26	2.2	0.63	5	0.7	1.8	0.83	8	0.7	2.2	0.71	8	0.8
27	2.2	0.63	5	0.7	2.2	0.71	3	0.8	1.5	0.63	3	0.9

In addition to these parameters, for entire experimental frame we assumed the expected demand rate  $\gamma = 0.5$ .

Table 2. Cost values

Cost values	$A_n$	$c_n$	$cd_n$	$c_h$	$c_l$
Station 1	10	1	2	3	4
Station 2	20	3	2		
Station 3	20	5	2		

According to this data set we obtained the following results

Table 3. Summary of results

Exp. No.	$\lambda_1^*$	$th_1$	$\lambda_2^*$	$th_2$	$\lambda_3^*$	$th_3$	$E_{SL}$	$E_{WIP}$	$E_{PT}$	$E_{TC}$
1	1.514	1.036	1.033	0.721	0.721	0.505	1.009	19.412	15.803	62.882
2	1.150	0.801	0.799	0.636	0.630	0.504	1.007	17.561	14.268	59.783
3	0.903	0.631	0.630	0.567	0.565	0.508	1.017	18.345	14.680	57.662
4	0.916	0.717	0.708	0.566	0.565	0.508	1.017	12.571	11.692	57.911
5	1.059	0.817	0.812	0.725	0.721	0.504	1.009	11.054	10.394	58.980
6	1.189	0.903	0.903	0.632	0.630	0.504	1.008	12.924	11.661	60.215
7	0.799	0.708	0.708	0.637	0.630	0.504	1.007	20.714	20.753	57.034
8	0.929	0.814	0.812	0.566	0.565	0.508	1.017	17.150	18.033	58.296
9	1.059	0.909	0.903	0.722	0.721	0.505	1.009	18.714	19.187	59.222
10	1.020	0.906	0.903	0.630	0.630	0.504	1.008	14.043	11.177	59.170
11	0.799	0.716	0.708	0.566	0.565	0.508	1.016	13.744	10.721	57.149
12	0.916	0.819	0.812	0.730	0.721	0.505	1.009	9.476	6.696	58.083
13	1.332	0.923	0.916	0.729	0.721	0.505	1.009	19.342	16.237	61.282
14	1.020	0.712	0.708	0.636	0.630	0.504	1.008	20.091	16.488	58.613
15	1.176	0.820	0.812	0.568	0.565	0.508	1.016	13.800	11.041	60.090
16	0.799	0.638	0.630	0.566	0.565	0.507	1.015	19.926	18.192	56.913
17	1.358	1.049	1.046	0.729	0.721	0.505	1.009	17.250	16.003	61.753

18	1.007	0.799	0.799	0.639	0.630	0.504	1.008	17.092	14.757	58.783
19	1.033	0.826	0.825	0.570	0.565	0.508	1.017	14.799	10.349	59.123
20	1.137	0.909	0.903	0.722	0.721	0.503	1.006	15.079	9.290	59.938
21	0.890	0.712	0.708	0.637	0.630	0.504	1.008	17.540	10.374	57.713
22	0.916	0.823	0.812	0.635	0.630	0.504	1.007	16.068	12.179	58.135
23	0.708	0.637	0.630	0.567	0.565	0.508	1.017	17.960	11.925	56.288
24	1.163	0.043	1.033	0.723	0.721	0.505	1.009	14.023	9.815	60.494
25	1.215	0.840	0.838	0.728	0.721	0.505	1.009	12.078	10.449	60.148
26	1.319	0.910	0.903	0.632	0.630	0.504	1.008	11.329	8.702	61.253
27	1.020	0.711	0.708	0.566	0.565	0.508	1.017	11.301	8.597	58.702

In this research we have developed an approach to determine optimal product recovery and disposal strategies for remanufacturing systems in which re-usability rate at each stage of the recovery operations are stochastic. The further research needs to incorporate other factors, viz. demand fluctuations, alternative procurement and disposition strategies, upgrading or downgrading opportunities for the returned products and multiple part cases.

