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# Constraints on Unparticle Physics from Solar and KamLAND Neutrinos

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Interest has been directed recently towards low energy implications of a non-trivial conformal sector of an effective field theory with an IR fixed point ( $\Lambda_U$ ), manifest in terms of “unparticles” with bizarre properties. We re-examine the implications of the limits on decay lifetimes of solar neutrinos for unparticle interactions. We study in detail the fundamental parameter space ( $\Lambda_U, M$ ) and derive bounds on the energy scale  $M$  characterizing the new physics. We work strictly within the framework where conformal invariance holds down to low energies. We first assume that couplings of the unparticle sector to the Higgs field are suppressed and derive bounds with  $\Lambda_U$  in the TeV region from neutrino decay into scalar unparticles. These bounds are significant for values of the anomalous dimension of the unparticle operator  $1.0 < d \lesssim 1.2$ . For a region of the parameter space, we show that the bounds are comparable to those arising from production rates at high energy colliders. We then relax our assumption, by considering a more natural framework which does not require *a priori* restrictions on couplings of Higgs-unparticle operators, and derive bounds with  $\Lambda_U$  in meV region from neutrino decay into vector unparticles. Such low scales for the IR fixed point are relevant in gauge theories with many flavors.

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A conformal hidden sector, which couples to the various gauge and matter fields of the Standard Model (SM), has been advocated by Georgi [1]. In the ultraviolet theory, the hidden sector couples to the SM through non-renormalizable interactions

$$\mathcal{L}_{UV} = \frac{O_{UV} O_{SM}}{M^{d_{UV}+n-4}}, \quad (1)$$

where  $M$  is the mass of the heavy exchanged particle, and  $O_{UV}$  and  $O_{SM}$  are hidden sector and SM operators with mass dimensions  $d_{UV}$  and  $n$ , respectively. The hidden sector has a non-trivial IR fixed point,  $\Lambda_U$ , below which the sector exhibits scale invariance and the operator  $O_{UV}$  mutates into an “unparticle” operator  $O_U$  with non-integral scaling dimension  $d$ . The couplings then become

$$\mathcal{L}_U = C_U \frac{\Lambda^{d_{UV}-d}}{M^{d_{UV}+n-4}} O_{SM} O_U, \quad (2)$$

where  $C_U$  is a dimensionless coupling constant.

The phenomenology of the unparticle has been explored by many groups [2] and lower bounds on  $\Lambda_U$  have already been derived by considering production rates at high energy colliders [3, 4] and unparticle emission from the core of SN1987 A [5]. If unparticle stuff exists, it could couple to the stress tensor and mediate a new force (ungravity) between massive particles. This would modify the inverse square law with  $r$  dependence in the range between  $1/r^{4+2\delta}$  ( $\delta > 0$ ), a region of the parameter space to be probed by future submillimeter tests of gravity [6].

As shown in [7], one of the bizarre implications of the conformal hidden sector is that neutrinos would become unstable: a neutrino mass eigenstate  $\nu_j$  can decay into another eigenstate  $\nu_i$  via  $\nu_j \rightarrow \nu_i \mathcal{U}$ , where  $\mathcal{U}$  is the invisible unparticle. In this Letter we re-examine the impact of solar and KamLAND neutrino data on the effective couplings between neutrinos and unparticle operators. We

derive bounds on the relevant parameter space ( $\Lambda_U, M$ ), and discuss how these bounds compare with existing limits.

