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AXION COUPLINGS AND EFFECTIVE CUT-OFFS
IN SUPERSTRING COMPACTIFICATIONS

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Abstract

We use the linear supermultiplet formalism of supergravity to study axion couplings and chiral anomalies in the context of field-theoretical Lagrangians describing orbifold compactifications beyond the classical approximation. By matching amplitudes computed in the effective low energy theory with the results of string loop calculations we determine the appropriate counterterm in this effective theory that assures modular invariance to all loop order. We use supersymmetry consistency constraints to identify the correct ultra-violet cut-offs for the effective low energy theory. Our results have a simple interpretation in terms of two-loop unification of gauge coupling constants at the string scale.

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1. Introduction

Over the past few years there has been considerable progress in understanding the structure of effective actions describing the physics of massless fields in four-dimensional superstring theory. The basic strategy [1] that has proven to be very successful has been to extract the relevant terms in the field-theoretical Lagrangian from the S-matrix elements computed within the full-fledged superstring theory. In this way many important quantities have been determined at the classical level, including the Kähler potentials and the gauge and Yukawa couplings for orbifold compactifications of heterotic superstrings.

Another important development was the observation that the duality symmetry [2] between small and large radius toroidal compactifications extends to a much larger symmetry group of the so-called target space modular transformations acting on the moduli fields [3]. This symmetry can be very helpful when studying the moduli-dependence of the effective actions for orbifold compactifications [4].

More recently, the program of reconstructing effective Lagrangians from string amplitudes has been pursued beyond the classical level, to higher genus in the string loop expansion. In particular, the moduli-dependence of the one-loop corrections to gauge couplings has been determined [5, 6] by computing the relevant string-theoretical amplitudes. These string loop corrections turn out to be manifestly invariant under the modular symmetry transformations. The results of [5, 6] provide important constraints on the form of the effective action describing the physics of massless string excitations. This is due to the fact [7, 8, 9] that the radiative corrections generated by quantum loops of massless particles violate the target space modular invariance. Since string theory is modular invariant to all loop order [10], the effective field theory action must contain some counterterms that cancel this “modular anomaly” in a way analogous to the Green-Schwarz mechanism [11]. In this paper we determine the appropriate combination of counterterms by matching the field-theoretical couplings of axionic moduli to the gauge bosons with the corresponding string-theoretical couplings. In addition, supersymmetry constraints permit us to identify

the correct ultra-violet cut-offs for a consistent Pauli-Villars regularization of the effective low energy theory.

Two classes of axions will be particularly important for our discussion: the universal axion and the model-dependent axionic moduli. The universal dilaton and axion are manifestations of the same superstring excitation that creates the graviton; hence they are present in all possible compactifications of the heterotic superstring theory. The axion corresponds to a two-index (Minkowski) antisymmetric field $b_{\mu\nu}$, and together with the dilaton and dilatino it forms one linear multiplet L [12] of supersymmetry. The effective tree-level supergravity Lagrangian for the dilaton sector is readily obtained [13] from the ten-dimensional Lagrangian by dimensional reduction, followed by the so-called duality transformation on the axion, from the two-index antisymmetric form to a pseudoscalar field. The dilaton supermultiplet is represented then by a scalar chiral superfield which is usually denoted by S . This allows a natural incorporation of the dilaton and axion into the Kähler structure of the locally supersymmetric four-dimensional sigma model describing the massless excitations of the compactified superstring. At the classical level, the Kähler potential has the form:

$$K = G - \ln(S + \bar{S}), \tag{1.1}$$

where G is an S -independent function of all other chiral superfields. The vacuum expectation value (VEV) of the dilaton¹ s determines the gauge coupling constant g :

$$\frac{1}{g^2} = \langle \text{Re } s \rangle \tag{1.2}$$

at the string scale M_s which is related to the Planck scale M_P by $M_s = gM_P$ [13].