## REFERENCES

1. Aksoy, H. K. and Gupta, S. M., "An open queueing network model for remanufacturing systems", *Proceedings of the 25th. Conference on Computers and Industrial Engineering*, pp. 62-65, New Orleans, LA, March 29-31, 1999.
2. Aksoy, H. K. and Gupta, S. M., "Buffer Allocation Plan for Cellular Remanufacturing Systems", *Computers and Industrial Engineering* (Accepted), 2001.
3. Brennan, L., Gupta, S. M. and Taleb, K. N., "Operations planning issues in an assembly/disassembly environment", *International Journal of Operations and Production Planning*, **14**(9), 57-67, 1994.
4. Ciarallo, F. W., Akella, R. and Morton, T. E., "A periodic review, production planning model with uncertain capacity and uncertain demand-optimality of extended myopic policies", *Management Science*, **40**, 320-332, 1994.
5. Guide, Jr., V. D. R. and Srivastava, R., "Buffering from material recovery uncertainty in a recoverable manufacturing environment", *Journal of the Operational Research Society*, **48**, 519-529, 1997.
6. Gungor, A. and Gupta, S. M., "Issues in environmentally conscious manufacturing and product recovery: a survey", *Computers and Industrial Engineering*, **36**, 811-853, 1999.
7. Gupta, S. M. and Kavusturucu, A., "Modeling of finite buffer cellular manufacturing systems with unreliable machines", *International Journal of Industrial Engineering*, **5**(4), 265-277, 1998.
8. Gupta, S. M. and Kavusturucu, A., "Production systems with interruptions, arbitrary topology and finite buffers", *Annals of Operations Research*, **93**, 145-176, 2000.
9. Johnson, M. R. and Wang, M. H., "Planning product disassembly for material recovery opportunities", *International Journal of Production Research*, **33**, 3119-3142, 1995.
10. Kavusturucu, A. and Gupta, S. M., "Analysis of Manufacturing flow lines with unreliable machines", *International Journal of Computer Integrated Manufacturing*, **12**(6), 510-524, 1999.
11. Kerbache, L. and Smith, J. M., "The Generalized Expansion Method for Open Finite Queueing Networks", *European Journal of Operational Research*, **32**, 448-461, 1987.
12. Kerbache, L. and Smith, J. M., "Asymptotic behavior of the expansion method for open finite queueing networks", *Computers and Operations Research*, **15**(2), 157-169, 1988.
13. Krikke, H. R., Van Harten, A., Schuur, P. C., "On a medium term product recovery and disposal strategy for durable assembly products", *International Journal of Production Research*, **36**, 111-139, 1998.
14. Lee, H. A. and Yano, C. A., "Production control in multistage systems with variable yield losses", *Operations Research*, **36**, 269-278, 1988.
15. Penev, K. D. and De Ron, A. J., "Determining of disassembly strategy", *International Journal of Production Research*, **34**, 495-506, 1996.
16. Phadke, M. S., *Quality Engineering Using Robust Design*. Englewood Cliffs, New Jersey, 1989.
17. Simpson, V. P., "Optimum solution structure for a repairable inventory problem", *Operations Research*, **26**, 270-281, 1978.

18. Taleb, K N., Gupta, S. M. and Brennan, L., "Disassembly of complex product structures with parts and materials commonality", *Production Planning and Control*, **8**, 255-269, 1997.
19. Teunter, R. H., "On finding optimal recovery/disposal and inventory strategies for returned products", *Preprint 11/9 University of Magdeburg*, 1999.
20. Thierry, M., Salomon, M., van Nunen, J., and van Wassenhove, L., " Strategic issues in product recovery management", *California Management Review* **37**(2), 114-135, 1995.
21. Yano, C. A., and Lee, H. L., "Lot-sizing with random yields: a review", *Operations Research*, **43**, 311-334, 1995.