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ANALYSIS AND VERIFICATION OF FATIGUE RELIABILITY VARIATION UNDER TWO-STAGE LOADING CONDITIONS.

MS thesis presentation

By

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ABSTRACT

The effective life of a specimen is calculated as the time from when the specimen is in its operating conditions to the time of its failure and reliability indicates probability of survival of the specimen or product. In material science, this effective life is represented as the number of loading cycles until the failure of the specimen. Life of any product or specimen is inversely proportional to the load (stress or strain) applied on it. Practically, external loading is mostly fluctuating and various levels of loads are applied on the same specimen causing the specimen to fail over time and this is referred to as fatigue failure under variable amplitude loading. Researches in the past have even shown that life prediction under variable amplitude loading under variable amplitude loading, it is important to derive a method which can predict and analyze reliability variations under various conditions.

In this study we have combined reliability prediction methods from past researches and presented a method to predict the fatigue reliability of a specimen under two-stage loading conditions with the help of failure data under constant amplitude loading, two dimensional probabilistic Miner's rule and Weibull analysis. Corresponding reliability values of the specimen at different stages are calculated. A significant relationship between reliability of the specimen and the change in stress level is derived with the help of test data results from simulation. The consistency of the obtained reliability values is examined by further calculating the Miner's verification coefficient and the variation in consistency is also studied.

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1. INTRODUCTION

Understanding, designing and determining the life of a product or specimen is very vital in any field. The mechanical life of a specimen or product has a diversified definition [1] but in materials science, it is most widely recognized as the number of loading cycles until the product or the specimen fails. For many years now statisticians, scientists, engineers in the field of mechanical and reliability have been consistently working in the field of life cycle engineering, reliability engineering, and fatigue analysis. The research works conducted in all these domains aid in deriving an efficient way to determine the reliability or life of a specimen or product.

Practically, life of any product or specimen deteriorates as time progresses. Life of a product in most cases is inversely proportionate to the load applied on it i.e. if the value of load applied on the product increases linearly then the remaining time of survival decreases. The changes in properties resulting from the application of cyclic loads are referred to as fatigue of materials. In simpler terms fatigue is understood as damage and failure of materials under cyclic loads. In this report fatigue is defined as a term which 'applies to changes in properties which can occur in a material due to the repeated application of stress leading to a crack or a failure'. Fluctuations in externally applied stresses or strains result in mechanical fatigue.

If the mean of the cyclic load applied is constant then the type of loading is referred to as constant amplitude loading. If the mean of the cyclic load applied changes after a period of time then the type of loading is referred to as variable amplitude loading. The study of fatigue properties of material under variable amplitude loading is more complicated when compared to constant amplitude loading since the variation in properties becomes more unpredictable and there is no specific pattern.

It is well known that the study of fatigue life of any product or specimen under variable amplitude loading requires the combination of several appropriate methods. In this paper, we will present a method to predict the fatigue reliability of a specimen under two-stage loading conditions [2] with the help of constant amplitude test data, two dimensional probabilistic Miner's rule [3 and 4] and Weibull analysis. Corresponding reliability values of the specimen at different stages are calculated. A significant relationship between reliability of the specimen and the change in stress level is derived with the help of test data results from simulation. The consistency of the obtained reliability values is examined by further calculating the Miner's verification coefficient [5 and 6] and the variation in consistency is also studied.

Let us first discuss the important terms and properties associated with life cycles and reliability of a specimen from which the significance of determining the reliability or life can be understood in the first few sections and then delve into fatigue life analysis.

LIFE OF A SPECIMEN AND RELIABILITY DEFINITION

All products and specimens manufactured have a specific life time. This life time represents the time until which the product or the specimen functions effectively. The life of a specimen can be represented in several appropriate ways respective to the type of specimen and load applied on it. The effective life of a specimen is calculated as the time from when the specimen is in its operating conditions to the time of its failure, hence it is a function of its operating conditions. The two important terms which need to be understood here are 'operating conditions' and 'failure'. In materials science, operating conditions are generally loading characteristics.

Similarly, the other equally significant term 'reliability' indicates the probability of survival over a period of time. Reliability [7] indicates the life of a specimen or a product. Hence, it is very important that we understand how to derive the reliability of a specimen. Knowing the reliability of a component or a specimen, it is easy to predict the failure time of that component. These two concepts form the base of our research 'Fatigue Reliability Prediction Under Variable Two-Stage Loading Conditions'.

1.1 LOAD, TYPES OF LOAD, FAILURE AND TYPES OF FAILURES

Load, types of load, failure and types of failures are all interconnected. Materials science helps understanding of these terms very clearly.

1.1.1 Load and types of loads

In this paper, load is generally referred to as mechanical load which means forces acting upon a body from a mechanical source. There are different ways in which a load can be applied upon a body. If the load acts along the body of the specimen in such a way that it pulls both the ends of the specimen in the opposite direction, the type of loading is referred to as tensile loading. If the force load acts in such a way that it compresses the specimen, it is compressive loading. A type of load which causes object to twist due to torque is called torsion load. Any translational or external load which causes shear stress upon the body is called shear load. The following illustrative diagrams can be help in better understanding.



1.1.2 Failures, types of failures and important terms associated with failures.

Failure of a component is the state of the component at which it has lost its potential to ever perform its designated operations. In mechanical terms failure is usually referred to as a fracture.

There are many different kinds of mechanical failure, and they include overload, impact, fatigue, creep, rupture, stress relaxation, stress corrosion cracking, corrosion fatigue and so on. Each produces a different type of fracture surface, and other indicators near the fracture surface(s). The way the product is loaded, and the loading history are also important factors which determine the outcome. The design geometry is also of critical importance as it influences crack growth.

From the above we understand that there are a lot of factors which influence the failure of a specimen or component. Failure is completely subjective. A failure can happen suddenly or gradually, mostly depending upon the material of the specimen. There are significant factors which are involved in both cases like tensile strength,

yield strength, neck formation, plastic deformation, elastic deformation and ultimate tensile strength [8].

Now with the understanding of above definitions and concepts we can further delve into the details of our research. In the next section, the objectives of the research will be clearly summarized.

2. RESEARCH OBJECTIVES AND SUMMARY

In this research study, we will present a method to estimate the reliability of a specimen under different two stage fatigue loading conditions and hence analyze the reliability variation with change in stress levels. Fatigue life [9] analysis of specimens has always been inherently challenging. The method involved in estimating the reliability of components under fatigue loading is not a conventional one. In this research we will propose an appropriate method to estimate the fatigue reliability under two-stage loading condition and then conduct variation analysis in reliability values under high and low values of stress. The corresponding variations in consistency are also studied with the help of Miner's verification coefficient.

Estimation of fatigue reliability in two-stage loading means the estimation of survival probability of the specimen after the first stage. Otherwise, it is referred to as the determination of number of cycles to failure in the second stage for a given percentage of reliability. Two-stage loading [2] refers to a loading condition experienced by a specimen subjected to two different level of stress at different times. In this study, we have generated failure data using simulation and provided the method a numerical example and verified the method. Further, a study on the Reliability variation has been done by bringing the change in stress level by altering the Weibull parameters accordingly in the second stage and generating five different sets of failure data. This study also contains the verification of the apt distribution to fit the failure data by comparing reliability, calculated residual life and the experimental residual life with the help of the verification coefficient from Miner's Rule. The variations in the verification coefficient is then observed to determine the level of accuracy of the method proposed at both high and low shape values (Weibull parameter) or high and low stress levels. In the later sections we will understand the details of the reliability estimation method and its verification.

