

January 01, 2003

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Recommended Citation

Gupta, Surendra M., "Optimal control of a remanufacturing system with consideration for product life cycle" (2003).. Paper 83.
<http://hdl.handle.net/2047/d10003216>

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Bibliographic Information

Nakashima, K. and, Gupta, S. M., "Optimal Control of a Remanufacturing System with Consideration for Product Life Cycle", ***Proceedings of the SPIE International Conference on Environmentally Conscious Manufacturing III***, Providence, Rhode Island, pp. 15-19, October 29-30, 2003.

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Optimal Control of a Remanufacturing System with Consideration for Product Life Cycle

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ABSTRACT

This paper deals with the cost management problem of a remanufacturing system with stochastic variability in the demand rate, the remanufacturing rate and the discard rate. We consider two types of inventories. One is the actual product inventory in the factory while the other is the virtual inventory that is still in use by the consumers. The state of the remanufacturing system is defined by considering the levels of both inventories. The cost function is composed of various costs such as the holding cost, backloging cost and other manufacturing costs. We obtain the optimal production policy that minimizes the expected average cost per period. Numerical results provide insights on the effects of the various costs on the optimal policy.

Keywords: Remanufacturing Systems, Product Life Cycle, Optimal Control.

1. INTRODUCTION

The escalating growth in consumer waste in recent years has started to threaten the environment. Currently product recovery is practiced in part because of the escalating deterioration of the environment and in part because of profit motives. Product recovery aims to minimize the amount of waste sent to landfills by recovering materials and parts from old or outdated products by means of recycling and remanufacturing. Product recovery includes collection, disassembly, cleaning, sorting, repairing, reconditioning, reassembling and testing [1], [5].

This paper deals with an optimal control problem of a remanufacturing system under stochastic variability. The system is modeled as a Markov Decision Process (MDP) [6], [15]. The MDP is a class of stochastic sequential process in which the reward and transition probability depend only on the current state of the system and the current action. The MDP models have been applied in such diverse fields as economics, communications, manufacturing and transportation. Each model consists of states, actions, rewards, and transition probabilities. In the engineering field, for example, the approach is used for controlling the production system [13], [14]. Choosing an action as production quantity in a state generates rewards and/or costs, and determines the state at the next decision epoch through a transition probability function. Then, we can obtain the optimal production policy that minimizes the expected average cost per period in the optimal production control problem. Here we consider two types of inventories. One is the actual product inventory in the factory while the other is the virtual inventory that is still in use by the consumers. We define the state of the remanufacturing system by considering both inventory levels. We can then obtain the optimal production policy that minimizes the expected average cost per period.

In section 2, we briefly present the relevant literature. In section 3, we consider a single-item remanufacturing system under stochastic demand. The system is formulated as an undiscounted MDP to determine the optimal control policy that minimizes the expected average cost per period. We then consider a numerical example and discuss the effect of changing the manufacturing rate on the cost of the remanufacturing system.

2. LITERATURE REVIEW

We present a brief review of the literature in the areas of product recovery modeling of remanufacturing systems with stochastic variability.

Gungor and Gupta [3] and Moyer and Gupta [9] reviewed the literature in the area of environmentally conscious manufacturing and product recovery. Minner [8] pointed out that there are the two well-known streams in product recovery research area. One is stochastic inventory control (SIC) and the other is materials requirements planning (MRP). In this paper, we restrict ourselves to SIC.

As for the periodic review models, Cohen et al. [2] developed the product recovery model in which the collected products are used as is. Inderfurth [7] discussed the effect of non-zero lead times for orders and recovery. Muckstadt and Isaac [10] dealt with continuous review models for remanufacturing systems with non-zero lead times and the traditional (Q, r) control policy. Van der Laan and Salomon [16] suggested push and pull strategies for the remanufacturing system. Guide and Gupta [4] presented a queueing model to study a remanufacturing system.

All the above studies, however, considered the demand and procurement to be independent. Nakashima *et al.* [11] dealt with a product recovery system with a single class of product life cycle. They proposed a new analytical approach to evaluate the system and gave numerical examples for various conditions. They also optimized the system using the minimum cost criterion [12].

