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A. Widom
Northeastern University

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Concepts of Intertial and Gravitational Mass

A. Widom
Physics Department, Northeastern University
110 Forsyth Street, Boston MA 02115
Email: allan.widom@gmail.com

Abstract. The general relativistic notion of gravitational and inertial mass is discussed from the general viewpoint of the tidal forces implicit in the curvature and the Einstein field equations within ponderable matter. A simple yet rigorously general derivation is given for the Tolman gravitational mass viewpoint wherein the computation of gravitational mass requires both a rest energy contribution (the inertial mass) and a pressure contribution. The pressure contribution is extremely small under normal conditions which implies the equality of gravitational and inertial mass to a high degree of accuracy. However, the pressure contribution is substantial for conformal symmetric systems such as Maxwell radiation, whose constituent photons are massless. Implications of the Tolman mass for standard cosmology and standard high energy particle physics models are briefly explored.

Any notions of a gravitational mass as a source for gravitational fields and inertial mass in general relativity[1] must derive their magnitude from the energy-pressure tensor $T_{\mu\nu}$ of ponderable matter. The continuous material media vector velocity v^μ and the scalar energy density ε may be computed via the unique time-like eigenvector of the energy-pressure tensor. In detail,

$$T_{\mu\nu}v^\nu = -\varepsilon v_\mu \quad \text{wherein} \quad v^\mu v_\mu = -c^2. \tag{1}$$

The energy-pressure tensor must then have the form

$$T_{\mu\nu} = \varepsilon\left[\frac{v_\mu v_\nu}{c^2}\right] + P_{\mu\nu} \quad \text{such that} \quad P_{\mu\nu}v^\nu = 0. \tag{2}$$

The anisotropic pressure tensor is thereby $P_{\mu\nu}$, which on average yields the scalar pressure P via

$$P = \frac{1}{3}P^\mu{}_\mu \quad \text{equivalent to} \quad T \equiv T^\mu{}_\mu = 3P - \varepsilon. \tag{3}$$

In summary,

$$T_{\mu\nu} = (\varepsilon + P)\left[\frac{v_\mu v_\nu}{c^2}\right] + Pg_{\mu\nu} + \Pi_{\mu\nu}, \quad \Pi_{\mu\nu}v^\nu = 0 \quad \text{and} \quad \Pi^\mu{}_\mu = 0, \tag{4}$$

represents the most general form for the energy-pressure tensor. From the above information contained in the energy-pressure tensor, we may discuss the differences between inertial and gravitational mass.

The inertial mass density of ponderable matter may be equated to the energy density in the local Lorentz rest frame of the continuous media, i.e.

$$\tilde{\rho} = \frac{\varepsilon}{c^2} \qquad \text{(inertial mass density).} \tag{5}$$

The definition of inertial mass density in Eq.(5) is more or less obvious. For example, if a point particle were moving along a space-time path, then consistent with Eq.(5) the inertial mass m would be computed from the particle vector momentum p^μ via the usual rule $-p^\mu p_\mu = m^2c^2$. For any finite inertial mass m, the scalar mass density $\tilde{\rho}$ definition in Eq.(5) amounts to the Einstein *rest energy* formula $m = \mathcal{E}/c^2$ but in a general relativistic form.

The proper definition of the *gravitational* mass density ρ of ponderable matter presents a more subtle problem. Tolman[2] studied the problem of how to define the observable gravitational mass M of a spherically symmetric large massive body. The required mass appears as the gravitational radius $R_s = (2GM/c^2)$ in the Schwartzchild metric solution to the Einstein field equations. It was concluded that the definition of the gravitational mass density within the spherical body required both the energy density ε and the pressure P according to the rule

$$\rho = \frac{\varepsilon + 3P}{c^2} = \tilde{\rho} + \frac{3P}{c^2} \qquad \text{(gravitational mass density).} \tag{6}$$

The difference between inertial mass density $\tilde{\rho}$ and gravitational mass density ρ is thereby due to the pressure P as described in the above Eq.(6).