The research is completely oriented on the failure data from simulation and the appropriate values assumed during the simulation of the data.

3. FATIGUE RELIABILITY ANALYSIS

Before we proceed any further into the paper, it is necessary we understand the core concept of this research, why it is important to do this research and how significantly does it contribute to the existing research.

Fatigue reliability analysis is the prime objective of this research. Many researches have been conducted in the field of fatigue reliability and still the area has a lot of spots to investigate on. Fatigue reliability analysis refers to the study of a specimen's life cycle which under fatigue loading. Reliability analysis completely defines a specimen's probability of survival at any given stage. The disciplined approach is to investigate the failure occurrence. In general reliability theory, the concept of a failure rate $\mu(t)$ is often used as an alternative description of the lifetime distribution of a specimen. Let 'T 'be the random lifetime of a specimen, $f_T(t)$ its probability density. The nonnegative function $\mu(t)$, defined as

 $\mu(t) = \frac{f_T(t)}{1 - F_T(t)}$, is called the failure intensity function or fatigue failure rate (or hazard function). $\mu(t)$ characterizes the failure time hence it is necessary to fit a suitable distribution and investigate the times to failure. Hence with the help of an appropriate probabilistic distribution and data from test results, we can analyze the time to failure of the specimen.

In this research we need to know how to perform reliability analysis of fatigue data. In our study, the loading conditions are variable amplitude (two-stage loading). This implies there are two different loading stages experienced by the same specimen. The type of strain experienced by the specimen is known as 'fatigue'.

Specimens are first subjected to stage 1 level loading for a specific period of time (in terms of no. of cycles), which is usually pre-determined. Then the remaining specimens which survived stage 1 level of loading is subjected to another stage 2 level loading until failure. The failure data is then collected. Using this test data and fitting it to an appropriate distribution completes the fatigue reliability analysis of the specimen.

In this research we have generated test data through simulation to analyze results. Appropriate values for the levels of stress have been assumed. The failure data is generated randomly, choosing a random distribution and a failure interval corresponding to the stress levels in terms of number of loading cycles. The details of these assumptions are explained under sections 6, 7 & 8 of this report.

4. PROBABILISTIC MINER'S RULE IN FATIGUE RELIABILITY ANALYSIS

The prediction of fatigue life under two-stage loading has always been obscure. Studying some of the successful works and combining methods done under two-stage loading condition, in this paper we have proposed an appropriate method to estimate the fatigue reliability under two-stage loading conditions. Hence we use the constant amplitude test data of the specimen to predict the fatigue life under variable amplitude loading. So the Probabilistic Miner's rule is used to predict the variable amplitude fatigue life given the constant amplitude test data.

Miner's rule is believed to be most researched area in the field of fatigue reliability analysis. This section is a brief description of the Two-Dimensional Miner's rule from the existing literatures.

In the following, *D* denotes fatigue damage, *S* denotes either stress amplitude S_a or mean stress S_m , n_{ij} represents the number of loading cycles applied in a multi stage loading block, N_c denotes fatigue life under constant amplitude loading, N_v denotes fatigue life under variable amplitude or stochastic time-history loading, *N* denotes cycles of fatigue loading in any form, N_{cp} , $N_{vp} \& n_{pij}$ are percentile constant amplitude loading, percentile variable amplitude loading and percentile number of loading cycles applied respectively.

In the first place, four basic assumptions of the evolution of nonlinear fatigue damage are proposed as follows [10 and 11]:

- 1. Monotonic increasing: dD/dN > 0; and $\delta(dD/dN)\delta S > 0$; for each individual in a specimen population under constant amplitude loading.
- 2. Noncoupling: for each individual specimen under variable amplitude loading, the damage path D-N in each loading stage is the same as the corresponding D-N path under constant amplitude loading.
- **3.** Separability: for each individual specimen, the fatigue damage growth ratio under constant amplitude loading can be described by a generally separable function $dD/dN = f^{-1}(D)g(Sa,Sm)$.
- **4.** Nonintersecting: for any two different individuals in a specimen population under constant amplitude loading, the two individuals' *Sa–Sm–Nc* surfaces do not intersect with each other in the range Sa > 0 and Nc > 0.

From the above four phenomenological assumptions about fatigue damage, a new random fatigue accumulative damage rule, namely, TPMiner has been established.

$$\sum_{i} \sum_{j} \left(n_{pij} / N_{cpij} \right) = 1, \ N_{\nu p} = \sum_{i} \sum_{j} n_{pij}, \tag{1}$$

$$\Pr\{N_{\nu} \le N_{\nu p}\} = 1 - p \tag{2}$$

Where $n_{pij} = n_p(S_{ai}, S_{mj})$ is the cycle number of (S_{ai}, S_{mj}) . In variable amplitude loading, $N_{cpij} = N_{cp}(S_{ai}, S_{mj})$ is the constant amplitude fatigue life corresponding to (S_{ai}, S_{mj}) , pdenotes reliability (survival probability), and n_p , N_{cp} and N_{vp} are percentiles with p, respectively.

In practical engineering, structural components usually are subjected to fluctuating load which is of stochastic time-history. Nevertheless, both the multistage loading block and the continuous one can be obtained by using rain-flow counting method [12].

More than 40 sets of test data have been employed by the author to verify TPMiner on the conditions of variable amplitude loading [13]. The results are very encouraging.

5. FATIGUE RELIABILITY ANALYSIS UNDER TWO-STAGE LOADING CONDITION

In this section we will discuss the analytical approach to estimate the fatigue reliability of the specimen under two-stage loading condition. First we need to understand what a two-stage loading condition is. Suppose we subject the specimen population to stress level I which is represented by (S_{a1}, S_{m1}) , where S_{a1} is the stress amplitude and S_{m1} is the mean stress. Then the same set of specimen population is subjected to stress level II which is represented by (S_{a2}, S_{m2}) . Each individual in the specimen population is run to n_1 number of cycles in stage I and then the specimens are run until failure in stage II or stress level II. While n_1 is a given number of cycles or is predetermined, n_2 is a random variable of the specimen population.

Now, in this situation when a specimen population is subjected to variable stress levels or two different stages of stress then there are several possible cases which have to be studied. The different cases are discussed below:

CASE I:

All the specimens in the population fail in the first stage. This happens when n_1 is very large. N_{c1}, N_{c2} are the constant amplitude lives of the specimen under stress level I and stress level II respectively. If $n_1 > N_{c1}$ then the entire specimen population fails in stage I itself. From experimental results, it can be verified that the constant amplitude lives follow log-normal distribution, i.e. lg $N_{c1} \sim N(\mu_1, \sigma_1)$ and lg $N_{c2} \sim N(\mu_2, \sigma_2)$, where μ and σ are the logarithmic mean and standard deviation respectively. For a given number of cycles n_1 , one can obtain

$$p_0 = \Pr\{N_{c1} > n_1\} = 1 - \varphi(\frac{\lg n_1 - \mu_1}{\sigma_1})$$
(3)

Where, φ is the standard normal distribution function, p_0 is the reliability, i.e. the percentage of the specimen population survive n_1 cycles at the first level (S_{a1}, S_{m1}) .

CASE II:

Some of the specimens from the entire population fail in stage I. In this case $p_0 < 1$. This happens when n_1 is anywhere close N_{c1} . This means some individuals of $(1 - p_0) \times 100\%$ of the population will fail at the first loading level and for those individuals $n_2 = 0$.