The model-dependent axions correspond to the pseudoscalar components of the chiral moduli superfields T whose vacuum expectation values determine the geometry (metric tensor) of the compactified dimensions. The symmetry group of modular transformations

¹We use upper case Roman and Greek letters for chiral supermultiplets and the corresponding lower case letters for their complex scalar components.

depends on the particular orbifold background, however it always contains at least one $SL(2, \mathbf{Z})$ subgroup which acts on a generic modulus T as:

$$T \rightarrow \frac{aT - ib}{icT + d}, \quad ad - bc = 1, \quad a, b, c, d \in \mathbf{Z}. \quad (1.3)$$

The orbifolds that have been discussed most extensively in the context of modular symmetries are the orbifolds with gauge group $E_8 \otimes E_6 \otimes U(1)^2$ [1, 7]. They contain three untwisted (1,1) moduli T^I , $I = 1, 2, 3$, which transform under $SL(2, \mathbf{Z})$ as in eq.(1.3). In order to make our discussion as explicit as possible, we consider here this one particular class of orbifolds. The corresponding Kähler potential is:

$$G = \sum_I g^I + \sum_A \exp(\sum_I q_A^I g^I) |\Phi^A|^2 + O(\Phi^4), \quad (1.4)$$

where $g^I = -\ln(T + \bar{T})^I$, and the exponents q_A^I depend on the particular matter field Φ^A as well as on the modulus T^I in question [7]. Our considerations can be generalized in a straightforward way to other orbifold models.

The transformation (1.3), supplemented by the appropriate transformations on the matter superfields Φ^A and chiral rotations on the gauginos λ :

$$\Phi^A \rightarrow \exp(-\sum_I q_A^I F^I) \Phi^A, \quad \lambda \rightarrow e^{-\frac{1}{4}(F - \bar{F})} \lambda, \quad (1.5)$$

where

$$F = \sum_I F^I = \sum_I \ln(icT^I + d), \quad (1.6)$$

effects a Kähler symmetry transformation

$$K \rightarrow K + F + \bar{F} \quad (1.7)$$

in the tree-level supergravity Lagrangian. The tree-level Lagrangian is invariant under such transformations.

It has been shown before by other authors [7, 8, 9] that the one-loop effective Lagrangian $\mathcal{L}_{1\text{-loop}}$ computed within the framework of the effective field theory of massless

string excitations is *not* invariant under the transformations (1.3, 1.5) because of chiral and conformal anomalies. The variation of the one loop Lagrangian [7, 8] under the modular transformation (1.3) is:

$$\delta\mathcal{L}_{1\text{-loop}} = \frac{1}{16\pi^2} \frac{1}{2} \sum_a \sum_I \alpha_a^I \int d^2\theta (W^\alpha W_\alpha)^a F^I + \text{h.c.} \quad (1.8)$$

Here, the summation extends over the indices a numbering the simple subgroups of the full gauge group, and the moduli indices I . The constants α_a^I will be specified in Section 3, where we derive $\mathcal{L}_{1\text{-loop}}$.

Since it is known that the full-fledged string theory is invariant under the orbifold modular transformations (1.3) to all orders of its loop expansion [10], the massless truncation of string theory is inconsistent unless the effective field theory is supplemented by counterterms whose variation cancels the modular anomaly. These counterterms can be interpreted as the result of integrating out massive fields, such as Kaluza-Klein excitations and the winding modes.

Two types of counterterms have been discussed in the literature in the context of modular anomaly cancellation. The first type [5, 14] is a simple gauge “kinetic” term:

$$\mathcal{L}_f = \frac{1}{4} \sum_a \int d^2\theta f_a (W^\alpha W_\alpha)^a + \text{h.c.}, \quad (1.9)$$

where the summation extends over the indices a numbering simple subgroups of the full gauge group, and the f_a are analytic functions of the chiral superfields. They transform as:

$$f_a \rightarrow f_a + \sum_I \eta_a^I F^I, \quad (1.10)$$

with the constants η_a^I adjusted in such a way as to cancel the modular anomaly, or at least a part of it. The second type of counterterm [8, 9] is the so-called Green-Schwarz term \mathcal{L}_{GS} which utilizes the linear multiplet representation for the dilaton supermultiplet [15, 16]. We will write down \mathcal{L}_{GS} after reviewing in Section 2 some basic properties of linear multiplets.

