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Optimal Production Policy for a Remanufacturing System with Virtual Inventory Cost

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ABSTRACT

This paper deals with a cost management problem of a remanufacturing system with stochastic demand. We model the system with consideration for two types of inventories. One is the actual product inventory in the factory. The other is the virtual inventory that is being used by the customer. For this virtual inventory, it should be required to consider an operational cost that we need in order to observe and check the quantity of the inventory. We call this the virtual inventory cost and model the system by including it. We define the state of the remanufacturing system by the two inventory levels. It is assumed that the cost function is composed of various cost factors such as holding, backlog and manufacturing costs. We obtain the optimal policy that minimizes the expected average cost per period. Numerical results reveal the effects of the factors on the optimal policy.

Keywords: Remanufacturing system, Product life cycle, Optimal control.

1. INTRODUCTION

The escalating growth in consumer waste in recent years has started to threaten the environment. Product recovery is mainly driven by the escalating deterioration of the environment and aims to minimize the amount of waste sent to landfills by recovering materials and parts from old or outdated products by means of recycling and remanufacturing. Product recovery includes collection, disassembly, cleaning, sorting, repairing and reconditioning broken components, reassembling and testing [1, 2, 6]. Here we focus on product recovery in a remanufacturing system with stochastic demand.

The system is formulated into a Markov Decision Process (MDP) [7, 17]. The MDP is a class of stochastic sequential processes in which the reward and transition probability depend only on the current state of the system and the current action. The MDP models have gained recognition in such diverse fields as economics, communications, transportation and so on. Each model consists of states, actions, rewards, and transition probability. In the engineering field, for example, the approach is used for controlling the production system [15, 16]. Choosing an action as production quantity in a state generates rewards and/or costs, and determines the state at the next decision epoch through a transition probability function. Then, we obtain the optimal production policy that minimizes the expected average cost per period in the optimal production control problem. Here we consider two types of inventories. One is the actual product inventory in a factory, and the other is the virtual inventory which is being used by customer. We define the state of the remanufacturing system by the two inventory levels. We can obtain the optimal production policy that minimizes the expected average cost per period. We also consider some scenarios under various conditions using design of experiments.

In section 2, we present a brief review of the literature on the remanufacturing systems. In section 3, we consider a single-item remanufacturing system under stochastic demand. The system is formulated as an undiscounted MDP to determine the optimal control policy that minimizes the expected average cost per period. Finally, we discuss the effect of factors on the optimal policy for the remanufacturing system under various conditions.

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2. LITERATURE REVIEW

We present a brief review of the literature in the areas of product recovery and remanufacturing systems with stochastic variability.

Gungor and Gupta [4] and Moyer and Gupta [11] reviewed the literature in the area of environmentally conscious manufacturing and product recovery. For an in-depth look at this area, see the recent book by Lambert and Gupta [9]. Minner [10] pointed out that there are the two well-known streams in product recovery research area. One is stochastic inventory control (SIC) and the other is material requirement planning (MRP). In this paper, we restrict ourselves to SIC.

As for the periodic review models, Cohen et al. [3] developed the product recovery model in which the collected products are used directly. Inderfurth [8] discussed effect of non-zero leadtimes for orders and recovery in the different model. As for continuous review models, Muckstadt and Isaac [12] dealt with a model for a remanufacturing system with non-zero leadtimes and a control policy with the traditional (Q, r) rule. Van der Laan and Salomon [18] suggested push and pull strategies for the remanufacturing system. Guide and Gupta [5] and Aksoy and Gupta [1] presented queueing models to study remanufacturing systems. They, however, considered that demand and procurement were independent in the inventory systems. Nakashima *et al.*[13] dealt with a product recovery system with a single class product life cycle. They proposed a new analytical approach for evaluating the system and provided numerical examples under various conditions. Furthermore, they optimized the system without virtual inventory cost [14].

3. OPTIMAL CONTROL OF THE SYSTEM

Let us consider a single process that produces a single item product. The finished products are stocked in the factory and are depleted according to the customer demand. Each product has its own remaining life time denoted by i $(i=M-1,\ldots,1)$ after it is sold. The remaining life time decreases one each period. Traditional inventory management focuses on only the inventory in the factory. In the remanufacturing system, however, we should focus on the outdated products that are collected from the customers. That is, the remanufacturing producers have to consider the products in use as the part of the future inventory. We here consider the products used by customers as virtual inventory. It is important not only to control the inventory on hand but also to manage the virtual inventory until the products are collected from customers and used in remanufacturing.