CASE III:

There are no failures in the first loading level or stage I. i.e. none of the specimens from the population fails until n_1 cycles. In this case $p_0 = 1$. This happens when n_1 takes a small value or when $n_1 < N_{c1}$. All the specimens are then run until failure in the II stage for n_2 cycles which is a random variable.

Case III is what would be studied in this research. Since this case represents several practical applications, it is necessary that we understand how to estimate the life in this case.

Let N_{ν} be the total variable fatigue life of the specimen population under two-stage loading, then

$$N_{\nu} = (n_1 + n_2), N_{c1} > n_1 \tag{4}$$

Further, let N_{vp} , N_{c1p} and N_{c2p} denote percentiles of p% survival of random variable N_v , N_{c1} and N_{c2} respectively. If $N_{c1p} > n_1$, then $n_{2p} = N_{vp} - n_1$. According to equation (1) of TP Miner's rule, it follows that

$$\frac{n_1}{N_{c1p}} + \frac{n_{2p}}{N_{c2p}} = 1, p < p_0(N_{c1p} > n_1)$$
(5)

When $p_0 = 1$, the reliability-based fatigue life prediction under two-stae loading can be achieved by directly using the above equation as follows

$$N_{vp} = n_1 + n_{2p} = n_1 + N_{c2p} \left(1 - \frac{n_1}{N_{c1p}}\right), \Pr\{N_v > N_{vp}\} = p$$
(6)

From a lot of test results in literature, it can be found that the conditional probability distribution of n_2 is not log-normal unless $p_0=1$. Particularly when $p_0 < 1$, the conditional probability of the residual life n_2 can be fitted by a three-parameter Weibull distribution [14].

$$\Pr\left\{\frac{n_2 \le n_{2p'}}{N_{c1} > n_1}\right\} = 1 - \exp\left\{-\left(\frac{n_{2p'} - x_0}{b}\right)^c\right\} = 1 - p'$$
(7)

Where $n_{2p'}$ is the conditional percentile with p' and c, b and x_0 are the Weilbull shape parameter, characteristic parameter and position parameter respectively.

Now setting
$$n_{2p'} = n_{2p} = N_{vp} - n_1$$
, one can obtain
 $p' = p/p_0$
(8)

Given a two-stage loading $(S_{a1}, S_{m1}, n_1; S_{a2}, S_{m2})$ as well as the probability distributions of the two constant amplitude fatigue lives $N_c(S_{a1}, S_{m1})$ and $N_c(S_{a2}, S_{m2})$, it is easy to perform reliability-based prediction of the residual life and the total life N_v .

6. SIMULATION ANALYSIS OF TEST DATA AND RESULTS

Now we know the method we are going to employ in analysis of fatigue data is the Probabilistic Miner's Rule. TPMiner'r rule will form the basis of the analytical analysis. However, there are other significant methods which might also prove effective. In order to perform the reliability

analysis, we need to first analyze the data we have generated. Since this is a probabilistic approach, we need to fit appropriate distributions for the data in order to further analyze it.

In the previous section we examined three different scenarios of fatigue conditions. Combining the methods employed in these 3 cases we can find out what distribution can be of best fit to failure data from two stage loading. According to case (I), if the entire specimen population fails in 1st loading stage itself then the failure times of the specimens are said to follow Log-Normal Distribution [15]. In this case the value of n_1 is lesser compared to the constant amplitude life of the specimen. In other words, the number of loading cycles applied is more than the total number of life cycles the specimen can survive for (under a specific stress level). In this case the reliability of the specimen at any given stage can be found using the following,

$$p_0 = \Pr\{N_{c1} > n_1\} = 1 - \varphi(\frac{\lg n_1 - \mu_1}{\sigma_1})$$
(9)

Where, p_0 is the probability of survival, p_0 is the reliability, i.e. the percentage of the specimen population survive n_1 cycles at the first level (S_{a1}, S_{m1}) , $\boldsymbol{\varphi}$ is the standard normal distribution function. N_{c1} is log-normally distributed with mean and variance μ_1 , σ_1 respectively.

This method holds good for reliability analysis under single stage fatigue loading but we want the specimen to pass through two different stages of loading and then perform the reliability analysis. Hence if we take into consideration of case (III), where we had discussed about the specimen being tested under two different stages of loading we can conclude that the failure time of specimens in the second stage alone is considered to follow 3P-Weibull distribution or otherwise the residual life, $n_{2p'}$ is said to follow 3P-Weibull distribution. The probability of survival under the second stage alone can be estimated as follows:

$$\Pr\left\{\frac{n_2 \le n_{2p'}}{N_{c1} > n_1}\right\} = 1 - \exp\left\{-\left(\frac{n_{2p'} - x_0}{b}\right)^c\right\} = 1 - p'$$
(10)

i.e., given the condition $N_{c1} > n_1$ (constant amplitude fatigue life in stage I is greater than the no. of cycles applied in stage I) then the probability of survival in the second stage can be estimated using the above formulation where $x_0, b \& c$ are the location parameter, scale parameter and the shape parameter respectively.

Now we can combine these two cases to derive the method for the reliability analysis under two stage fatigue loading condition under these assumptions:

- 1. There are no failures in stage 1
- 2. Best fit distribution.

The estimation of reliability represents the probability of survival of a specimen after passing through both the stages. Now, we know how to calculate the survival probabilities individually

for each stage. Combining both these methods and using the principles of TP Miner's rule we can frame the reliability estimation methods for two stage loading as follows:

Generally, if n_1 takes such a small value that $p_0=1$; then the whole population of specimen will not fail at the first loading level. However, when n_1 becomes large to a certain degree one can obtain $p_0 < 1$; that means some individuals of $(1 - p_0) * 100\%$ of the population will fail at the first loading level, and for those individuals $n_2 = 0$:

Let
$$N_{\nu}$$
 be the total fatigue life of specimen population under two-stage loading, then
 $N_{\nu} = N_{c1}, N_{c1} \le n_1; N_{\nu} = n_1 + n_2, N_{c1} > n_1.$ (11)

Further, let N_{vp} , N_{c1p} and N_{c2p} denote percentiles of p% survival of random variables N_v , N_{c1} , N_{c2} respectively. If $N_{c1p} > n_1$, then $n_{2p} = N_{vp} - n_1$. According to Eq. (1) of TPMiner, it follows that

$$\frac{N_{vp}}{N_{c1p}} = 1, p \ge p_0 (N_{c1p} \le n_1)$$
(12)

$$\frac{n_1}{N_{c1p}} + \frac{n_{2p}}{N_{c2p}} = 1, p \le p_0(N_{c1p} \ge n_1)$$
(13)

Consequently when $p_0=1$, the reliability-based fatigue life prediction under two-stage loading can be achieved by directly using Eq. (13) as follows:

$$N_{vp} = n_1 + n_{2p} = n_1 + N_{c2p} \left(1 - \frac{n_1}{N_{c1p}}\right)$$

$$\Pr\{N_v > N_{vp}\} = p$$
(14)

However, when $p_0 < 1$, the fatigue life of the population should be divided into two parts. And Eq. (12) applies to the part of $p \ge p_0$ while Eq. (13) applies to the other part.