In Section 4 we will exploit the linear formalism to relate the axial couplings given in Section 3 to recent results from string loop calculations [6]. This will allow us to determine the correct choice of counterterm, up to fluctuations about the vacuum configuration for the effective field theory. In Section 5 we use supersymmetry consistency constraints to determine the ultra-violet cut-offs for the effective low energy theory. We also write down the full one-loop effective gauge coupling constant, including the contributions from quantum loops of massless particles as well as from the counterterms. The result has an interesting interpretation in terms of the two-loop renormalization group equation. These results can have interesting implications for threshold corrections in attempts to extract the string parameters from data on coupling constant unification.

We should mention at this point that some results of Sections 3 and 4 have already appeared elsewhere. In particular, the correct combination of counterterms was identified in [8, 9], although with little explanation. Since the problem of the moduli-dependence of effective actions has been obscured in the literature, which includes a number of incorrect statements, we address it here in a more systematic way. The novel part of this paper consists of a detailed discussion of axionic couplings, the determination of ultra-violet cut-offs and the implications for two-loop coupling constant unification.

2. The Linear Multiplet Formalism

The general formalism for linear multiplets has been described in Ref. [16]. In this section we review the salient properties. The standard linearity condition is $\mathcal{D}\mathcal{D}L = \overline{\mathcal{D}}\overline{\mathcal{D}}L = 0$, where \mathcal{D} and $\overline{\mathcal{D}}$ are the supersymmetric covariant derivatives. The dilaton l enters as the $\theta = \bar{\theta} = 0$ component of L . For further discussion, it is very convenient to include in the linear multiplet the Yang-Mills Chern-Simons form

$$\Omega_{\mu\nu\rho} = A_{[\mu}^a F_{\nu\rho]a} - \frac{1}{3}c_{abc}A_{\mu}^a A_{\nu}^b A_{\rho}^c, \quad (2.1)$$

where c_{abc} are the structure constants of the Yang-Mills group. This can be done by

employing the so-called modified linearity conditions [16]:

$$\mathcal{D}\mathcal{D}L = -\sum_a(\overline{W}W)^a ; \quad \overline{\mathcal{D}}\overline{\mathcal{D}}L = -\sum_a(WW)^a . \quad (2.2)$$

As a result, the Chern-Simons form enters together with the field strength of the axion field $b_{\mu\nu}$ as the $\theta\bar{\theta}$ component of L : $[\mathcal{D}_\alpha, \overline{\mathcal{D}}_{\dot{\alpha}}]L|_{\theta=\bar{\theta}=0} = -2\sigma_{\mu\alpha\dot{\alpha}}\mathcal{H}^\mu$, where

$$\mathcal{H}^\mu = \frac{1}{2}\epsilon^{\mu\nu\rho\lambda}(\partial_\nu b_{\rho\lambda} - \frac{1}{2}\Omega_{\nu\rho\lambda}) + \frac{1}{2}\lambda^a\sigma^\mu\bar{\lambda}_a . \quad (2.3)$$

The gauge invariance of a multiplet satisfying the modified linearity condition is ensured by imposing the appropriate transformation properties for $b_{\mu\nu}$.

The Lagrangian describing one linear multiplet coupled to supergravity and matter in the presence of a Yang-Mills Chern-Simons form is:

$$\mathcal{L}_{lin} = -3 \int d^4\theta EF(Z, \bar{Z}, L) , \quad (2.4)$$

where F is a function of chiral and antichiral superfields (Z and \bar{Z}), and the linear superfield L . We work in the Kähler superspace formulation [17] in which the supervierbein E depends implicitly on the Kähler potential $K(Z, \bar{Z}, L)$. In order to exhibit the relation between the linear and chiral representations for the dilaton supermultiplet, it is convenient to consider

$$\mathcal{L}_{lin} = -3 \int d^4\theta E \left[F(Z, \bar{Z}, L) + \frac{1}{3}(L + \Omega)(S + \bar{S}) \right] , \quad (2.5)$$

where L is now an unconstrained superfield and S is a chiral superfield. Here, Ω denotes the full Chern-Simons superfield [15]: $\mathcal{D}\mathcal{D}\Omega = \sum_a(WW)^a$. The superfield S plays the role of a Lagrange multiplier whose equations of motion enforce the modified linearity constraint (2.2). After eliminating S by using its classical equations of motion one arrives at the Lagrangian (2.4).