Observation of solar neutrinos suggest the disappearance of  $\nu_e$  while propagating within the Sun ( $\sim 2$  s) or between the Sun and Earth ( $\sim 500$  s). Specifically, data collected by the Sudbury Neutrino Observatory (SNO) [8] in conjunction with data from SuperKamiokande (SK) [9] show that solar  $\nu_e$ 's convert to  $\nu_\mu$  or  $\nu_\tau$  with CL of more than  $7\sigma$ . On the other hand, the KamLAND Collaboration [10] has measured the flux of  $\bar{\nu}_e$  from distant reactors and find that  $\bar{\nu}_e$ 's disappear over distances of about 180 km. The combined analysis of solar and KamLAND neutrinos is consistent at the  $3\sigma$  CL, with best-fit point and  $1\sigma$  ranges:  $\delta m_{\odot}^2 = 8.2_{-0.3}^{+0.3} \times 10^{-5} \text{ eV}^2$  and  $\tan^2 \theta_{\odot} = 0.39_{-0.04}^{+0.05}$  [11]. The striking agreement between solar and KamLAND data determines a unique solution in the mass-mixing parameter space, dubbed the Mikheyev-Smirnov-Wolfenstein Large Mixing Angle (LMA) solution [12]. This provides evidence that solar neutrinos in the energy range  $5 \text{ MeV} < E_\nu < 15 \text{ MeV}$  are created as nearly pure  $\nu_2$  mass eigenstates. Moreover, for LMA their propagation in the Sun is completely adiabatic, and hence neutrinos emerge as pure  $\nu_2$  eigenstates, where  $m_2 > m_1$ . The mass ordering of neutrinos, however, is not uniquely determined. There are two possible mass ordering that we denote as normal and inverted, which without any loss of generality, can be chosen as  $m_1 < m_2 < m_3$  and  $m_3 \ll m_1 \approx m_2$ , respectively. For simplicity here we assume that the lightest neutrino is massless and take  $m_2 \approx 9 \text{ meV}$  and  $m_3 \approx 50 \text{ meV}$  for the normal, and  $m_2 \approx m_1 \approx 50 \text{ meV}$  for the inverted hierarchy.

It has long been realized that the solar neutrino flavor ratios predicted by the standard oscillation phenomenology can be modified if processes such as neutrino decay

occur [13]. Indeed, since neutrinos leave the Sun in a single mass eigenstate, there is no ambiguity concerning flavor mixes at the source [14]. However, in the limit that neutrino masses are degenerate, a daughter neutrino  $\nu_i$  produced in a hypothetical decay will carry the full energy of the parent neutrino  $\nu_j$ , and would be detected by experiments on Earth. Therefore, the replacement of  $\nu_j$  with an active daughter  $\nu_i$  of about the same energy could camouflage the characteristics of decay. This would be specially pertinent for  $\nu_2 \rightarrow \nu_1 \mathcal{U}$ , where both  $\nu_2$  and  $\nu_1$  have large  $\nu_e$  projections. All in all, if neutrino masses are non-degenerate, the Earth-Sun baseline defines a  $\nu_2$  lifetime limit,  $\tau/m_2 \gtrsim 10^{-4}$  s/eV [15], for neutrinos decaying into invisible unparticles.

Before proceeding, we stress that for any conformal field theory the conformal dimensions of the unparticle operators are bounded from unitarity as  $d \geq 1 + s$ , where  $s$  is the spin of the operator [16]. Thus, for a rank one tensor operator  $d > 2$  and for a rank two  $d > 3$ . Because of this, vector and higher tensor operators are less dominant in the unparticle scheme [17].

In the mass basis, the interaction between neutrinos and scalar unparticle operators can be written as  $\lambda_{\nu}^{ij} \bar{\nu}_i \nu_j O_{\mathcal{U}}/\Lambda_{\mathcal{U}}^{d-1}$ , where

$$\lambda_{\nu}^{ij} = C_{\mathcal{U}} (\Lambda_{\mathcal{U}}/M)^{d_{UV}-1} \quad (3)$$

is the coupling constant. The total decay rate is found to be [7]

$$\Gamma_j = A_d \frac{|\lambda_{\nu}^{ij}|^2 m_j}{16\pi^2 d (d^2 - 1)} \left( \frac{m_j^2}{\Lambda_{\mathcal{U}}^2} \right)^{d-1}, \quad (4)$$

where

$$A_d \equiv \frac{16\pi^{5/2}}{(2\pi)^{2d}} \frac{\Gamma(d+1/2)}{\Gamma(d-1)\Gamma(2d)}. \quad (5)$$