In fatigue tests under two-stage loading, many researchers focus their attention on the residual life n_2 . From a lot of test results in literature, it can be found that the conditional probability distribution of n_2 is not log-normal unless $p_0 = 1$. When $p_0 < 1$, the conditional probability of the residual life is fitted by the 3-parameter Weibull Distribution rather than log-normal distribution. Therefore,

$$\Pr\left\{\frac{n_2 \le n_{2p'}}{N_{c1} > n_1}\right\} = 1 - \exp\left\{-\left(\frac{n_{2p'} - x_0}{b}\right)^c\right\} = 1 - p'$$
(15)

Where $n_{2p'}$ is the conditional percentile with p', and c,b and x_0 are the Weibull shape parameter, characteristic parameter and position parameter, respectively.

Now setting $n_{2p'} = n_{2p} = N_{\nu p} - n_1$, one can obtain,

$$p' = \Pr\{N_{\nu} > N_{\nu p}\} / \Pr\{N_{c1} > n_1\} = p/p_0$$
(16)

For the proof of the above equation please refer to [16]

Now following the procedure below and using the equations from (9) to (16) we can obtain the reliability of the specimen under two stage loading condition

- 1. Assumption of appropriate stress levels (S_{a1}, S_{m1}) and (S_{a2}, S_{m2}) .
- 2. Assumption of the constant amplitude lives $N_{c1} \& N_{c2}$. Which in the simulation model are assumed with an appropriate mean and a standard deviation. Table (1) shows the constant amplitude lives. Also the value of n_1 is assumed to be lesser than N_{c1} . Hence $p_0 = 1$.
- 3. Generate random Weibull data (100 samples) assuming values for the three parameters of Weibull. (Assumptions made here correspond with the assumptions made for $N_{c1} \& N_{c2}$).
- 4. Now the software generates random failure data (time to failure in terms of no. of cycles).
- 5. Now the appropriate 3 parameters of Weibull for this random failure data are estimated by fitting (distribution fitting) this set of data to Weibull distribution in EasyFit. 3 parameters of Weibull can also be estimated by other means [17].
- 6. Now we have the 3 parameters of Weibull for the first set of failure data.
- Arrange the failure data in ascending order and the data will now represent no. of cycles to 1st failure, no. of cycles to 2nd failure,..., no. of cycles to 100th failure.
- 8. By using Eq. (9), calculate p_0
- 9. By using Eq. (15), calculate p' for each of $n_2(n_{2p'})$ of the failure data from simulation.
- 10. According to Eq. (16), calculate $p = p'p_0$ corresponding to each $n_{2p'}$.
- 11. Now we have the reliability at every stage (for every value of $n_{2p'}$). Now assume another stress value for (S_{a2}, S_{m2}, N_{c2}) and appropriately alter the parameters of Weibull and generate the second set of random failure data. While altering the parameters, the Weibull characteristics [18] have to be taken into consideration since the parameter alteration is the direct reflection of stress change.
- 12. Do the 3 parameter Weibull distribution fitting to the second set of failure data and estimate the 3 parameters of Weibull. Repeat steps 7-10 five times to obtain 5 different sets of data.
- 13. Now we have the reliability for the second set of data. Similarly generate 5 sets of random data assuming 5 different values for (S_{a2}, S_{m2}) by changing the Weibull parameters according to Weibull characteristics and estimate the reliability for each $n_{2p'}$. Table (2) shows the calculated reliability values at each stage for 5 different sets of failure data (low values of stress or shape parameter (1.0-2.0)).

ASSUMPTIONS MADE FOR LOW LEVELS OF STRESS

DEFINITION	ΝΟΤΑΤΙΟΝ	ASSUMED VALUES
DEFINITION	NOTATION	ASSUMED VALUES
Constant Amplitude Fatigue life (Level I)	N _{c1}	$\log N_{c1} \sim N(4.962, 0.03)$
Constant Amplitude Fatigue lives (different level II's)	N _{c2}	$\begin{array}{l} \text{Log } N_{c2} \sim \text{N}(4.6573, 0.04) \\ \text{Log } N_{c2} \sim \text{N}(4.585, 0.03) \\ \text{Log } N_{c2} \sim \text{N}(4.5357, 0.066) \\ \text{Log } N_{c2} \sim \text{N}(4.5102, 0.063) \\ \text{Log } N_{c2} \sim \text{N}(4.4698, 0.069) \end{array}$
Probability of survival in stage I	p_0	$p_0 = 1$
No. of loading cycles applied in stage I	n_1	$n_1 = 80000$
Shape parameter	C	C=1.3085, 1.443, 1.5658, 1.6131, 1.8416

TABLE 1

C = 1.3085		C = 1.443		C = 1.5658		C = 1.613	1	C = 1.8416	
Failure tin	Reliability	Failure tin	Reliability	Failure tin	Reliability	Failure t	in Reliability	Failure tir	Reliability
14498.12	0.998009	12133.09	0.997595	10276.01	0.995144	9960.67	1 0.995047	8744.226	0.997501
14746.03	0.991985	12350.69	0.991768	10493.42	0.98878	10024.1	9 0.993375	9066.692	0.990844
14968.82	0.985353	12686.15	0.979565	10516.32	0.988006	11391.	1 0.92607	10683.85	0.909873
16869.59	0.90901	12686.47	0.979552	12071.22	0.905303	11482.7	4 0.919989	11082.14	0.880328
16924.71	0.906505	14488.81	0.880618	12394.88	0.882933	11509.2	3 0.918203	11796.97	0.820514
17608.23	0.874736	14818.83	0.8591	12455.33	0.878622	12660.7	8 0.83102	11919.88	0.809507
18888.4	0.813095	14961.15	0.849637	13862.02	0.770128	12673.0	5 0.830012	12124.95	0.790756
18898.85	0.812586	16825.34	0.720412	13878.84	0.768766	13031.7	8 0.80008	13352.8	0.67151
18953.92	0.809904	16838.66	0.719476	14096.02	0.751109	14597.3	3 0.66261	13434.25	0.663354
20730.45	0.723534	16839.75	0.7194	14953.17	0.680702	14849.1	5 0.640302	13443.61	0.662415
21065.21	0.707466	18934.57	0.575794	15017.69	0.675395	15143.8	8 0.614341	14854.22	0.521936
21572.59	0.683338	19004.45	0.571199	15177.94	0.66223	16112.1	1 0.531124	14909.17	0.516598
23254.6	0.605883	19089.78	0.565611	15315.3	0.650971	16224.4	2 0.52175	15062.41	0.501803
23985.32	0.573698	21689.83	0.409393	14953.17	0.680702	16342.8	5 0.511941	16452.13	0.375736
24179.39	0.565317	21826.85	0.401999	15017.69	0.675395	16112.1	1 0.531124	16510.29	0.370842
25836.87	0.496777	22269.81	0.378705	15177.94	0.66223	16224.4	2 0.52175	16516.85	0.370292
25963.71	0.491764	22393.42	0.372373	15315.3	0.650971	16342.8	5 0.511941	18803.22	0.207742
26163.13	0.483951	25399.58	0.240735	16878.72	0.526472	17847.9	3 0.395221	18993.87	0.1969
28040.4	0.414576	25407.02	0.240461	17132.46	0.507206	17955.1	4 0.387535	19091.05	0.191533
28465.84	0.399908	25958.26	0.220835	17176.06	0.50393	18061.3	8 0.380008	18803.22	0.207742
29685.75	0.359999	27464.95	0.173667	17389.48	0.488035	18224.4	0.368628	18993.87	0.1969
30797.87	0.326351	28241.33	0.152804	18662.28	0.398834	19513.0	9 0.286351	19091.05	0.191533
32211.15	0.287233	28476.27	0.14692	19078.77	0.371909	19933.5	8 0.262483	20244.79	0.13574
32394.61	0.282445	29122.48	0.131709	19180.45	0.365513	20029.3	3 0.257253	20704.39	0.117371
38523.58	0.156579	29292.57	0.127935	21438.42	0.242018	22010.	5 0.165434	20790.09	0.114172
38628.32	0.15494	35661.62	0.039524	21472.54	0.240422	23158.5	2 0.125418	20837.85	0.11242
38766.96	0.152792	36441.86	0.033861	21840.33	0.223718	23444.	6 0.116784	21445.9	0.091922
45119.18	0.078631	36784.31	0.031616	26557.88	0.079762	29320.7	4 0.022257	23688.57	0.040817
45727.72	0.073606	37290.27	0.028548	26801.31	0.075241	30202.5	4 0.016842	26466.83	0.012872
45789.59	0.073111	40695.62	0.014032	32997.81	0.014603	30396.7	8 0.015823	26564.06	0.012327
53449.63	0.030809	41779.78	0.011101	36792.64	0.004669	30702.	5 0.014332	31111.35	0.00131
			-	TABLE 2 (1 <	C < 2) RELIA	BILITY ESTI	MATION		

