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Charged Lepton Oscillations and (g-2) Measurements

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ABSTRACT

Traditional analysis of $g = 2(1 + \kappa)$ experiments for charged leptons use a classical spin vector picture. For muons, we here employ a more exact Dirac quantum four component spinor theory. Survival probabilities (including wave packet effects) are computed. These oscillate with the frequency $\Omega = (\kappa e B / M c)$ as has been assumed in previous muon $(g - 2)$ experimental analyses; i.e. muon survival probability oscillations are already at the root of previous successful $(g - 2)$ measurements. Further oscillations should also be observed if mixed neutrino mass matrices were to enter into the reactions (e.g. $\pi^+ \rightarrow \mu^+ + \nu_\mu$) producing muons.

1. Introduction

Measurements of the charged lepton anomalous magnetic moments[1-5], i.e. the g -factor

$$g = 2(1 + \kappa), \quad (1a)$$

are of considerable interest. The calculation of κ as a function of the coupling strength $\alpha = (e^2/\hbar c)$,

$$\kappa = \left(\frac{\alpha}{2\pi}\right) + \dots, \quad (1b)$$

gives one some confidence (when compared with experiment) that perturbative quantum electrodynamics (and perhaps some other gauge theories) makes sense. New precision measurements of κ for the muon have been proposed[6] for obtaining additional insights into strong and weak interactions[7].

The analysis of $(g - 2)$ measurements often proceeds from classical equations[8,9]. For example, a charged lepton is considered to follow a classical orbit in a magnetic field

$$\frac{du^\mu}{d\tau} = \left(\frac{e}{Mc}\right)F^{\mu\nu}u_\nu. \quad (2a)$$

Even the spin-1/2 pseudo-vector is considered to follow a classical equation of motion

$$\frac{ds^\mu}{d\tau} = \left(\frac{(1 + \kappa)e}{Mc}\right)F^{\mu\nu}s_\nu + \left(\frac{\kappa eu^\mu u_\lambda}{Mc^3}\right)F^{\lambda\nu}s_\nu. \quad (2b)$$

Such *classical spin* equations can serve only as a rough guide to the inherent *quantum spin* interference exhibited in laboratory high precision measurements of $(g - 2)$.

Consider the precision measurements of $(g - 2)$ for the muon[2-5]. Muons (produced by the decay (say) $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$) are injected into a ring with a uniform applied magnetic field \mathbf{B} . The experimental quantum survival probability of the muon in the ring to decay, (via $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$ with a detected electron energy above a threshold value) has been fit to the theoretical functional form

$$P_{\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e}(t) = e^{-Mc^2\Gamma t/E} \left(\frac{1 + A \cos(\Omega t + \phi)}{1 + A \cos\phi}\right). \quad (3)$$

In Eq.(3), M , E , and Γ^{-1} represent (respectively) the mass, energy, and intrinsic lifetime of the muon; t is time in the laboratory reference frame, and

$$\Omega = \left(\frac{\kappa e B}{Mc}\right) \quad (4)$$

is the experimental frequency measured in the muon survival probability due to quantum mechanical amplitude interference. The frequency in Eq.(4) is central for the experimental determination of κ in Eq.(1). Eq.(3) for the muon survival probability (as determined by a detected energetic electron) is central for the $(g - 2)$ measurement. Eq.(3) represents a quantum mechanical charged lepton oscillation which follows from the Dirac equation and

cannot be fully understood in purely classical terms. This point will be discussed in detail in the work which follows.

Starting from the work of Schwinger[10,11], the magnetic moment anomaly parameter κ was defined by the manner in which the vacuum renormalized lepton mass depended on an applied magnetic field. For example, the Dirac-Schwinger equation for a muon moving in a magnetic field has the form

$$\left(-i\hbar\gamma^\mu d_\mu + \mathcal{M}c - i(\hbar\Gamma/2c)\right)\psi(x) = 0, \quad (5a)$$

where the gauge derivative is defined as

$$d_\mu = \partial_\mu - i\left(\frac{eA_\mu}{\hbar c}\right), \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad (5b)$$

and the renormalized mass (matrix) has the form

$$\mathcal{M}c^2 = Mc^2 - \left(\frac{\kappa\hbar e}{4Mc}\right)\sigma^{\mu\nu}F_{\mu\nu}. \quad (5c)$$

