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Sagar V. Kamarthi
Northeastern University

Surendra M. Gupta
Northeastern University

Vadde Srikanth
Northeastern University

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Laboratory for Responsible Manufacturing

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Contact Information

Dr. Surendra M. Gupta, P.E.
Professor of Mechanical and Industrial Engineering and
Director of Laboratory for Responsible Manufacturing
334 SN, Department of MIE
Northeastern University
360 Huntington Avenue
Boston, MA 02115, U.S.A.

(617)-373-4846 **Phone**
(617)-373-2921 **Fax**
gupta@neu.edu **e-mail address**

<http://www.coe.neu.edu/~smgupta/> **Home Page**

Multi-Scale Registration Algorithm for Alignment of Meshes

Srikanth Vadde, Sagar V. Kamarthi*, Surendra M. Gupta
Department of Mechanical Industrial and Manufacturing Engineering
Northeastern University, 360 Huntington Avenue, Boston, MA 02115

ABSTRACT

Taking a multi-resolution approach, this research work proposes an effective algorithm for aligning a pair of scans obtained by scanning an object's surface from two adjacent views. This algorithm first encases each scan in the pair with an array of cubes of equal and fixed size. For each scan in the pair a surrogate scan is created by the centroids of the cubes that encase the scan. The Gaussian curvatures of points across the surrogate scan pair are compared to find the surrogate corresponding points. If the difference between the Gaussian curvatures of any two points on the surrogate scan pair is less than a predetermined threshold, then those two points are accepted as a pair of surrogate corresponding points. The rotation and translation values between the surrogate scan pair are determined by using a set of surrogate corresponding points. Using the same rotation and translation values the original scan pairs are aligned. The resulting registration (or alignment) error is computed to check the accuracy of the scan alignment. When the registration error becomes acceptably small, the algorithm is terminated. Otherwise the above process is continued with cubes of smaller and smaller sizes until the algorithm is terminated. However at each finer resolution the search space for finding the surrogate corresponding points is restricted to the regions in the neighborhood of the surrogate points that were at found at the preceding coarser level. The surrogate corresponding points, as the resolution becomes finer and finer, converge to the true corresponding points on the original scans. This approach offers three main benefits: it improves the chances of finding the true corresponding points on the scans, minimize the adverse effects of noise in the scans, and reduce the computational load for finding the corresponding points.

Keywords: Mesh alignment, multi-scale registration, reverse engineering.

1. INTRODUCTION

Design in recent years has increasingly focused on making models that are *organic* in shape. To model an organic shape, one has to take a completely different approach from the conventional design techniques. The process of capturing the shape of a physical object in digital form starts by scanning the object from various directions through a scanning device. This scanning process produces dense sets of point clouds. By employing certain computational techniques on the point clouds, surfaces that mathematically represent the object are produced. Once the CAD model of the object is created, the designers can use it to improve the existing design, study its manufacturing complexity, estimate its mechanical properties using finite element analysis methods, and so on. Further, this surface model of the object can be sent to a downstream CAD/CAM/CAE package, either for creating a rapid prototype of the object or manufacturing the object itself.

1.1 Background

Merging each and every registered scan with the scan next to it will give a complete representation of the object in digital form. There are two general approaches to achieve this goal: surface matching methods¹ and feature-based methods^{2,3}. The present work takes the feature-based approach because it is more promising, though challenging. Therefore this paper proposes a feature-based method in which the common regions in the scans are identified by finding on the object's surface the features that are invariant to the orientation and the translation of the coordinate system. Typical examples of surface features that are invariant to the orientation and the translation of the object's coordinate system are principle curvature, mean curvature, Gaussian curvature and angle between normals. Surface features that are a combination of the above listed examples are splash⁴, synthetic signatures^{3,5}, simplex angle⁶, spin image⁷, and surface point signature⁸. A feature-based method first computes an invariant feature for all points in both the scans. If the difference between the feature values of any two points across the scan pair is below a certain threshold, then those two