7. RELIABILITY VARIATION ANALYSIS

Now we have estimated the reliability at all the stages of failure for five different stress values. Now we can analyze how reliability varies with change in stress levels.

- 1. First we need to choose a common interval for all five set of failure data between which we will analyze the reliability variation.
- 2. Once we choose an interval, using the 3 parameters of Weibull we estimated earlier for the 1st set of failure data, we estimate the value of reliability using Eq (9), (15) & (16) for each of the failure time in the interval.

- 3. Similarly, using 3 parameters of Weibull for the 2nd set of failure data, we estimate reliability values for each failure time in the interval. Same steps are followed to obtain reliability values for the 5 different stress levels (here represented by 5 sets of Weibull parameters).
- 4. Now, plot the graph Cycle time vs Reliability as shown in figure (1).
- 5. Table (3) shows the different values calculated between the failure time interval 15000-20000 and figure (1) shows the reliability variation graph.



	C = 1.3085	C = 1.443	C = 1.5658	C= = 1.6131	C = 1.8416
Failure time	Reliability	Reliability	Reliability	Reliability	Reliability
15070	0.982074	0.842336	0.671095	0.620829	0.501074
15080	0.981742	0.841662	0.670273	0.61995	0.500114
15090	0.981409	0.840989	0.669451	0.619071	0.499154
15100	0.981075	0.840314	0.66863	0.618193	0.498196
15110	0.980739	0.83964	0.667808	0.617315	0.497237
15120	0.980402	0.838965	0.666987	0.616437	0.49628
16330	0.932936	0.755218	0.569168	0.513001	0.386123
16340	0.932504	0.754516	0.568378	0.512176	0.385267
16350	0.932072	0.753813	0.567589	0.511351	0.384412
16360	0.931639	0.753111	0.5668	0.510527	0.383558
16370	0.931205	0.752409	0.566012	0.509703	0.382705
16380	0.930771	0.751707	0.565224	0.50888	0.381853
16950	0.905353	0.71166	0.521027	0.462983	0.334986
16960	0.904896	0.710958	0.520265	0.462197	0.334194
16970	0.90444	0.710256	0.519504	0.461411	0.333404
16980	0.903983	0.709555	0.518743	0.460626	0.332615
16990	0.903525	0.708853	0.517983	0.459842	0.331827
17000	0.903068	0.708152	0.517223	0.459058	0.33104
17010	0.90261	0.707451	0.516464	0.458275	0.330254
17020	0.902152	0.706749	0.515705	0.457492	0.329469
17030	0.901693	0.706048	0.514947	0.456711	0.328685
17040	0.901234	0.705347	0.514189	0.45593	0.327903
18650	0.824691	0.594671	0.399646	0.339945	0.216758
18660	0.824205	0.594003	0.398985	0.339288	0.216161
18670	0.82372	0.593336	0.398324	0.338633	0.215566
18680	0.823234	0.592669	0.397665	0.337978	0.214971
19970	0.760364	0.509532	0.31829	0.260485	0.147739
19980	0.759878	0.508912	0.31772	0.259938	0.147288
19990	0.759391	0.508293	0.317151	0.259392	0.146839
20000	0.758905	0.507674	0.316582	0.258847	0.146391

TABLE 3 (ESTIMATION OF RELIABILITY BETWEEN THE INTERVAL 15000-20000)

8. STUDY ON VERIFICATION COEFFICIENT AND ACCURACY

We have now estimated reliability and studied the variations considering 3P-Weibull distribution as an appropriate fit for the failure data. In this section we will check for the accuracy of the method proposed using Miner's verification coefficient. In the method we used above we assumed that 3P-Weibull distribution would be the most apt fit the failure data presented. This is mostly due to the reason that, 3P-Weibull distribution has the most versatile characteristics. It is necessary to understand the nature of 3P-Weibull distribution and how it best relates to failure data.

The characteristic exhibited by the 3P-Weibull distribution cooperates with the nature failure data but we need to check if the distribution also provides accurate results. This is done by

verifying the results. Considering the results we obtained from the previous section, we will determine the Miner's verification coefficient using the following equation.

$$\frac{n_1}{N_{c1p}} + \frac{n_{2p}}{N_{c2p}} = \beta, \text{ where } \beta \text{ is the verification coefficient}$$
(17)

The value of β is found at each stage i.e. for every single failure data in the table. This value of β provides the verity. Several researches and tests have been performed to determine the range of β [19]. Several tests proved several conclusions.

Considerable test data has been generated in an attempt to verify Miner's Rule. Most test cases use a two step load history. The results of Miner's original tests showed that the damage criterion X corresponding to failure ranged from 0.61 to 1.45. Other researchers have shown variations as large as 0.18 to 23.0, with most results tending to fall between 0.5 and 2.0. In most cases, the average value is close to Miner's proposed value of 1.0.

From the above we can conclude that, it is best if the value of β is close to unity then the values determined are acceptable. The step wise procedure involved in determining the value of β is as follows

- 1. The failure data generated in the previous section and the results obtained from simulation are carried over for determining the value of β .
- 2. Now we already have the failure data and the corresponding reliability's at each stage.
- 3. The table (1) shows the different values assumed in the simulation model
- 4. Now calculate $N_{c1p} \& N_{c2p}$ corresponding to each $n_{2p'}$. Since we know that $N_{c1p} \& N_{c2p}$ are Log-normally distributed, using the equation

$$p = 1 - \varphi(\frac{\lg N_{c1} - \mu_1}{\sigma_1})$$
 & $p = 1 - \varphi(\frac{\lg N_{c2} - \mu_2}{\sigma_2})$

and substituting calculated values of reliability from table (2) for p and assumed values of mean and standard deviations for $N_{c1} \& N_{c2}$ from table (1), we can determine the corresponding values of $N_{c1p} \& N_{c2p}$.

- 5. Now using the equation (17) we can determine the verification coefficient β .
- 6. Now corresponding to each $N_{c1p} \& N_{c2p}$ we can also calculate the residual life n_{2pm} using the equation

$$n_{2pm} = N_{c2p} \left(1 - \left(\frac{n_1}{N_{c1p}}\right)\right) \tag{18}$$

Now we have calculated the verification coefficient for the five different sets of failure data corresponding to five different stress values. In order to compare the verification coefficients between the different sets of failure data, we extend table (3) where we had found the reliability for a specific interval 15000-20000 cycles for the different set of Weibull parameters which

represent the stress levels. Steps 4 - 6 of verification calculation procedure is repeated for these values of reliability as shown in table (4). Figure (2) shows the different ranges of verification coefficients corresponding to different stress levels.