Remanufacturing preserves the product's or the part's identity, and performs the required disassembly and refurbishing operations to bring the product to a desired level of quality at some remanufacturing cost. On the other hand, we define normal manufacturing as producing the products using new resources. The number of products produced by normal manufacturing at period t, $P(t)$ is chosen as an action, k, i.e., $k=P(t)$. Products are produced by normal manufacturing and/or remanufacturing with the parts taken back from the customers. All production begins at the start of the period and all products are completed by the end of the period. All the products bought by customers are new. We assume that the number of finished products and that of the products bought by customers are $I(t)$ and $J_i(t)(i=M-1,\ldots,1)$, respectively. We take $J_i(t)$ as virtual inventory. For this virtual inventory, we consider an operational cost that needed to observe and check the quantity of the inventory. We call it the virtual inventory cost and model the system by including it. If a backlog occurs, $I(t)$ is a negative value. Demand in successive periods, $D(t)$, is an independent random variable with known identical distribution. When sold, products are remanufactured at the remanufacturing rate, λ_i (i=M-1, …, 1), with some remanufacturing cost, and products in use are discarded at rate, μ with some out-of-date cost. It is assumed that $\lambda_i < 1$ and $\lambda_1 + \mu \le 1$.

The state of the system is denoted by

$$s(t) = \left(I(t), J_{M-1}(t), \ldots, J_1(t)\right). \tag{1}$$

Figure 1 shows a remanufacturing system with single product life cycle.

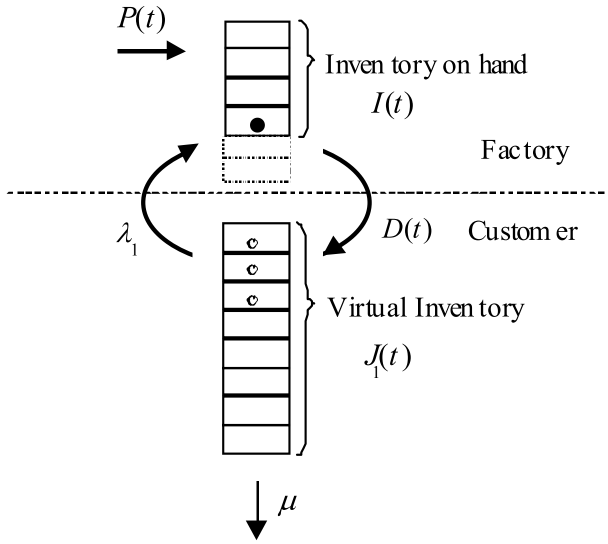

Figure 1: A remanufacturing system with single product life cycle

The transition of the each inventory is given by

$$I(t+1) = I(t) + k + \sum_{i=1}^{M-1} \lambda_i J_i(t) - D(t), \tag{2}$$

$$J_{M-1}(t+1) = \min\{[I(t)]^+ + P(t) + \sum_{i=1}^{M-1} \lambda_i J_i(t), D(t) + [I(t)]^+\}, \tag{3}$$

$$J_i(t+1) = (1-\lambda_{i+1}) J_{i+1}(t) \quad \text{for } i=M-1, \ldots, 2 \tag{4}$$

and

$$J_1(t+1) = (1-\lambda_1 - \mu) J_1(t) + (1-\lambda_2) J_2(t) \tag{5}$$

where $[x]^+=\min\{0, x\}$. The action space in $s(t)$, $K(s(t))$ is defined by

$$K(s(t)) = \{\, 0, \ldots, \ \max\{0, Imax-I(t)-\sum_{i=1}^{M-1} \lambda_i J_i(t)\} \,\}. \tag{6}$$

The transition probability of the system is given by

$$P_{s(t)s(t+1)}(k) = \begin{cases} \mathbf{Pr}\{D(t) = d\} & \textbf{if } s(t+1) = (I(t) + k + \sum_{i=1}^{M-1} \lambda_i J_i(t) - d, \\ & \ldots, (1-\lambda_1 - \mu) J_1(t) + (1-\lambda_2) J_2(t), \\ \mathbf{0} & \text{otherwise.} \end{cases} \tag{7}$$