Our purpose is to discuss in detail the notion of charged lepton oscillations as they have appeared in previous $(g-2)$ measurements employing the quantum mechanical Dirac-Schwinger Eqs.(5), rather than the classical Eqs.(2). The quantum mechanical superposition of amplitudes viewpoint is by far the more fundamental. In Sec.2, the Dirac-Schwinger Eqs.(5) will be derived from the usual charged current definition of κ , i.e.

$$J^\mu(x) = ec\bar{\psi}(x)\gamma^\mu\psi(x) + \left(\frac{\kappa\hbar e}{2M}\right)\partial_\nu(\bar{\psi}(x)\sigma^{\mu\nu}\psi(x)). \quad (6)$$

It will be shown that the second term on the right hand side of Eq.(6) gives rise to the second term on the right hand side of Eq.(5c). The exact energy spectrum of a charged lepton (with $\kappa \neq 0$) in a uniform magnetic field will be derived in Sec.3 from the Dirac equation. In Sec.4, the survival amplitude for a muon in a magnetic field will be computed from solutions of the Dirac equation and the experimental modulation frequency Eqs. (3) and (4) will be derived. The general nature of charged muon oscillations (including those induced by neutrino oscillations) is discussed in the concluding Sec.5.

2. The Dirac-Schwinger Equation

The action for a spin-1/2 charged particle may be written as

$$S = \int d^4x \bar{\psi}(i\hbar\gamma^\mu\partial_\mu - Mc)\psi + \frac{1}{c^2} \int d^4x J^\mu A_\mu. \quad (7)$$

From Eqs.(6) and (7) it follows that

$$S = \int d^4x \bar{\psi}(i\hbar\gamma^\mu d_\mu - Mc)\psi - \left(\frac{\kappa\hbar e}{2Mc^2}\right) \int d^4x \bar{\psi}\sigma^{\mu\nu}\psi\partial_\nu A_\mu, \quad (8)$$

where d_μ is defined in Eq.(5b) and integration by parts has been performed on the second term in Eq.(8). From Eqs.(5c) and (8), one may write the action

$$S = \int d^4x \bar{\psi}(i\hbar\gamma^\mu d_\mu - \mathcal{M}c)\psi, \quad (9)$$

from which the Dirac-Schwinger Eq.(5a) follows. However, implicit in the above considerations is the complex mass replacement rule

$$Mc^2 \rightarrow Mc^2 - i(\hbar\Gamma/2), \quad (10)$$

which will be used throughout this work to describe muon decay rates.

For a particle in a uniform magnetic field, the mass matrix in Eq.(5c) has two eigenvalues,

$$\mathcal{M}w_\pm = \mathcal{M}_\pm w_\pm, \quad (11)$$

where w_\pm are four component spinors and

$$\mathcal{M}_\pm c^2 = Mc^2 \mp \left(\frac{\kappa\hbar eB}{2Mc}\right) = Mc^2 \mp \left(\frac{\hbar\Omega}{2}\right). \quad (12)$$

The mass splitting is thus determined by Eq.(4) as

$$\Delta\mathcal{M}c^2 = |\mathcal{M}_+ - \mathcal{M}_-|c^2 = \hbar\Omega. \quad (13)$$

In analogy to K -meson[12] and B -meson physics, there may be a temptation to employ a phase shift $\theta = -(c^2/\hbar)(\mathcal{M}_+ - \mathcal{M}_-)\tau = \Omega\tau$ in terms of the proper time of the classical Eqs.(2). Such temptation is ill advised since the experimental phase interference in Eq.(3) involves laboratory time t with the phase Ωt and not the ‘‘proper time’’ $\tau = (Mc^2/E)t$ with phase $\Omega\tau$. This kind of error does not arise if one merely solves the Dirac-Schwinger Eqs.(5). And this we shall now proceed to do.