The duality transformation from the linear to the chiral representation amounts to applying the equations of motion obtained by varying \mathcal{L}_{lin} , eq.(2.5), with respect to L , in order to express L in terms of the remaining superfields. After eliminating the linear

multiplet, one obtains the supergravity Lagrangian with the correctly normalized Einstein term provided that the function F satisfies the condition:

$$F(Z, \bar{Z}, L) + \frac{1}{3}L(S + \bar{S}) = 1. \quad (2.6)$$

This condition, combined with the equations of motion for L , yields a differential equation for the function F and the Kähler potential K , with the solution

$$F(Z, \bar{Z}, L) = 1 + \frac{1}{3}LV(Z, \bar{Z}) + \frac{L}{3} \int \frac{dL}{L} \frac{\partial K}{\partial L}(Z, \bar{Z}, L), \quad (2.7)$$

where the “constant of integration” V is an arbitrary real function of the matter superfields. Here we will consider the special case in which the Kähler potential has the form:

$$K(Z, \bar{Z}, L) = G(Z, \bar{Z}) + k(L), \quad (2.8)$$

where G is an arbitrary function of chiral matter superfields. Then the constraints (2.6, 2.7) give

$$\int \frac{dL}{L} k' = -[S + \bar{S} + V(Z, \bar{Z})], \quad (2.9)$$

where $k' = dk/dl$. This equation determines the functional dependence of L :

$$L = L(X), \quad X \equiv S + \bar{S} + V(Z, \bar{Z}). \quad (2.10)$$

When the theory is written in terms of the chiral multiplet S , the Lagrangian reads:

$$\mathcal{L}_{lin} = -3 \int d^4\theta E - \int d^4\theta E \Omega(S + \bar{S}), \quad (2.11)$$

The first term is the standard supergravity Lagrangian with the Kähler potential $K = G(Z, \bar{Z}) + k[L(X)]$ [8]. The second term gives rise to a gauge kinetic term [18] of the form (1.9) with a group-independent function $f_a = S$. This is exactly the tree-level kinetic gauge term [13] of superstring supergravity! In fact, the full tree-level Lagrangian is given by \mathcal{L}_{lin} corresponding to the function F determined from the Kähler potential of the form (2.8), with G of eq.(1.4), $k(L) = \ln(L)$, and the “constant of integration” $V = 0$.

Indeed, eq.(2.9) then gives $L = (S + \bar{S})^{-1}$; therefore the Kähler potential has the right S -dependence.

3. Anomalies in the Effective Field Theory

In this section we evaluate the one-loop corrections that induce modular anomalies. The results are obtained from standard results for the effective field theory in terms of component fields in the Kähler covariant [17] formulation of supergravity.²

The modular transformations act on fermions as field-dependent (local) chiral rotations (1.5). The invariance of the tree-level Lagrangian under such transformations, which can be thought of as a combination of the usual Kähler supergravity transformation and field reparametrizations, is ensured by the presence of the appropriate fermion connection in the kinetic energy terms.³ The modular anomaly includes both the chiral and conformal anomalies of ordinary field theories. The chiral anomaly arises through the axial part A_ρ of the (matrix valued, moduli-dependent) fermion connection. The conformal anomaly arises [19, 20, 14, 21] through the dependence of the cut-off(s) Λ of the effective supergravity theory on the moduli fields which transform nontrivially under (1.3); these cut-offs may be different for different sectors of the theory.

Using standard results, the one-loop correction to the Yang-Mills Lagrangian is⁴

$$\mathcal{L}_{1-loop} = \frac{1}{16\pi^2} \frac{1}{4} [3\text{Tr}_G F^2 \ln \Lambda_G^2 - \text{Tr}_M F^2 \ln \Lambda_M^2 - 4\text{Tr}(F\tilde{F} \frac{1}{\partial^2} \partial^\rho A_\rho)] + O(\varphi), \quad (3.1)$$

where φ represents gauge nonsinglet fields, $F_{\mu\nu} = F_{\mu\nu}^a T_a$ is the (matrix valued) gauge field strength and $\text{Tr}_{G(M)}$ means the trace over the gauge (matter) representation of the

²The commonly used formulation [18] of supergravity is obtained from the Kähler covariant one by fixing the Kähler gauge with the choice $F(Z) = \ln W(Z)$ in the Kähler transformation (1.7).