Now, using the bound on the neutrino lifetime, we can constrain the  $(|\lambda_{\nu}^{ij}|, \Lambda_{\mathcal{U}})$  parameter space from

$$\frac{16\pi^2 d (d^2 - 1)}{A_d |\lambda_{\nu}^{ij}|^2 m_j^2} \left( \frac{\Lambda_{\mathcal{U}}^2}{m_j^2} \right)^{d-1} > 1.5 \times 10^{11} \text{ eV}^{-2}. \quad (6)$$

Note that these constraints hide the dimension  $d_{UV}$  of the Banks-Zaks (BZ) fields [18].

To provide further probes of new physics we re-introduce the parametrization given in Eq. (3). When combined with Eq. (6), we obtain the constraint

$$M > D^{1/[2(d_{UV}-1)]} \left( \frac{\Lambda_{\mathcal{U}}}{m_j} \right)^{(1-d)/(d_{UV}-1)} \Lambda_{\mathcal{U}}, \quad (7)$$

where

$$D = 1.7 \times 10^7 \left( \frac{m_j}{9 \text{ meV}} \right)^2 C_{\mathcal{U}}^2 \frac{A_d}{16\pi^2 d (d^2 - 1)}. \quad (8)$$

Since  $d > 1$ , the lower bound on  $M$  rises faster than linearly for small  $\Lambda_{\mathcal{U}}$ ; then for large  $\Lambda_{\mathcal{U}}$ , falls below the

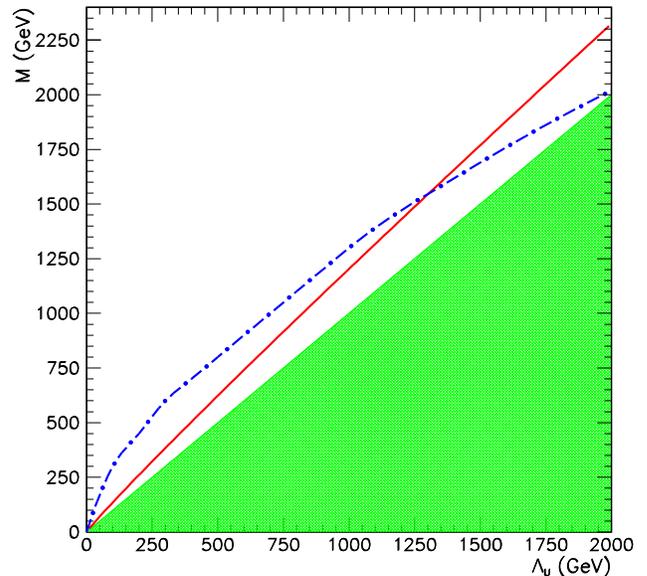


FIG. 1: Bounds from solar and KamLAND neutrinos (solid line) and from  $e^+e^- \rightarrow \mu^+\mu^-$  (dot-dashed line) [3] on the fundamental parameter space  $(\Lambda_{\mathcal{U}}, M)$  for a scalar unparticle operator, with  $d = 1.1$ ,  $d_{UV} = 3$ , and couplings to the Higgs bilinear suppressed. The regions below the contours are excluded. The shaded region is excluded by the requirement  $M > \Lambda_{\mathcal{U}}$ . In our calculations we have taken  $C_{\mathcal{U}} = 0.1$ .