*Correspondence – e-mail: sagar@coe.neu.edu; phone: 617-373-3070; fax: 617-373-2921

points are accepted as a possible corresponding point pair. At least three such corresponding point pairs are necessary to find suitable rotation and translation matrices to align the scans together. One of the scans in the pair is rotated and translated to align it with the second one in the pair. After such a transformation, the distance between the transformed scan and the reference scan is computed using an error function. If the error is within an acceptable limit, the alignment (or registration) of the scan pair is concluded; otherwise the process is repeated with a new set of corresponding point pairs. A feature-based method provides an accurate scan alignment if the correct corresponding point pairs are identified.

1.2 Research objectives

This paper presents an algorithm to align a given pair of scans of the object that share a common surface. The proposed technique first uses pose-invariant surface features, *viz.* uni-Gaussian-curvature regions or low and high curvature regions for aligning scans. This paper attempts to address some of the unresolved issues in this process:

- The algorithm should make no assumption about the shape of the object. In other words, it should work on almost all kinds of geometric shapes.
- It should make the alignment possible even if the matching scans have relatively flat regions in the overlap regions.
- The algorithm should align meshes with features computed at broader resolution and refine the alignment with features computed at finer resolution.
- The algorithm should be insensitive to noise in meshes.

1.3 Motivation

Scanning and reverse engineering methods have wide ranging applications from medical imagery to movies: estimating the geometry of an object in robotic applications³, applications such as industrial object recognition, face recognition, navigation and manipulation in natural environments⁶, animation and visual simulation⁹, building CAD model representations of objects such as human faces, and sculptured objects and in creating virtual museums and virtual reality environment¹⁰, the inspection of parts, entertainment industries, image tiling, and automated inspection of in-situ engine components whose surface information is acquired from various viewpoints using range scanners¹¹, the analysis of deformation under stress or collision damage¹², architectural design to represent a prototype digital model¹³, biology^{14,15}, chemistry¹⁶, crystallography¹⁷, dental applications¹⁸, digital film-making¹⁹, orthodontics²⁰, and nuclear waste cleanup²¹.

2. MESH ALIGNMENT ALGORITHM

This section presents the mesh alignment algorithm. Before outlining the steps of the algorithm some important terms and definitions are presented below.

2.1 Definitions

Scan: A scan is a point cloud which represents the surface information of the object as viewed in the scanning direction. A scan is a direct output of a scanning device.

Mesh: A mesh refers to a scan when the points in the scan are triangulated.

A Pair of meshes: Any two meshes are referred to as a pair of meshes if they are being aligned or considered for alignment.

A pair of corresponding points: Let p_i be a point on Mesh 1 and q_j be a point on Mesh 2. The points p_i and q_j are referred to as a pair of corresponding points if $|K(p_i) - K(q_j)| < \delta$, for a predefined threshold δ ; $K(p)$ is the Gaussian curvature computed at a point p .

2.2 Outline of the algorithm

The following are the steps of the algorithm to align two meshes:

Phase I: Let M_1 and M_2 be the set of points on Mesh 1 and Mesh 2 respectively. Compute the Gaussian curvatures $K(p_i)$ and $K(q_j)$ for every $p_i \in M_1$ and for every $q_j \in M_2$ using the procedure developed by Meyer *et al.*²².

Phase II: Compare $K(p_i)$ and $K(q_j)$ to find J the set of pairs of corresponding points.

Phase III: Compute the rotation matrix R and the translation matrix T that will align Mesh 1 and Mesh 2 from the pairs of corresponding points in set J using the Singular Value Decomposition method developed by Arun *et al.*²³.

Details of each phase of the algorithm are presented in a previous work²⁵.