Table (4) shows the variation in reliability and the ranges, interval of verification for low values of shape parameter (or low values of stress in stage II). We will further analyze the reliability variations and variations in verification coefficients for higher values of shape parameter (or higher values of stress in stage II)

NOTE: All the values and parameters corresponding to the stress at Stage I remain unaltered at all times.

The interval of the low shape parameters (1.0 and 2.0) was estimated. Now by altering the assumed values (3 parameters of Weilbull distribution) corresponding to the new values assumed for N_{c2} 's while generating random Weibull data, we can generate failure data with higher values for shape parameter as shown in table (6). The table (5) below shows the different assumptions made while generating failure data for higher values of shape parameter.

	C = 1.3085	C = 1.443	C =	1.5658		C = 1.6131		C = 1.842	16	
F.T	Reliabi V.C (β) Resd	Life Reliabi V.C (β) R	esd Life Rel	iabi V.C (β)	Resd Life	ReliabilV.C (β)	Resd Life	Reliabil	V.C (β)	Resd Lif
15070	0.9821 0.8402 2613	32.8 0.8423 0.9071	19963 0.6	5711 0.8416	25854	0.6208 0.8628	23589	0.5011	0.8686	23031
15080	0.9817 0.8403 2613	0.8417 0.9073	19963 0.6	5703 0.8418	25855	0.62 0.863	23590	0.5001	0.8687	23032
15090	0.9814 0.8404 2613	35.5 0.841 0.9075	19963 0.6	695 0.8419	25855	0.6191 0.8631	23591	0.4992	0.8689	23033
15100	0.9811 0.8406 2613	36.9 0.8403 0.9077	19963 0.6	686 0.8421	25856	0.6182 0.8633	23591	0.4982	0.869	23033
15110	0.9807 0.8407 2613	0.8396 0.9079	19964 0.6	678 0.8422	25857	0.6173 0.8634	23592	0.4972	0.8692	23034
15120	0.9804 0.8408 2613	39.5 0.839 0.908	19964 0	.667 0.8423	25858	0.6164 0.8636	23593	0.4963	0.8693	23035
16330	0.9329 0.8571 2624	44.4 0.7552 0.9304	19997 0.5	692 0.8592	25947	0.513 0.8821	23671	0.3861	0.8881	23130
16340	0.9325 0.8572 26	0.7545 0.9306	19997 0.5	684 0.8593	25948	0.5122 0.8822	23671	0.3853	0.8883	23130
16350	0.9321 0.8574 2624	45.6 0.7538 0.9308	19997 0.5	676 0.8594	25948	0.5114 0.8824	23672	0.3844	0.8884	23131
16360	0.9316 0.8575 2624	46.2 0.7531 0.931	19997 0.5	668 0.8596	25949	0.5105 0.8826	23673	0.3836	0.8886	23132
16370	0.9312 0.8576 2624	46.8 0.7524 0.9312	19998 0	.566 0.8597	25950	0.5097 0.8827	23673	0.3827	0.8887	23133
16380	0.9308 0.8578 2624	47.4 0.7517 0.9314	19998 0.5	652 0.8598	25951	0.5089 0.8829	23674	0.3819	0.8889	23133
16950	0.9054 0.8656 2627	78.9 0.7117 0.9419	20011 0	.521 0.8678	25989	0.463 0.8916	23708	0.335	0.8977	23176
16960	0.9049 0.8658 2627	79.4 0.711 0.9421	20011 0.5	203 0.8679	25990	0.4622 0.8917	23709	0.3342	0.8979	23177
16970	0.9044 0.8659 2627	79.9 0.7103 0.9423	20012 0.5	5195 0.8681	25991	0.4614 0.8919	23709	0.3334	0.898	23177
16980	0.904 0.866 2628	0.7096 0.9425	20012 0.5	5187 0.8682	25991	0.4606 0.892	23710	0.3326	0.8982	23178
16990	0.9035 0.8662 2628	0.7089 0.9427	20012 0	.518 0.8683	25992	0.4598 0.8922	23710	0.3318	0.8983	23179
17000	0.9031 0.8663 2628	81.4 0.7082 0.9429	20012 0.5	5172 0.8685	25993	0.4591 0.8923	23711	0.331	0.8985	23180
17010	0.9026 0.8664 2628	0.7075 0.9431	20013 0.5	5165 0.8686	25993	0.4583 0.8925	23712	0.3303	0.8986	23180
17020	0.9022 0.8666 2628	82.3 0.7067 0.9432	20013 0.5	5157 0.8687	25994	0.4575 0.8927	23712	0.3295	0.8988	23181
17030	0.9017 0.8667 2628	0.706 0.9434	20013 0.5	5149 0.8689	25995	0.4567 0.8928	23713	0.3287	0.8989	23182
17040	0.9012 0.8669 2628	33.3 0.7053 0.9436	20013 0.5	0.869	25995	0.4559 0.893	23713	0.3279	0.8991	23183
18650	0.8247 0.8892 2634	49.5 0.5947 0.9735	20046 0.3	8996 0.8914	26097	0.3399 0.9176	23803	0.2168	0.924	23297
18660	0.8242 0.8894 2634	49.9 0.594 0.9737	20046 0	.399 0.8915	26097	0.3393 0.9177	23804	0.2162	0.9241	23298
18670	0.8237 0.8895 2635	50.2 0.5933 0.9739	20046 0.3	983 0.8917	26098	0.3386 0.9179	23804	0.2156	0.9243	23299
18680	0.8232 0.8897 2635	50.6 0.5927 0.9741	20047 0.3	8977 0.8918	26098	0.338 0.918	23805	0.215	0.9244	23299
19970	0.7604 0.9077 2639	91.7 0.5095 0.9981	20070 0.3	3183 0.9097	26173	0.2605 0.9377	23871	0.1477	0.9442	23387
19980	0.7599 0.9078 26	0.5089 0.9983	20070 0.3	3177 0.9099	26174	0.2599 0.9379	23872	0.1473	0.9444	23388
19990	0.7594 0.908 2639	92.3 0.5083 0.9985	20070 0.3	8172 0.91	26174	0.2594 0.938	23872	0.1468	0.9446	23388
20000	0.7589 0.9081 2639	92.6 0.5077 0.9987	20070 0.3	8166 0.9102	26175	0.2588 0.9382	23873	0.1464	0.9447	23389
TABLE 4 (ESTIMATION OF VERIFICATION COEFFICIENT BETWEEN 15000-20000 CYCLE NO.)										(



ASSUMPTIONS MADE FOR HIGH LEVELS OF STRESS

DEFINITION	NOTATION	ASSUMED VALUE
Constant Amplitude Fatigue Life in Stage I	N _{c1}	$\log N_{c1} \sim N(4.962, 0.03)$

Different Constant Amplitude	N_{c2} 's	$\log N_{c2} \sim N(4.0565, 0.0366)$
Fatigue Lives in Stage II		$\log N_{c2} \sim N(4.0113, 0.0412)$
		$\log N_{c2} \sim N(3.9965, 0.0261)$
		$\log N_{c2} \sim N(3.939, .0286)$
		$\log N_{c2} \sim N(3.8043, 0.03113)$
No. of Loading cycles applied	n_1	$n_1 = 80000$
in Stage I	-	-
Shape parameter	С	C=3.6417, 4.272, 4.368,
		4.6607, 5.3276