3. Energy Wave Functions

The Dirac-Schwinger Hamiltonian in the laboratory frame in which there is an applied uniform magnetic field is given by

$$H = c\boldsymbol{\alpha} \cdot \boldsymbol{\Pi} + \beta(Mc^2 - (\kappa\hbar e/2Mc)\boldsymbol{\sigma} \cdot \mathbf{B}), \quad (14)$$

where

$$\boldsymbol{\Pi} = \mathbf{p} - (e/c)\mathbf{A}, \quad [\Pi_i, \Pi_j] = i(\hbar e/c)\epsilon_{ijk}B_k, \quad (15)$$

and

$$\boldsymbol{\alpha} = \gamma_5\boldsymbol{\sigma}. \quad (16)$$

We seek solutions to

$$H\psi(\mathbf{r}) = E\psi(\mathbf{r}). \quad (17)$$

With the matrix ρ_2 defined as

$$\rho_2 = i\beta\gamma_5, \quad \rho_2^2 = 1, \quad (18a)$$

one notes that

$$H\rho_2 + \rho_2H = 0. \quad (18b)$$

Given a particle solution in Eq.(17) with $E > 0$, one may also construct anti-particle (negative energy) solutions

$$H\rho_2\psi(\mathbf{r}) = -E\rho_2\psi(\mathbf{r}). \quad (19)$$

Here we consider only positive energy ($E > 0$) eigenstates.

Note that orbital angular momentum and spin angular momentum about the z -axis (chosen parallel to the magnetic field) are not separately conserved; i.e. with $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ and $\mathbf{S} = \hbar\sigma/2$,

$$[H, S_z] = -[H, L_z] = 2ic\gamma_5(\mathbf{\Pi} \times \mathbf{S})_z. \quad (20)$$

On the other hand the total angular momentum along the magnetic field axis is conserved

$$J_z = L_z + S_z, \quad [H, J_z] = 0. \quad (21)$$

Another conserved quantity is Π_z ,

$$[H, \Pi_z] = 0. \quad (22)$$

For motions in a plane perpendicular to the magnetic field $\Pi_z\psi(\mathbf{r}) = 0$, i.e. there is zero motion along the magnetic field axis.

Note that for the classical Eqs.(2) of motion in the plane (zero motion along the magnetic field axis) we have $(dS_z/dt)_{classical} = 0$. However, in the quantum mechanical treatment $(dS_z/dt)_{quantum} = (i/\hbar)[H, S_z] \neq 0$. The classical equations of motion thereby contain integrals of motion *not* present in the physical quantum mechanical treatment. The point is that a particle with *total* spin $s = (\hbar/2)$ is *not* a classical spinning particle. A classical spinning particle has a total spin angular momentum *large* on the scale of \hbar .

If one employs cylinder coordinates $\mathbf{r} = (\rho, \phi, z)$, and the Landau gauge for the vector potential in $\mathbf{B} = \nabla \times \mathbf{A}$,

$$A_\rho = 0, \quad A_\phi = (B\rho/2), \quad A_z = 0, \quad (23)$$

then for zero motion along the magnetic field axis

$$\Pi_z\psi = -i\hbar(\partial\psi/\partial z) = 0, \quad (24)$$

so that

$$\psi = \psi(\rho, \phi) = u_\downarrow(\rho)e^{im\phi} + u_\uparrow(\rho)e^{i(m-1)\phi}. \quad (25)$$

In Eq.(25), $u_{\downarrow,\uparrow}(\rho)$ are four component spinors and conservation of total angular momentum has been invoked; i.e. $[H, J_z] = 0$ allows us to choose the eigenfunctions in Eq.(17) to also be eigenfunctions of $J_z = L_z + S_z$. Thus, the choice

$$J_z\psi(\rho, \phi) = \hbar(m - (1/2))\psi(\rho, \phi), \quad (m = 0, 1, 2, \dots), \quad (26)$$

determines the coherent superposition of spin up and spin down spinors in Eq.(25) which enter into high energy experimental measurements of $(g - 2)$.