3. LIMITATIONS OF MESH ALIGNMENT ALGORITHM

The authors in their earlier work²⁵ observed that mesh alignment algorithm presented in the previous section has some issues. The Gaussian curvature computed at a point, being too fine a feature, poses problems in finding corresponding points. Finer features computed at the points of a scan may not be unique. Many points may have identical Gaussian curvature values. This will lead to the problem of labeling a pair of points as corresponding points while they are not in reality. The presence of noise in scans further poses challenges in finding the corresponding points. If there is noise around a point, the Gaussian curvature computed will be incorrect. Because of this the algorithm may fail to recognize a pair of points as corresponding points, while they actually are. Since the scans being aligned consist of large number of points the comparison of the Gaussian curvatures across the scans is computationally intensive.

In order to address the issues listed above the authors propose a multi-scale registration algorithm. The multi-scale registration algorithm attempts to find pairs of corresponding points by comparing broader features that are more likely to be unique. This method is expected to be insensitive to the presence of noise in a mesh. This way the mesh alignment algorithm will be able to find precise registration. By virtue of its design the algorithm will require significantly less computational time.

4. MULTI-SCALE REGISTRATION ALGORITHM

Features such as Gaussian curvature and mean curvature which are defined in the neighborhood of a point are narrowly local in space. They do not capture spatially broader features like bumps, dents, hills, and valleys which are more effective for comparison of scans for alignment. Such broader features that capture the shape of the scan can be obtained by adopting a multi-resolution approach. In a multi-resolution approach the intent is to start with features that are defined at a broader resolution on a scan and move to features of finer and finer resolution in a step by step manner. Once features on a broader resolution match across scans being aligned, the features at finer resolutions are evaluated to find more accurate registrations.

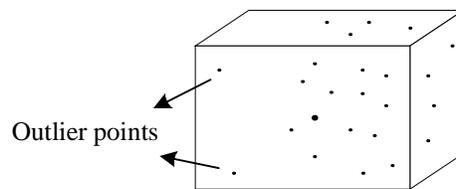


Figure 1: Identification of outlier points in cube

The proposed multi-scale registration approach is more likely to be insensitive to noise in a mesh, see Figure 1. Since in this approach the points that lie in a cube are approximated by the centroid, the noise present in the scan will have minimal effect on the computation of the Gaussian curvature at the centroid.

The central idea behind multi-scale registration algorithm is as follows. The volume that encloses a scan is divided into cubes of equal and fixed size. The point or the points that lie in a cube are approximated by the cube centroid. The centroids of all the cubes that encase a scan are considered to approximate the scan. In other words the set of centroids act as the surrogate scan. Using the Gaussian curvatures computed at the cube centroids, the alignment of the surrogate scans is performed according to the algorithm presented in section 2. This process results in a set of surrogate corresponding points and surrogate transformation parameters. These transformation parameters are then used to align the original scans and to compute the corresponding registration error. Depending on this registration error, one can decide whether or not to execute the algorithm at the next finer resolution. To move to the next finer resolution only the cubes that contain surrogate corresponding points at the current resolution are subdivided into cubes of half the size; and all other cubes that do not contain surrogate corresponding points are ignored. At the finer resolution all the cubes that do not encase any scan points are discarded. The mesh alignment algorithm is applied again on the surrogate scans populated by the centroids of the cubes at the finer resolution. As the algorithm moves from one resolution to the next

resolution, search space on the scan is narrowed, but the search is carried out at finer resolutions. This approach offers three benefits: improve the chances of finding true corresponding points, minimize the adverse effects of scan noise, and reduce the computational load significantly.

4.1 Multi-scale registration algorithm

Let V represent the binding box of a scan/mesh M , see Figures 2 and 3. Let it be divided into cubes of size a . Let c_k be the set of scan points that lie within the k^{th} cube in the binding box, see Figure 4. The stepwise procedure of multi-scale registration is presented below.