TABLE 5

C = 3.6417		C = 4	.272			C = 4.368			C = 4.6607			C = 5.3276	
Failure tin	Reliability	Failu	ure tin	Reliability	/	Failure tin	Reliability		Failure tin	Reliability	,	Failure Tir	Reliability
3839.093	0.988624	36	66.13	0.993085		3390.577	0.996966		3167.27	0.99599		3142.526	0.994577
3848.268	0.987994	374	6.971	0.98824		3702.377	0.975258		3431.578	0.975149		3341.054	0.977312
3879.978	0.985623	388	3.802	0.97467		3703.273	0.97514		3488.247	0.966128		3367.688	0.973253
4437.323	0.871826	39	77.59	0.960185		3770.748	0.964994		3719.239	0.901782		3678.32	0.87019
4439.588	0.871009	425	6.426	0.881624		3978.982	0.914015		3757.225	0.885939		3714.405	0.849297
4457.795	0.864324	426	5.878	0.87784		4043.134	0.891005		3775.262	0.877817		3717.435	0.84744
4855.752	0.667267	432	9.576	0.850197		4095.428	0.869325		3914.679	0.801269		3786.116	0.800997
4857.089	0.66646	436	6.849	0.832259		4275.815	0.773125		3921.627	0.796799		3791.336	0.797121
4869.087	0.659178	437	1.611	0.829873		4309.157	0.751685		3924.766	0.79476		3817.507	0.776943
4884.404	0.649795	454	9.675	0.725407		4319.364	0.744901		3951.677	0.776752		3876	0.727412
5036.707	0.55222	455	0.404	0.724921		4462.366	0.63989		3968.883	0.764753		3987.602	0.617183
5050.598	0.543035	464	8.197	0.655792		4471.817	0.632365		4089.186	0.670789		3994.978	0.609267
5075.201	0.5267	466	8.937	0.640229		4615.792	0.511587		4105.15	0.657101		4013.994	0.588557
5076.262	0.525993	467	8.296	0.633114		4674.678	0.460278		4115.016	0.648513		4091.895	0.500147
5087.325	0.518623	467	8.567	0.632907		4677.997	0.45738		4125.278	0.639483		4093.876	0.497844
5327.033	0.360244	481	1.854	0.526659		4687.082	0.449448		4267.714	0.505972		4108.952	0.48027
5327.426	0.359993	484	4.099	0.50002		4847.764	0.312811		4279.788	0.494185		4110.55	0.478403
5327.655	0.359847	485	6.919	0.48938		4857.761	0.304745		4291.356	0.482862		4218.742	0.352355
5335.771	0.354683	492	6.385	0.431627		4869.598	0.295291		4316.133	0.45855		4222.792	0.347741
5342.328	0.350526	498	6.618	0.382059		4947.931	0.235763		4466.717	0.313852		4289.854	0.273886
5498.619	0.25689	499	2.847	0.376994		4963.896	0.224362		4468.062	0.312619		4305.295	0.257745
5570.509	0.21818	516	8.106	0.243416		4972.008	0.218673		4595.113	0.204952		4320.462	0.242275
5572.933	0.216931	516	8.187	0.24336		4973.28	0.217788		4603.08	0.198894		4362.529	0.201601
5596.391	0.205035	517	0.818	0.241531		5090.445	0.144367		4609.917	0.193768		4372.257	0.192704
5806.276	0.115487	536	2.321	0.127667		5142.28	0.117369		4611.5	0.192591		4386.162	0.180341
5844.703	0.102484	538	9.441	0.114883		5161.464	0.108258		4758.293	0.100833		4400.29	0.16822
6194.303	0.027426	57	53.87	0.017872		5181.544	0.09923		4794.838	0.083511		4523.409	0.082775
6506.363	0.005649	577	8.291	0.015259		5795.514	0.001486		5225.298	0.003153		4895.833	0.002164
				TARI F	6 13 6	< C < 5 3) R	FUARILITY	FSTIM	ΔΤΙΟΝ				

Table (7) shows the different values of reliability estimated at each stage for high values of stress. Table (8) shows the variations in verification coefficient and different values of reliability estimated for the failure data (high values of shape parameter)

	C = 3.	.6417		C = 4	.272		C = 4	.368		C = 4	.6607		C = 5	.3276	
Failure	tin Reliabil	V.C (β)	Resd life	e Reliabi	V.C (β)	Resd life	Reliabi	V.C (β)	Resd lif	e Reliabi	V.C (β)	Resd Life	Reliabi	V.C (β)	Resd Lif
4000	0.9736	0.8733	6019.6	0.956	0.8901	5639	0.9069	0.9393	4764.4	0.7421	0.9762	4267.9	0.6038	1.0983	3177.9
4005	0.973	0.8736	6020	0.955	0.8904	5639.5	0.9051	0.9396	4764.7	0.7384	0.9766	4268.2	0.5984	1.0988	3178.1
4065	0.9647	0.8771	6024.6	0.9419	0.8942	5644.6	0.8823	0.9442	4767.6	0.691	0.9817	4271.3	0.5312	1.1056	3181.2
4160	0.9482	0.8827	6031.6	0.9157	0.9001	5652.6	0.8388	0.9514	4772.3	0.6082	0.9897	4276.4	0.4204	1.1163	3186.1
4240	0.9308	0.8874	6037.4	0.888	0.905	5659.3	0.7949	0.9574	4776.2	0.5329	0.9964	4280.7	0.3283	1.1253	3190.5
4260	0.9259	0.8885	6038.8	0.8802	0.9063	5660.9	0.7829	0.9589	4777.2	0.5135	0.9981	4281.8	0.3061	1.1275	3191.6
4265	0.9246	0.8888	6039.1	0.8782	0.9066	5661.3	0.7798	0.9593	4777.4	0.5086	0.9985	4282.1	0.3006	1.1281	3191.9
4370	0.8946	0.895	6046.4	0.8307	0.9131	5670	0.7098	0.9673	4782.6	0.4058	1.0073	4287.9	0.1948	1.1398	3197.9
4375	0.893	0.8953	6046.7	0.8282	0.9134	5670.4	0.7062	0.9676	4782.9	0.4009	1.0077	4288.2	0.1902	1.1403	3198.2
4380	0.8914	0.8956	6047.1	0.8256	0.9137	5670.8	0.7026	0.968	4783.1	0.3961	1.0081	4288.5	0.1858	1.1409	3198.5
4450	0.8672	0.8997	6051.8	0.7875	0.9181	5676.6	0.6496	0.9733	4786.6	0.3293	1.014	4292.5	0.1293	1.1486	3202.7
4455	0.8654	0.9	6052.2	0.7846	0.9184	5677	0.6457	0.9737	4786.9	0.3247	1.0144	4292.8	0.1257	1.1492	3203.1
4465	0.8616	0.9006	6052.8	0.7787	0.919	5677.8	0.6378	0.9744	4787.4	0.3154	1.0152	4293.4	0.1187	1.1503	3203.7
4585	0.8117	0.9077	6060.8	0.7013	0.9264	5687.7	0.5382	0.9835	4793.5	0.2128	1.0252	4300.5	0.0537	1.1635	3211.3
4590	0.8094	0.908	6061.1	0.6978	0.9267	5688.2	0.5339	0.9838	4793.7	0.2089	1.0256	4300.8	0.0517	1.1641	3211.7
4595	0.8071	0.9083	6061.5	0.6943	0.927	5688.6	0.5296	0.9842	4794	0.205	1.0261	4301.1	0.0498	1.1646	3212
4600	0.8048	0.9086	6061.8	0.6908	0.9273	5689	0.5252	0.9846	4794.2	0.2012	1.0265	4301.4	0.0479	1.1652	3212.3
4760	0.7229	0.918	6072.3	0.5689	0.9372	5702.5	0.3862	0.9966	4802.6	0.1	1.0397	4311.5	0.0112	1.1826	3223.4
4765	0.7201	0.9183	6072.6	0.5649	0.9375	5702.9	0.3819	0.997	4802.9	0.0975	1.0401	4311.8	0.0106	1.1832	3223.8
4775	0.7145	0.9189	6073.3	0.5568	0.9381	5703.8	0.3734	0.9977	4803.4	0.0926	1.041	4312.4	0.0095	1.1843	3224.5
4780	0.7117	0.9192	6073.6	0.5527	0.9384	5704.2	0.3692	0.9981	4803.7	0.0903	1.0414	4312.8	0.009	1.1848	3224.9
4880	0.6525	0.925	6080.1	0.4702	0.9446	5712.8	0.2871	1.0056	4809.1	0.0513	1.0496	4319.4	0.0027	1.1956	3232.4
4885	0.6494	0.9253	6080.5	0.466	0.9449	5713.2	0.2832	1.006	4809.4	0.0498	1.05	4319.7	0.0025	1.1962	3232.8
4890	0.6463	0.9256	6080.8	0.4619	0.9452	5713.7	0.2793	1.0064	4809.6	0.0483	1.0504	4320.1	0.0023	1.1967	3233.2
4895	0.6433	0.9259	6081.1	0.4577	0.9455	5714.1	0.2754	1.0067	4809.9	0.0468	1.0509	4320.4	0.0022	1.1972	3233.6
4950	0.6086	0.9292	6084.7	0.4121	0.9488	5718.9	0.2343	1.0109	4812.9	0.0326	1.0554	4324.2	0.001	1.2031	3237.9
4955	0.6054	0.9295	6085.1	0.408	0.9491	5719.4	0.2307	1.0112	4813.2	0.0315	1.0558	4324.5	0.0009	1.2037	3238.3
4990	0.5828	0.9315	6087.3	0.3793	0.9513	5722.5	0.2063	1.0138	4815.2	0.0246	1.0587	4326.9	0.0005	1.2074	3241.2
4995	0.5796	0.9318	6087.7	0.3752	0.9516	5722.9	0.2029	1.0142	4815.5	0.0237	1.0591	4327.3	0.0005	1.208	3241.6
5000	0.5763	0.9321	6088	0.3712	0.9519	5723.4	0.1996	1.0146	4815.8	0.0229	1.0595	4327.7	0.0005	1.2085	3242
		TA	ABLE 7 (ES	STIMATIO	N OF VE	RIFICATIO	N COEFF	ICIENT E	BETWEEN	4000-500	0 CYCLE	NO.)			