From Eqs.(14), (15), (23), and (25) we find the exact energy spectrum

$$E_{\pm,N} = \sqrt{M_B^2 c^4 + 2\hbar ecBN \mp (\kappa\hbar eB/M_B c)} \sqrt{M_B^2 c^4 + 2\hbar ecBN}, \quad (27a)$$

$$M_B = M \sqrt{1 + \left(\frac{\kappa\hbar eB}{2M^2 c^3}\right)^2}, \quad (27b)$$

where $N = n_\rho + |m|$ and n_ρ is an integer counting nodes in the radial wave functions. If there were a momentum component p_z parallel to the magnetic field, then Eq.(27) would be modified to

$$E_{\pm,N}(p_z) = \sqrt{M_B^2 c^4 + c^2 p_z^2 + 2\hbar ecBN \mp (\kappa\hbar eB/M_B c)} \sqrt{M_B^2 c^4 + 2\hbar ecBN}. \quad (28)$$

Eq.(28) describes rigorously the exact energy spectrum of a Dirac spin-1/2 particle with a g -factor defined as $g = 2(1 + \kappa)$ moving in a uniform magnetic field $B = |\mathbf{B}|$.

For magnetic fields of laboratory size (say less than a few Tesla), the inequality $\kappa\hbar eB \ll M^2 c^4$ holds by an overwhelmingly large margin. Thus, *very accurately* M_B may be replaced by M , the lifetime of the energy eigenstate to weak muon decays may be determined by

$$\Gamma_{\pm,N} = \left(\frac{Mc^2}{E_{\pm,N}}\right)\Gamma, \quad (29)$$

and the Bohr transition frequency

$$E_{-,N} - E_{+,N} = \hbar\Omega, \quad (30)$$

where Ω is given by Eq.(4) independent of N , i.e. independent of the energy $E_{\pm,N}$ itself. In deriving the transition rate $\Gamma_{\pm,N}$ for the weak decay of the muon energy eigenstate, Eqs.(10) and (27a) were employed with $\hbar\Gamma \ll Mc^2$ also satisfied again by a very wide margin. Within the same high accuracy approximation one may write

$$E_{\pm,N+n} - E_{\pm,N} = \left(\frac{n\hbar ecB}{E_{\pm,N}}\right), \quad 1 \leq |n| \ll N. \quad (31)$$

Eqs.(29) and (30) constitute the quantum mechanical basis for Eq.(3) which lies at the root of the experimental analysis of the muon $(g - 2)$ measurement. More general statements can be deduced about lepton polarization in magnetic fields on the basis of the *fundamental differences* between the quantum and the classical viewpoints. According to the classical Eqs.(2) of motion, it is quite possible for a high energy particle with $E \gg Mc^2$ and $\kappa \neq 0$ moving in a circular orbit in a plane normal to \mathbf{B} to have the spin polarized parallel to the magnetic field. For a quantum Dirac particle in a high energy (on the mass scale) eigenstate and with $\kappa \neq 0$, the spin polarization is required by the Dirac-Schwinger

Hamiltonian to be (almost) perpendicular to the magnetic field. We have discussed above the reasons for this difference. For the classical equations of motion S_3 is conserved. For the quantum equations of motion, S_3 is not conserved. This has an effect on lepton beam polarizations in high energy machines, e.g. LEP. We note (in this regard) that (at LEP) the electron cyclotron frequency $\Omega_c = (ecB/E)$ is certainly not large when compared with the anomaly frequency $\Omega = (\kappa eB/mc)$ because even though $\kappa \ll 1$, the energy $E \gg mc^2$ is quite high.

4. The Muon Survival Amplitude

Survival amplitudes are conventionally defined as

$$\mathcal{S}(t) = \langle \psi(0) | \psi(t) \rangle, \quad (32)$$

i.e. the amplitude that a quantum object with a wave function $\psi(0)$ at time zero will still be in the same state after a time t . We take the scalar product for the muon to be

$$\mathcal{S}(t) = \int d^3r \psi^\dagger(\mathbf{r}, 0) \psi(\mathbf{r}, t). \quad (33)$$

For the muon moving in a uniform magnetic field

$$\psi(\mathbf{r}, t) = \sum_{s=\pm} \sum_N \exp(-\Gamma_{s,N}t/2) \exp(-iE_{s,N}t/\hbar) c_{s,N} \psi_{s,N}(\mathbf{r}), \quad (34)$$

so that

$$\mathcal{S}(t) = \sum_{s=\pm} \sum_N |c_{s,N}|^2 \exp(-\Gamma_{s,N}t/2) \exp(-iE_{s,N}t/\hbar). \quad (35)$$