³The kinetic energy terms for fermions are normalized canonically: $\mathcal{L}_{kin}^{\text{fermion}} = -i\bar{\lambda}\bar{D}\lambda + \dots$

⁴When matter fields φ are included in the effective Lagrangian (3.2), the contributions of chiral supermultiplet loops are modified [22] by $\text{Tr}_M F^2 \rightarrow \text{Tr}_M f^2$, $\text{Tr}_M F\tilde{F} \rightarrow \text{Tr}_M f\tilde{f}$, where $f_{\mu\nu} = [D_\mu, D_\nu]_M = (F_{\mu\nu})_M + O(\varphi)$.

generators T_a . Provided that the axial connection and the cut-offs satisfy the conditions

$$A_\rho^{G(M)} = \frac{i}{4}(\partial_\rho z^m \frac{\partial}{\partial z^m} B_{G(M)} - h.c.), \quad \ln \Lambda_G^2 = \frac{B_G}{3}, \quad \ln \Lambda_M^2 = -B_M, \quad (3.2)$$

eq.(3.1) takes the form

$$\mathcal{L}_{1-loop} = \frac{1}{16\pi^2} \frac{1}{4} \text{Tr} \left\{ [F^2 - iF\tilde{F}] \frac{\partial^\rho}{\partial^2} \left(\partial_\rho z^m \frac{\partial}{\partial z^m} B \right) \right\} + h.c. \quad (3.3)$$

which is contained in the component field expansion of the superspace result [23]

$$\mathcal{L}_{1-loop} = \frac{1}{16\pi^2} \frac{1}{8} \int d^4\theta \text{Tr} \left\{ W^2 \frac{\mathcal{D}^2}{\square} B \right\} + h.c. \quad (3.4)$$

Since the coefficient of the axial anomaly is unambiguous, the supersymmetry constraint (3.2) can be used to determine the correct ultra-violet cut-offs. We return to this point in Section 5. Here we will simply use the fact that the anomalous Lagrangian (3.4) is uniquely determined from the knowledge of the axial vector couplings.

First consider the gauge sector. For gauginos, the axial part of the connection is

$$A_\mu^G = \frac{i}{4}(\partial_\mu z^m \frac{\partial}{\partial z^m} K - h.c.). \quad (3.5)$$

Due to the specific gauge kinetic term, eq.(1.9) with $f_a = S$, the chiral gaugino current $\lambda^a \sigma^\mu \bar{\lambda}_a$ couples also to another axial vector field:

$$A'_\mu{}^G = \frac{i}{2} \left\{ \partial_\mu z^m \frac{\partial}{\partial z^m} \ln(s + \bar{s}) - h.c. \right\}. \quad (3.6)$$

The corresponding radiative correction of the form (3.1) induces an additional coupling of $\text{Im } s$ to gauge bosons. In Section 4 we will argue that the loop diagrams involving gauge bosons cancel this radiative correction. This means that the total gauge sector contribution is of the form (3.1), with the axial vector field of eq.(3.5).

Next consider matter-loop contributions. We will evaluate the effective Lagrangian (3.4) at vanishing VEVs for gauge nonsinglet scalar fields. In this case only gauge nonsinglet chiral fermion loops contribute to the chiral anomaly. The fermion matter connection is [here the indices $\alpha = (A, a)$ and $\beta = (B, b)$ label gauge nonsinglet complex scalar fields

φ_a^A and φ_b^B , respectively, where a and b are the gauge indices; the index m runs over all scalar fields z^m]:

$$(A_\mu^M)^\alpha_\beta = -\frac{i}{4}(\delta_\beta^\alpha \partial_\mu z^m \frac{\partial}{\partial z^m} K - 2\Gamma_{\beta m}^\alpha \partial_\mu z^m - h.c.), \quad \Gamma_{\beta m}^\alpha \partial_\mu z^m = K^{\alpha\bar{n}} K_{\bar{n}\beta m} \partial_\mu z^m, \quad (3.7)$$

where the second term assures invariance under reparametrization of the field variables.