1. Fit a binding box V_1 to Mesh 1 and V_2 to Mesh 2.
2. Divide V_1 and V_2 into cubes of size a .
3. From the cubes in V_1 choose the set of cubes W_1 each of which encloses at least one scan point. Ignore all cubes in V_1 that do not belong to W_1 . The cubes in W_1 encase the points in Mesh 1 at a resolution characterized by the cube size. Similarly find the set of cubes W_2 for Mesh 2.
4. Let L_1 and L_2 be the sets of centroids of cubes in W_1 and W_2 respectively. The sets L_1 and L_2 act as the surrogate scans for Mesh 1 and Mesh 2 respectively.
5. Apply the mesh alignment algorithm presented in section 2 on point sets L_1 and L_2 . This process results in a set of surrogate corresponding points and surrogate transformation parameters. If the process leads to an insufficient number of surrogate corresponding points then go to step 8.
6. Apply the surrogate transformation from the previous step to align Mesh 1 and Mesh 2; and compute the resulting registration error. If the registration is within the desired limits, then go to step 10 to terminate the algorithm; otherwise proceed to the next step.
7. From the sets W_1 and W_2 discard all the cubes that do not enclose a surrogate corresponding point.
8. Let F_1 be the set of cubes obtained by dividing each cube in W_1 into cubes whose size is half that of the cubes in W_1 . Similarly find the set F_2 . If the cube size is smaller than a predefined lower limit, then go to step 10 to terminate the algorithm; otherwise proceed to the next step.
9. Redefine the set W_1 : from the cubes in F_1 choose the set of cubes W_1 each of which encloses at least one scan point. Now the cubes in W_1 encase the points of Mesh 1 in the neighborhood of the surrogate corresponding points if they were identified in step 5. Similarly redefine the set W_2 . Go to step 4.
10. Terminate the algorithm.