3. OPTIMIZATION OF THE SYSTEM

Consider a process that produces a single item product. The finished products are stocked in the factory and are used to satisfy consumers' demands. Each product has a life of M periods and a remaining lifetime after it is sold denoted by i ($i=M-1, \dots, 1$). The remaining lifetime decreases by one time unit each period. Traditional inventory management focuses on inventory in the factory only. In a remanufacturing system, however, we need to consider the products that are yet to be collected from the consumers. That is, the remanufacturing companies should consider the products that are still in use by the consumers as virtual inventory. It is important to manage the virtual inventory to be collected as well as the inventory on hand for use in remanufacturing.

Remanufacturing preserves the product's identity, and performs the required disassembly and refurbishing operations using most of the old parts to bring the product to a desired level of quality at some remanufacturing cost. On the other hand, the parts used in regular manufacturing process are new. The number of products by normal manufacturing in period t , $P(t)$ is chosen as an action, k , that is $k=P(t)$. Thus, the products are produced via normal manufacturing using new parts and/or via remanufacturing using the parts from the products taken back from the consumers. All production begins at the start of the period and all products are completed by the end of the period. All the products bought by consumers are new. We assume that the number of finished products and that of the products bought by consumers are $I(t)$ and $J_i(t)$ ($i=M-1, \dots, 1$), respectively. If backlog occurs, $I(t)$ is a negative value. Demand in successive periods, $D(t)$ is an independent random variable with known and identical distribution. When sold, products are remanufactured at the remanufacturing rate, λ_i ($i=M-1, \dots, 1$) with remanufacturing cost, and products in use are discarded at the discarded rate, μ with out-of-date cost. It is supposed that $\lambda_i < 1$ and $\lambda_1 + \mu \leq 1$.

The state of the system is denoted by

$$s(t) = (I(t), J_{M-1}(t), \dots, J_1(t)) \quad (1)$$

Figure 1 shows a remanufacturing system with single product life cycle.

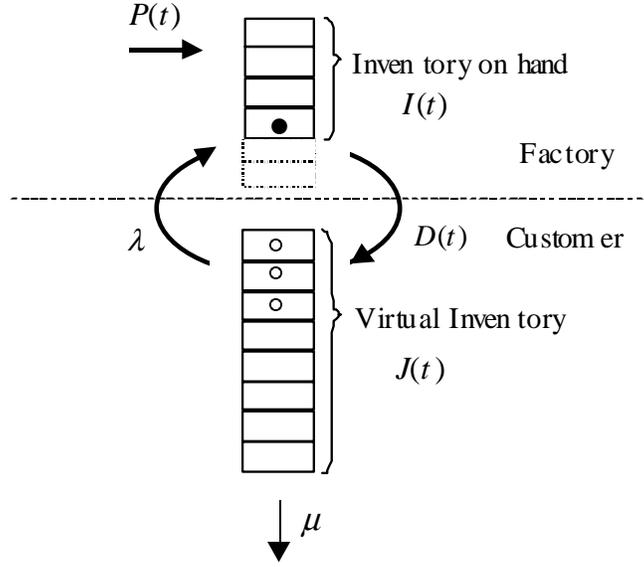


Figure 1: A remanufacturing system with single product life cycle

The transition of the each inventory is given by

$$I(t+1) = I(t) + k + \sum_{i=1}^{M-1} \lambda_i J_i(t) - D(t) \quad (2)$$

$$J_{M-1}(t+1) = \min\{[I(t)]^+ + P(t) + \sum_{i=1}^{M-1} \lambda_i J_i(t), D(t) + [I(t)]^+\} \quad (3)$$

$$J_i(t+1) = (1 - \lambda_{i+1}) J_{i+1}(t) \text{ for } i=M-1, \dots, 2 \quad (4)$$

$$J_1(t+1) = (1 - \lambda_1 - \mu) J_1(t) + (1 - \lambda_2) J_2(t) \quad (5)$$

where $[x]^+ = \min\{0, x\}$. The action space in $s(t)$, $K(s(t))$ is defined by

$$K(s(t)) = \{ 0, \dots, \max\{0, I_{\max} - I(t) - \sum_{i=1}^{M-1} \lambda_i J_i(t)\} \}. \quad (6)$$