To proceed further, we restrict our attention to the class of orbifolds discussed in the Introduction, with the corresponding Kähler potential parametrized by a set of constants q_A^I , see eq.(1.4). The following equation, which follows from eq.(2.9), is particularly useful in evaluating the reparametrization invariance connection coefficients $\Gamma_{\beta m}^\alpha$:

$$K_m = \begin{cases} -L & \text{if } z^m = s \\ G_m - LV_m & \text{if } z^m \neq s. \end{cases} \quad (3.8)$$

It will become clear in the next section that it is sufficient to consider a limited class of “integration constants” V which are all equal in the limit of vanishing VEVs for gauge nonsinglet fields:

$$V = \omega \sum_I g^I + \sum_A p_A \exp(\sum_I q_A^I g^I) |\Phi^A|^2 + O(\Phi^4) = \omega G + O(\Phi^2), \quad (3.9)$$

with the constant ω to be determined from the requirement of modular anomaly cancellation at the vacuum; the form of the $O(\Phi^2)$ terms is dictated by modular covariance. After evaluating $\Gamma_{\beta m}^\alpha$, eq.(3.7) becomes:

$$(A_\mu^M)^\alpha_\beta = -\frac{i}{4} \delta_b^a \left\{ \delta_B^A \partial_\mu z^m \frac{\partial}{\partial z^m} [K - 2 \sum_I q_A^I g^I - 2 \ln(1 - p_A L)] - h.c. \right\} \quad (3.10)$$

Summing the gauge and matter contributions to the effective Lagrangian (3.1), and using the appropriate cut-offs as determined by eq.(3.2), the result can be interpreted as a term in the expansion of the following superspace expression:

$$\begin{aligned} \mathcal{L}_{1-loop} = \frac{1}{16\pi^2} \frac{1}{8} \sum_a \int d^4\theta (W^\alpha W_\alpha)^a \frac{\mathcal{D}^2}{\square} \left\{ - \sum_I \alpha_a^I g^I \right. \\ \left. + (C_G^a - C_M^a) k(L) + 2 \sum_A C_A^a \ln(1 - p_A L) \right\} + h.c., \quad (3.11) \end{aligned}$$

where

$$\alpha_a^I = -C_G^a + \sum_A (1 - 2q_A^I) C_A^a. \quad (3.12)$$

Here C_G^a denotes the quadratic Casimir operator in the adjoint representation of the gauge subgroup labeled by a and $C_M^a = \sum_A C_A^a = \sum_A \text{Tr}(T_a^A)^2$, with T_a^A denoting the gauge group generator for the representation of Φ^A . It is convenient to rewrite eq.(3.12) as

$$\alpha_a^I = -C + b_a^I, \quad (3.13)$$

where $C = 30$ is the Casimir operator in the adjoint representation of E_8 . The constants b_a^I were determined in [7] by using the Kähler potential exponents q_A^I previously computed in [1] from the appropriate string amplitudes. It turns out that $b_a^I = 0$, unless the modulus T^I corresponds to an internal plane which is left invariant under some orbifold group transformations; this may happen only if an $N = 2$ supersymmetric twisted sector is present. This means that for a large class of orbifolds, including the familiar Z_3 and Z_7 orbifolds which contain $N = 1$ and $N = 4$ sectors only, the constants $\alpha_a^I = -C$ are gauge group independent. This fact will have important implications for the structure of anomalous counterterms.

4. Higher Genus Supergravity

In this section, we shall discuss the implications of recent computations [6] of higher genus axion couplings for the structure of field-theoretical Lagrangians describing orbifold compactifications beyond the classical approximation. We will determine the appropriate combination of counterterms which are necessary in order to restore the modular invariance broken by the loop contributions of eq.(3.11).

The variation of the one-loop Lagrangian under the modular transformations (1.3) can be computed from eq.(3.11). The result has already been written in eq.(1.8), with the constants α_a^I of eqs.(3.12, 3.13). Notice that due to the assumed modular invariance of the

linear multiplet L , only the first term in eq.(3.11) contributes to this modular anomaly. We have already discussed in the Introduction one class of counterterms, eq.(1.9). We begin this section by discussing another type – the so-called Green-Schwarz counterterm [8, 9].