line  $M = \Lambda_{\mathcal{U}}$ ; thereafter the lower bound on  $M$  is simply  $\Lambda_{\mathcal{U}}$ . Equating the lower bound in Eq. (7) to  $\Lambda_{\mathcal{U}}$ , we obtain the crossover point

$$\Lambda_{\mathcal{U}}^{\text{cross}} = D^{1/[2(d-1)]} m_j, \quad (9)$$

independent of  $d_{UV}$ . For a qualitative view, we restrict ourselves to the cases where the BZ [18] operator is a dimension-3 fermion bilinear ( $d_{UV} = 3$ ) or a dimension-4 gauge invariant gluon bilinear. (The dimension-3 case corresponds to the chiral order parameter which is known to have an anomalous dimension near  $d = 1$  [19].) If it is desired to obtain a bound in the TeV region, it is required that the crossover point occur at  $\Lambda_{\mathcal{U}} \gtrsim 1$  TeV. A straightforward numerical exercise shows that this occurs only when the anomalous dimension  $1 < d < 1.2$  (for  $C_{\mathcal{U}} < 1$ ). However, we know from the analysis of [20] that if (a) the scalar operator has  $d < 2$  and (b) it couples to the Higgs field, then conformal invariance must be broken at energies below the electroweak scale. This would vitiate our bounds, because we have assumed conformal invariance down to the meV scale. Thus, the only way to retain bounds in the TeV region is to assume that  $d \sim 1$  and that, for some reason, the coupling of the scalar unparticle to the Higgs field vanishes. In that case, illustration of the results are presented in Figs. 1 and 2, for the case  $d = 1.1$ ,  $C_{\mathcal{U}} = 0.1$ , and  $d_{UV} = 3, 4$ ; respectively. It is evident that the results are little changed by the variation in  $d_{UV}$ . The crossover point in this case lies

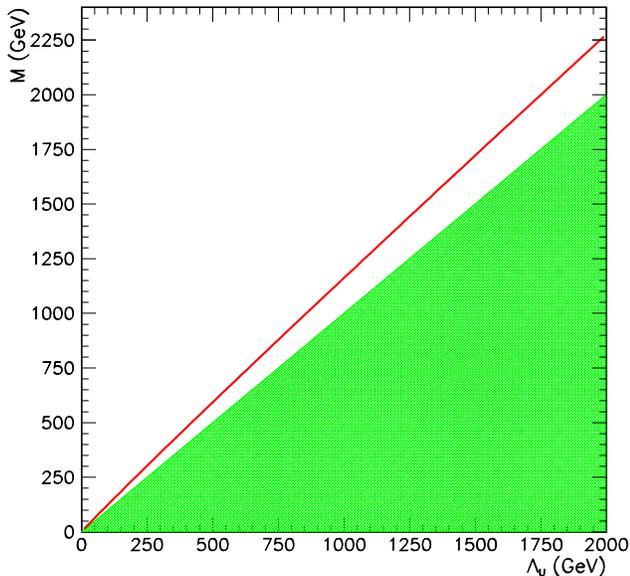


FIG. 2: Bounds from solar and KamLAND neutrinos on the fundamental parameter space  $(\Lambda_U, M)$  for a scalar unparticle operator, with  $d = 1.1$ ,  $d_{UV} = 4$ , and couplings to the Higgs bilinear suppressed. The region below the contour is excluded. The shaded region is excluded by the requirement  $M > \Lambda_U$ . As in Fig. 1, we have taken  $C_U = 0.1$ .

well above the TeV range. For comparison, constraints on the  $(\Lambda_U, M)$  parameter space from production rate at colliders are also shown. For  $1 < d \lesssim 1.2$ , collider bounds are comparable to those from neutrino decay into unparticles. These bounds leave open a substantial window for discovery of unparticle stuff at the LHC.

In what follows we consider a more natural framework, which does not require *a priori* restrictions on couplings of Higgs-unparticle operators, and we derive bounds on the  $(\Lambda_U, M)$  parameter space from neutrino decay into vector unparticles. As mentioned above, in this case the dimension of the BZ field at the IR fixed point runs to  $d > 2$ . The coupling to the Higgs field is then naturally suppressed [3], and we can retain conformality to low scales. The total decay rate of neutrinos into vector unparticles is given by [7]

$$\Gamma_j = 3 A_d \frac{|\lambda_\nu^{ij}|^2 m_j}{16\pi^2 d(d-2)(d+1)} \left( \frac{m_j^2}{\Lambda_U^2} \right)^{d-1}, \quad (10)$$

and the associated equation constraining the  $(|\lambda_\nu^{ij}|, \Lambda_U)$  parameter space reads

$$\frac{16\pi^2 d(d-2)(d+1)}{3 A_d |\lambda_\nu^{ij}|^2 m_j^2} \left( \frac{\Lambda_U^2}{m_j^2} \right)^{d-1} > 1.5 \times 10^{11} \text{ eV}^{-2}. \quad (11)$$