The Survival probability $P(t) = |\mathcal{S}(t)|^2$ is then

$$P(t) = \sum_{s,N,s',N'} |c_{s,N}|^2 |c_{s',N'}|^2 \exp(-\gamma_{s,N,s',N'}t/2) \cos(\omega_{s,N,s',N'}t), \quad (36a)$$

where

$$\gamma_{s,N,s',N'} = \Gamma_{s,N} + \Gamma_{s',N'}, \quad \hbar\omega_{s,N,s',N'} = E_{s,N} - E_{s',N'}. \quad (36b)$$

From Eqs.(29), (30), (31), and (36) one finds

$$P(t) = \frac{1}{2} \int dW(E) \int dW(E') \exp(-\gamma_{E,E'}t/2) (\cos(\omega_{E,E'}t) + \cos(\omega_{E,E'}t + \Omega t)), \quad (37a)$$

where $dW(E)$ is the probability that the muon has an energy in the interval dE ,

$$\gamma_{E,E'} = \Gamma \left(\frac{Mc^2}{E} + \frac{Mc^2}{E'} \right), \quad \omega_{E,E'} = \left(\frac{eB}{Mc} \right) \left(\frac{Mc^2}{E} - \frac{Mc^2}{E'} \right), \quad (37b)$$

and Ω is defined in Eq.(4). Eq.(37a) is more simply written as

$$P(t) = \frac{1}{2} K(t) (1 + \cos(\Omega t)), \quad (38a)$$

where

$$K(t) = \int dW(E) \int dW(E') \exp(-\gamma_{E,E'} t/2) \cos(\omega_{E,E'} t) \quad (38b)$$

describes the spreading of the muon wave packet moving around in the magnetic field. Such wave packet spreading depends on the precise nature of muon energy distribution $dW(E)/dE$. For a Gaussian muon energy distribution, with mean energy $\bar{E} = \langle E \rangle$ and deviations from the mean $\delta E = \sqrt{\langle E^2 \rangle - \langle E \rangle^2} \ll \bar{E}$,

$$K(t) = \exp\left(\frac{-\Gamma M c^2 t}{\bar{E}}\right) \exp\left(\frac{-t^2}{2t_s^2}\right),$$

$$t_s = \left(\frac{1}{\sqrt{2}\delta\omega}\right), \quad \left(\frac{\delta\omega}{\omega}\right) = \left(\frac{\delta E}{\bar{E}}\right), \quad \omega = \left(\frac{eBc}{\bar{E}}\right). \quad (38c)$$

Comparing Eqs.(3) and (4), which have been previously employed[2-5] for the analysis of muon ($g-2$), to our central Eqs.(38) we note some similarities and some differences: (i) Our $\cos(\Omega t)$ does not have an arbitrary phase ϕ as in $\cos(\Omega t + \phi)$. The reason for this is that as theorists we may declare that the initial wave function is known at exactly “time zero”. The experimentalists actually have to build the clock and provide the beam (quite different from a mere theoretical pronouncement), and “time zero” has fluctuated a bit in past experiments. This has been reported as a fluctuating (from day to day) phase ϕ . (ii) Eq.(3) has a coefficient $A \leq 1$ while Eqs.(38) correspond to $A = 1$. The reason for this is firstly that we have assumed polarization perfectly in the plane perpendicular to the magnetic field and nothing in an experiment is perfect. Secondly, we have calculated in Eq.(38) the total survival probability while Eq.(3) refers to a muon decay ejecting an electron above a high energy threshold. The value of A depends at least in part on experimental cuts in the reported electron counts. (iii) Eq.(3) contains only the intrinsic exponential decay, while our Eqs.(38) also contains wave packet spreading effects modeled assuming a Gaussian energy distribution. The reason is that wave packet spreading effects have to be treated using a quantum mechanical wave (Dirac-Schwinger) equation. The spreading of the wave packet is in a time $t_s \sim (1/\delta\omega)$. This should not adversely effect the accuracy of measured κ values for $\delta E/\bar{E} \leq 10^{-3}$ in laboratory magnetic fields of order 1.5 Tesla.