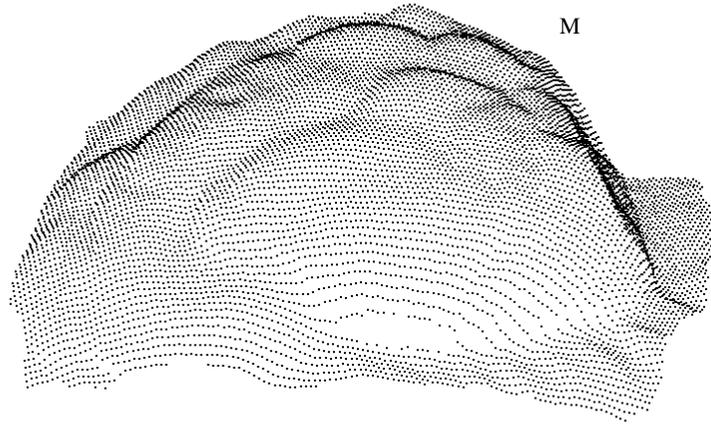


Figure 2: Scan M

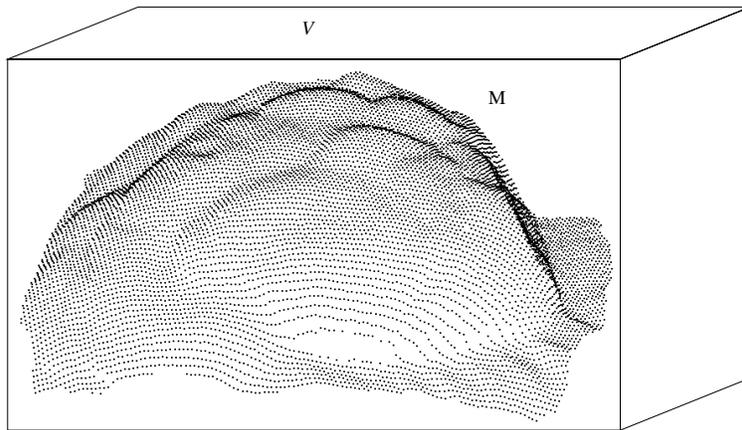


Figure 3: Binding box V of scan M

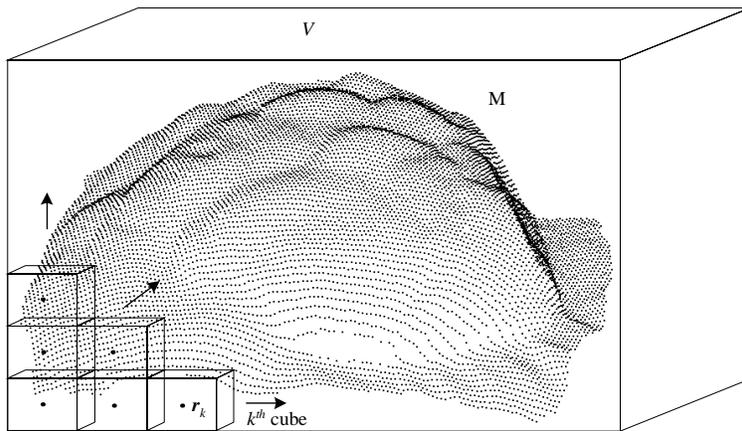


Figure 4: Approximation of scan M by cubes

4.2 Implementation issues

Unlike in a original scan, the points in a surrogate scan (sets L_1 and L_2) are not triangulated. Therefore to compute the Gaussian curvature at a cube centroid, the knowledge of its neighboring cubes should be maintained during implementation using an appropriate data structure. One can expect that the computational time for the multi-scale registration algorithm to align a pair of scans will be comparatively less than that of the mesh alignment algorithm presented in section 2. The reason is explained as follows. At the starting resolution the density of points in a surrogate scan is less than the density of points in the counterpart original scan. At every subsequent resolution the surrogate scan will contain only points that are located in the neighborhood of the corresponding points while its counterpart original scan will contain points located all through the surface. The number of points in a surrogate scan are far fewer than those in its counterpart original scan. Therefore the task of comparing points across a pair of surrogate scans requires much less computational time relative to what is required for the original scans.

5. ISSUES WITH MULTI-SCALE REGISTRATION ALGORITHM

Prior to executing the multi-scale registration method one has to determine the starting and the ending resolution. The cube size characterizes the resolution: larger the cube size coarser the resolution and smaller the cube size finer the resolution. The global and local variations in the surface of the mesh would dictate the starting cube size (upper limit) and the ending cube size (lower limit). The size of the global hills and valleys determine the upper limit of the cube size and the finer hills and valleys determine the lower limit of the cube size. An inherent limitation of the multi-scale registration algorithm is, it expects one to choose a predetermined threshold δ to find corresponding points. Therefore for a given pair of meshes one has to perform hit and trial experiments to determine a reasonable δ .

6. CONCLUSIONS AND FUTURE WORK

In this paper a multi-scale registration algorithm for alignment of meshes is proposed. The multi-scale registration algorithm starts with broader features to isolate the regions where the corresponding points are likely to exist and then steadily moves to finer and finer features for finding actual corresponding points. The points in the scan are encased by an array of cubes and the scan is itself approximated by the cube centroids; in the sense the set of cube centroids act as the surrogate scan. Using the Gaussian curvatures computed at the points in the surrogate scans the surrogate corresponding points are identified and the surrogate translation and rotation matrices are estimated. These translation and rotation matrices are applied on the original scans and the resulting registration error is checked to see if it is within the desired limits. This process is repeated at finer and finer resolutions until the desired small registration error is achieved. This approach leads to improved registration accuracy, better tolerance to noise, and reduced computational load.

Future work will focus on implementing the multi-scale registration algorithm. The performance of the algorithm will be studied on a set of test scans of real world objects. The performance of the algorithm will be compared with those of the existing algorithms. The upper limit of the cube size, which depends on the shape of the mesh surface, is difficult to determine, and requires further work. The lower limit of the cube size needs to be somehow related to the average distance between points on scan. A meaningful method to determine the predetermined threshold for comparing the Gaussian curvature values will be developed.

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