The expected cost per period in state $s(t)$ under k, $r(s(t),k)$ is given by

$$r(s(t),k) = C_N k + \sum_{i=1}^{M-1} C_{Ri} \lambda_i J_i(t) + C_H [I(t)]^+ + C_B [-I(t)]^+ + Cv_i \sum_i Ji(t) + C_O \mu J_1(t) \tag{8}$$

where the parameters are as follows:

C_N : the normal manufacturing cost of a new product
C_{Ri} : the remanufacturing cost of a product
C_H : the holding cost per unit
C_{vi} : the virtual inventory cost per unit product of which remaining life time is i
$C_{\boldsymbol{B}}$: the backlog cost per unit
C_O : the out-of-date cost per unit

Denote by S and $|S|$ a set of all possible states and the total number of the states, respectively. Let us number the states s_n by s (= 1,…, $|S|$). An undiscounted MDP that minimizes the expected average cost per period, g is formulated as the following optimality equation:

$$g + v_s = \min_{k \in K(s)} \left\{ r(s,k) + \sum_{s' \in S} P_{ss'}(k) v_{s'} \right\} (s \in S). \tag{9}$$

where v_s denotes the relative value when the production system starts from state s [7]. An optimal production policy is determined as a set of k that minimizes the right-hand side of equation (9) for each state s using the policy iteration method [7, 17].

4. NUMERICAL RESULTS

In this section, we show the numerical results on the system with $M=3$. The distribution of the demand is given by

$$\Pr\{D_n = D - \frac{1}{2}Q + j\} = \binom{Q}{j}\left(\frac{1}{2}\right)^Q, (0 \le j \le Q),$$

where D=2 and Q is an even number and variance(σ^2) is $Q/4$. The maximum number of actual inventory and that of virtual inventory are 5, 4 and 4, respectively. The maximum number of backlog demand is set as -5. It is assumed that C_H =1, C_N =1, $C_{\boldsymbol{B}}$ =10, C_O =10, C_{R1} =4, C_{R2} =2,C_{v1} =3, C_{v2} =5 and σ^2=0.5. The remanufacturing rate of virtual inventory 1, λ_1, is 0.2.

Figure 2 illustrates the behavior of the expected average cost with varying remanufacturing rates for virtual inventory 2. As the rate increases, the expected cost tends to decrease.

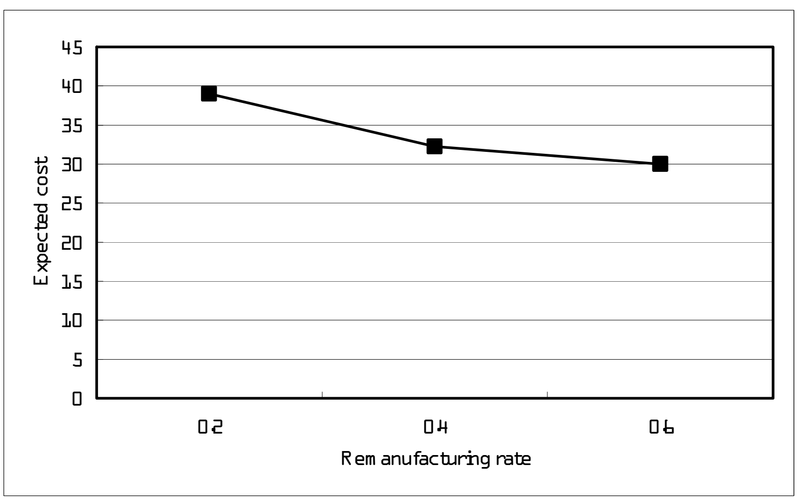

Figure 2: Expected minimum average cost

5. CONCLUSION

This paper dealt with a cost management problem of the remanufacturing system with stochastic variability. We modeled the system as a Markov decision system by considering two types of inventories. We optimized the remanufacturing system to obtain the minimum expected cost including virtual inventory cost. We also considered some scenarios under various conditions and discussed the effects of the factors on the remanufacturing system. The numerical investigation illustrated the remanufacturing management.

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