9. DISCUSSION & COMMENTS BASED ON RESULTS

In this section we will discuss the various results we obtained from the tables and graphs.

Discussion on variations for low values of shape parameter (low levels of stress in stage II)

Figure (3) & (4) are based on the values from the table (7). As the title suggests the graph shows the variation in verification coefficient and reliability for low values of shape parameter (1.0 - 2.0) respectively between the cycle number intervals 15000-20000.





Clearly from the graph we can say that when c = 1.443 the values of the verification coefficients are closest to unity. However the range (difference between maximum and minimum value) of verification coefficient is also found to be the largest for this value of shape parameter. Higher the range of the verification coefficient, higher is the percentage of error (Range of the verification coefficient is approximately equal to 0.1, which indicates large variations). The range for all the other values of shape parameter is almost equal (range of VC is approximately equal to 0.07). A smaller value of range indicates more consistency in accuracy. This indicates that the slope is more when c = 1.443 than for other values of c.

As for the variation in reliability, we can see that for the lowest value of shape parameter (c = 1.3085), the values of reliability are maximum. The different values of the shape parameters are representations of different stress levels in stage II of loading. We can conclude that as stress level in stage II increases the values of reliability decreases. Also it is clearly seen that the reliability range increases with increase in shape parameter which makes the curves

steeper for higher values of shape parameter. The table (8) below shows the reliability ranges and the verification coefficient ranges for different values of c. This indicates, for higher values of stresses in stage II, the rate of decrease in reliability increases with respect to cycle number (faster failure rate).

SHAPE PARAMETER	RELIABILITY RANGES	VERIFICATION				
		COEFFICIENT KANGES				
C = 1.3085	0.984358 - 0.7589	0.8390 - 0.9081				
C = 1.443	0.847037 - 0.507674	0.9071 - 0.9987				
C = 1.5658	0.67685 - 0.316582	0.8416 - 0.9102				
C = 1.6131	0.62699 - 0.258847	0.8628 - 0.9382				
C = 1.8416	0.507812 - 0.146391	0.8686 - 0.9447				

TABLE 8

Discussion on variations for high values of shape parameter (higher values of stress in stage II)

Previously we studied the variations in reliability and verification coefficient for low stress values in stage II. In this section we will understand the variations for higher values of stress in stage II (high values of shape parameter, 3.6 - 5.4) between the cycle number intervals 4000-5000

From figure (4), we can say that higher the values of the shape parameter, the value of the verification coefficient is high. Here, when c = 4.6607 & 4.368, the values of verification coefficients are consistently close to unity and also the range is narrow for these values. Also for lower values of shape parameter the range of the verification coefficient becomes narrow. The narrower the ranges of V.C are, the more consistent the values are. This can also be observed even in the previous analysis for values of shape parameter between 1.0 and 2.0. Hence we can conclude that, lower the values of shape parameter, the verification coefficient becomes more consistent.

From figure (3), we can interpret the reliability variation increases with increase in shape parameter. Similar to the previous case where we observed the increase in rate of decrease in reliability for higher values of c, even in this case for higher values of the shape parameter, the reliability curves become steeper, making the reliability ranges wider. Hence even for higher values of the shape parameter, the rate of decrease in reliability increases. The table (9) below shows the ranges of reliability & verification coefficient for different values of shape parameter (higher stress values in stage II).

SHAPE PARAMETER	RELIABILITY RANGES	VERIFICATION COEFFICIENT RANGES
C=3.6417	0.973574 - 0.576315	0.8733 - 0.9321
C=4.272	0.955966 - 0.371194	0.8901 – 0.9519
C=4.368	0.906898 - 0.19603	0.9393 - 1.0146
C=4.6607	0.742105 - 0.022851	0.9762 - 1.0595
C=5.3276	0.603838 - 0.000453	1.0983 - 1.2085

TABLE 9

Comparing table (8) & table (9) or figures (1) & (3) of low & high shape values respectively, we can say that the reliability ranges are wider in later. From which we can conclude that failure rate increases for higher values of shape parameter. Also in table (9) for high shape values, there is a sudden decrease in reliability when the shape parameter increased from 4.368 to 4.6607, their corresponding stress levels give an idea on the strength of the material tested. A lot can be understood about the nature of the material from the above analysis.

Thus we have successfully presented a model to derive the reliability, verification coefficient and residual life and they were all analyzed at different levels of stresses. In this study we have analyzed reliability considering the three-parameter Weibull distribution. However, we can further study reliability fitting data to other distributions which can facilitate a fatigue failure rate and compare the best fit distribution with the help of verification coefficient ranges.

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