5. Conclusions

From the viewpoint of quantum mechanics, the reason for the oscillations of any total survival amplitude

$$\mathcal{S}_{tot}(t) = \langle \Psi | e^{-iHt/\hbar} | \Psi \rangle, \quad (39)$$

is in the nature of the energy probability distribution

$$dW_{tot}(E) = \langle \Psi | \delta(E - H) | \Psi \rangle dE, \quad (40)$$

i.e.

$$\mathcal{S}_{tot}(t) = \int dW_{tot}(E) e^{-iEt/\hbar}. \quad (41)$$

For example, a two peaked Gaussian distribution in energy ($\Delta \ll \bar{E}$),

$$dW_{two\ peak}(E) = (1/2)\sqrt{1/2\pi\Delta^2} \sum_{s=\pm 1} \exp(-(E - \bar{E} + s\hbar\omega_o/2)^2/2\Delta^2)dE, \quad (42a)$$

gives rise to an oscillating in time survival amplitude

$$\mathcal{S}_{two\ peak}(t) = \cos(\omega_o t/2)\exp(-i\bar{E}t/\hbar)\exp(-\Delta^2 t^2/2\hbar^2). \quad (42b)$$

For another example, if the total energy distribution is a convolution of two energy distributions

$$\frac{dW_{tot}(E)}{dE} = \int d\epsilon \frac{dW_1(E + \epsilon)}{dE} \frac{dW_2(\epsilon)}{d\epsilon}, \quad (43a)$$

then the survival amplitude is the product of two survival amplitudes

$$\mathcal{S}_{tot}(t) = \mathcal{S}_1(t)\mathcal{S}_2(t). \quad (43b)$$

Thus Eq.(38a) represents a convolution of a normal weak muon decay with a two peaked (at $\bar{E} \pm (\hbar\Omega/2)$) distribution separated by $\Omega = (\kappa eB/Mc)$, i.e. the observed muon ($g - 2$) oscillation frequency.

Consider the notion of neutrino mass mixing[13]. In two previous experiments[14,15], attempts were made to find neutrino mass matrix mixing by observing a muon propagating in a magnetic field. One tried to measure a double peak in the muon energy distribution using a time scale very short compared with Ω^{-1} . The point was that the neutrino mass from $\pi^+ \rightarrow \mu^+ + \nu_\mu$ would induce in the muon energy distribution a double peak from (say) two possible masses for the neutrino. These two possible masses would yield two possible muon energies, and thus exhibit a double peak in the muon energy distribution. Such a double peak induced by neutrino masses did not actually appear within the accuracy of the previous experiments. However, in those experiments one attempted to measure the muon energy distribution *directly*. We are here suggesting that a *more sensitive* probe of energy splitting is of the type already successful in muon ($g - 2$) experiments; i.e. one should look for the double peak in energy via the muon survival probability oscillation in time. The resulting extra oscillation should appear in a modified form of Eq.(3)

$$P_{\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e}(t) \approx e^{-Mc^2\Gamma t/E} (1 - \sin^2(2\theta)\sin^2(\Omega't/2)) \left(\frac{1 + A\cos(\Omega t)}{1 + A} \right), \quad (44a)$$

where θ is the two flavor neutrino mixing angle, and

$$\hbar\Omega' \approx E_\pi \left(\frac{\Delta m}{M_\pi} \right)^2. \quad (44b)$$

We have assumed that the pion has decayed into a forward moving muon (in the laboratory frame) and neutrino with $E_\pi \gg M_\pi c^2$. The neutrino masses (differing by Δm) are assumed very small on the scale of pion and muon masses. For a freely moving muon (i.e. $\Omega = 0$), only the neutrino induced oscillations survive as previously discussed[16].

Finally, there are certainly two characteristic frequencies associated with $(g - 2)$ measurements, i.e. the anomaly frequency $\Omega = (\kappa e B / M c)$ and the cyclotron frequency $\Omega_c = (e c B / E)$. The cyclotron frequency Ω_c has been measured directly from emitted radiation. To some extent, the anomaly frequency Ω should also enter into the radiation spectrum, although the authors are not aware of any direct measurements of this sort. If there is also neutrino mass mixing, then a third frequency Ω' enters into the problem as well.

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