Let us pause to give a *simple* yet quite *general* proof of the Tolman gravitational mass density Eq.(6) which is free of the specific spherical symmetries of the Schwartzchild metric. Within ponderable matter with a flow velocity v^μ, there will exist tidal forces due to the gravitational curvature. The tidal force tensor is well known[3] to be given by

$$\Phi_{\lambda\sigma} = R_{\lambda\mu\sigma\nu} v^\mu v^\nu. \tag{7}$$

The trace of the tidal force tensor may be evaluated employing the Einstein field equations,

$$R_{\mu\nu} = \frac{8\pi G}{c^4}\left\{T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right\}, \tag{8}$$

in the form

$$\Phi^\lambda{}_\lambda = R_{\mu\nu} v^\mu v^\nu = \frac{4\pi G}{c^2}\left\{2T_{\mu\nu}\frac{v^\mu v^\nu}{c^2} + T\right\}. \tag{9}$$

Eqs.(1), (3) and (9) imply the tidal force tensor trace

$$\Phi^\lambda{}_\lambda = \frac{4\pi G}{c^2}(\varepsilon + 3P) = 4\pi G\rho. \tag{10}$$

In that Eq.(10) is merely the general relativistic version of the Newtonian gravitational field equation $\nabla^2\Phi = 4\pi G\rho$ in a very thinly disguised form, the proof of the Tolman gravitational mass density Eq,(6) has been completed.

Since the relativistic stability of ponderable matter requires that $3P \leq \varepsilon$, we have an inequality between inertial and gravitational mass densities

$$\rho \leq 2\tilde{\rho}. \tag{11}$$

We note in passing that a *cosmological* term, when added into the Einstein field equations, may be viewed as a uniform but *negative* pressure in Eq.(4). Negative pressure metastable states of matter are available in laboratories. Furthermore, $\rho < 0$ is *not* forbidden by any known general relativistic theorem. For the ordinary stable continuous matter around us, the positive pressure contribution to the gravitational mass is extremely small so that $\tilde{\rho} \approx \rho$ holds true to a sufficient degree of accuracy. For the Maxwell radiation field, one has $\varepsilon = 3P$ so that $\rho = 2\tilde{\rho}$ for a gas of photons. That the gravitational mass is twice the inertial mass holds true for all model systems in which the constitutive particles are massless.

The $\rho = 2\tilde{\rho}$ result holds true for field theoretical models exhibiting conformal symmetry. For example, the massless gluons which help model strong interactions contribute an energy density and pressure related by $\varepsilon = 3P$. If one were to build an inertial mass with say a glue-ball made up of massless constituent gluons, then one would also build up a gravitational glue-ball mass at twice the inertial mass value. But as stated above, the masses in our neighborhood obey $\tilde{\rho} \approx \rho$. It is thereby evident that glue-balls contribute very little to the observed neighborhood gravitational masses. Let us consider this type of result in somewhat more detail.

In the standard model of matter[4], one begins with an $SU(3) \times SU_{\text{left}}(2) \times U(1)$ field theory with conformal symmetry even for the quark and lepton sectors of the theory. The conformal symmetry is broken by a conjectured Higgs field which grows masses on some of the elementary particles, specifically $(Z, W^{\pm}, e, \mu, \tau)$ in the electroweak interaction sector and the quarks (u, d, c, s, t, b) in the strong interaction sector. For the model to hold true and also give the observed *gravitational* as well as inertial masses, i.e. $\rho \approx \tilde{\rho}$ without a factor of two, one must hold the Higgs field responsible for growing macroscopic gravitational mass as well as inertial mass on the elementary constituent particles. The gravitational implications of the Higgs nechanicsm of growing inertial and gravitational masses on elementary particles have yet to be fully explored.

References

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