The transition probability of the system is given by

$$P_{s(t)s(t+1)} = \begin{cases} \Pr\{D(t) = d\} & \text{if } s(t+1) = (I(t) + k + \sum_{i=1}^{M-1} \lambda_i J_i(t) - d, \\ & \dots, (1 - \lambda_1 - \mu) J_1(t) + (1 - \lambda_2) J_2(t), \\ 0 & \text{otherwise.} \end{cases} \quad (7)$$

The expected cost per period in state $s(t)$ under k , $r(s(t), k)$ is given by

$$r(s(t), k) = C_N k + \sum_{i=1}^{M-1} C_{Ri} \lambda_i J_i(t) + C_H [I(t)]^+ + C_B [-I(t)]^+ + C_O \mu J_1(t) \quad (8)$$

where the parameters are as follows:

- C_N : the normal manufacturing cost of a new product
- C_{Ri} : the remanufacturing cost of a product
- C_H : the holding cost per unit
- C_B : the backlog cost per unit
- C_O : the out-of-date cost per unit

Denote by S and $|S|$ a set of all possible states and the total number of the states, respectively. Let number the state s_n by $s (=$

1...|S|). An undiscounted MDP that minimizes the expected average cost per period, g is formulated as the following optimality equation:

$$g + v_s = \underset{k \in K(s)}{\text{Min}} \left\{ r(s, k) + \sum_{s' \in S} P_{ss'}(k) v_{s'} \right\} (s \in S). \quad (9)$$

where v_s denotes the relative value when the production system starts from state s [6]. An optimal production policy is determined as a set of k that minimizes the right-hand side of equation (9) for each state s using policy iteration method [6], [15].

4. COMPUTATIONAL RESULTS

In this section, we show the numerical results on the system with $M=2$. The distribution of the demand is given by

$$\Pr\{D_n = D - \frac{1}{2}Q + j\} = \binom{Q}{j} \left(\frac{1}{2}\right)^Q, (0 \leq j \leq Q) \quad (10)$$

where $D=2$ and Q is an even number and $\text{variance}(\sigma^2)$ is $Q/4$. The maximum number of actual inventory and that of virtual inventory are 5 and 10, respectively. The maximum number of backlog demand is set as -5. It is assumed that $h=1$, $c=1$, $b=10$, $\delta =10$, $\theta =2$, $\sigma^2 =0.5$ and the discarded rate is 0.2.

Figure 2 illustrates the behavior of the expected average cost under varying remanufacturing rates. As the rate increases, the expected cost tends to decrease.

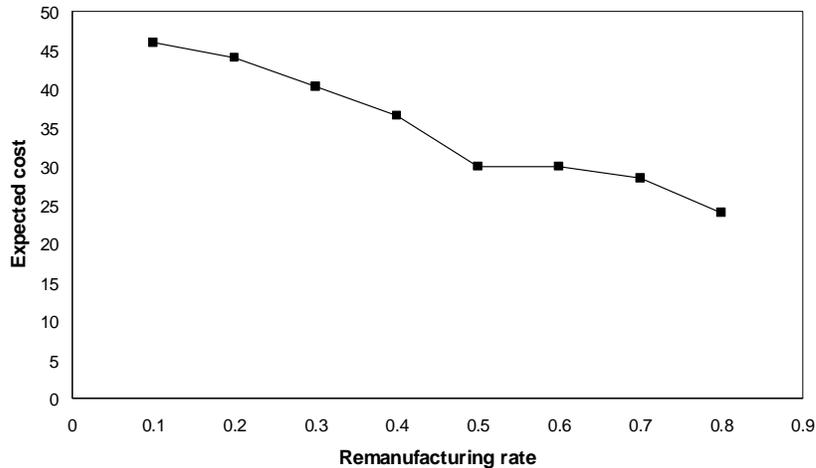


Figure 2: Expected average cost

5. CONCLUSIONS

This paper dealt with a cost management problem of a remanufacturing system with stochastic variability. We modeled the system as a Markov decision process with consideration for two types of inventories. We optimized the remanufacturing system to obtain the minimum expected cost. A numerical example showed the behavior of the remanufacturing system.

6. ACKNOWLEDGMENT

The authors would like to express their gratitude to Mr. Arimitsu, a PhD student at Osaka Institute of Technology, for his computational help.

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