A sample result, for  $d_{UV} = 3$ ,  $d = 2.1$ ,  $C_U = 0.1$  is shown in Fig. 3. Such low scales for the IR fixed point

are possible in gauge theories with many flavors [19].

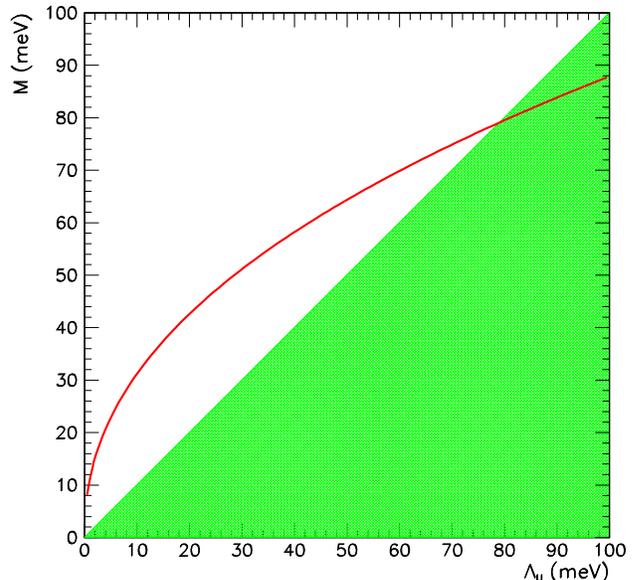


FIG. 3: Bounds from solar and KamLAND neutrinos on the fundamental parameter space  $(\Lambda_U, M)$  for a vector unparticle operator, with  $d = 2.1$  and  $d_{UV} = 3$ . The region below the contour is excluded. The shaded region is excluded by the requirement  $M > \Lambda_U$ .

In closing we comment on the potential of the Pierre Auger Observatory [21] to probe the  $(\Lambda_U, M)$  plane, using cosmic baselines for measuring neutrino lifetimes. For a normal mass hierarchy, the relative cosmic neutrino flux  $(\phi_\alpha)$  on Earth would be given by the flavor  $(\alpha)$  projection of the sole surviving (lightest) mass-eigenstate,  $|U_{\alpha 1}|^2$ ; a result that is independent of neutrino energy and source dynamics [22]. Because of the  $\nu_\mu$ - $\nu_\tau$  interchange symmetry, each mass eigenstate contains an equal fraction of  $\nu_\mu$  and  $\nu_\tau$ . Unitarity plus the condition  $|U_{\mu 1}|^2 = |U_{\tau 1}|^2$  leads to earthly ratios of  $2|U_{e1}|^2 : (1 - |U_{e1}|^2) : (1 - |U_{e1}|^2)$ . There is then a single flavor ratio to be determined, which we take to be  $\phi_e : \phi_\tau$ , as it can be inferred at Auger from the ratio of measured rates of quasi-horizontal and Earth-skimming events [23]. Substituting the measured value of  $|U_{e1}|^2$  [25], one finds a flavor ratio  $\phi_e : \phi_\mu : \phi_\tau = 6 : 1 : 1$ . This result is in striking contrast to the expectation for stable neutrinos [24]. Since cosmic neutrinos propagate for distances  $L \gtrsim 100$  Mpc, future Auger observations can be used to probe neutrino lifetimes at the level  $\tau/m \sim L/E_\nu \sim 10^{-2} \text{ s/eV}$ , increasing sensitivity to the unparticle stuff by about half an order of magnitude.

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