In the linear multiplet formulation [16], the Green-Schwarz counterterm corresponds to the part of \mathcal{L}_{lin} involving the “constant of integration” V , *c.f.* (2.7):

$$\mathcal{L}_{GS} = - \int d^4\theta E L V . \quad (4.1)$$

If the linear multiplet is invariant under modular transformations while V varies as

$$V \rightarrow V + h(Z) + \bar{h}(\bar{Z}) , \quad (4.2)$$

then

$$\delta\mathcal{L}_{GS} = -\frac{1}{4} \sum_a \int d^2\theta (W^\alpha W_\alpha)^a h(Z) + h.c. + \dots , \quad (4.3)$$

where we neglected the terms containing the gravitino field and the terms vanishing in the flat gravitational background limit. Consider for instance the function V of eq.(3.9), which transforms with $h = \omega \sum_I F^I$. Then the variation (4.3) has a form very similar to eq.(1.8); therefore the corresponding Green-Schwarz counterterm may contribute to the modular anomaly cancellation. Notice that the assumed invariance of L under modular transformations requires through (2.10) that its dual S transforms as

$$S \rightarrow S - h , \quad (4.4)$$

therefore in the chiral formulation it is the tree level gauge kinetic term [the second term in (2.11)] which gives rise to the *r.h.s.* of eq.(4.3).

In general, the modular anomaly can be cancelled by some linear combination of \mathcal{L}_f (1.9) and \mathcal{L}_{GS} (4.1). The question as to which combination actually does appear in the counterterm can be answered by comparing the results of recent string computations of the axion couplings to gauge bosons with the similar effective supergravity computations.

We are interested in the couplings of scalar particles to gauge bosons. The explicit form of the Lagrangian \mathcal{L}_{lin} (2.4) expressed in terms of the component fields is rather complicated; however, only three types of terms will be relevant for further considerations. These are: 1) scalar kinetic terms and tree-level scalar couplings to gauge bosons, 2) gauge kinetic terms, and 3) pseudoscalar couplings to the chiral matter currents, which induce the couplings to gauge bosons via the usual anomaly diagrams discussed in the previous section.

1) The kinetic energy terms for the scalar fields are:

$$\mathcal{L}_{kin} = -\frac{k'}{4l} \partial^\mu l \partial_\mu l + \frac{k'}{4l} \mathcal{H}^\mu \mathcal{H}_\mu + (IV - G)_{i\bar{j}} \partial^\mu z^i \partial_\mu \bar{z}^{\bar{j}} + \frac{1}{2} \mathcal{H}^\mu \mathcal{V}_\mu, \quad (4.5)$$

where

$$\mathcal{V}_\mu = -i(V_j \partial_\mu z^j - h.c.). \quad (4.6)$$

Here, we list all terms involving \mathcal{H}^μ that give rise to the kinetic energy terms for the universal axion. It turns out that the last term in (4.5) contains also the only tree-level coupling of axionic moduli to gauge fields, through the Chern-Simons form in eq.(2.3). In order to extract these couplings, one expands V in the fluctuations of scalar fields about their vacuum expectation values. Then the integration by parts yields

$$\frac{1}{2} \mathcal{H}^\mu \mathcal{V}_\mu = \frac{i}{2} (V_j z^j - V_{\bar{j}} \bar{z}^{\bar{j}}) \partial_\mu \mathcal{H}^\mu = -\frac{i}{8} (V_j z^j - V_{\bar{j}} \bar{z}^{\bar{j}}) [F_{\mu\nu}^a \tilde{F}_a^{\mu\nu} - 2\partial_\mu (\lambda^a \sigma^\mu \bar{\lambda}_a)] + \dots, \quad (4.7)$$

where z denotes now the fluctuation of the scalar component of Z and the derivatives are evaluated at the vacuum expectation values.

2) The gauge kinetic terms are:

$$\mathcal{L}_{gauge} = -\frac{1}{4} \sum_a \Delta_a^{lin} (F_{\mu\nu}^a F_a^{\mu\nu} + 2i\lambda^a \not{D} \bar{\lambda}_a + 2i\bar{\lambda}_a \not{D} \lambda^a), \quad (4.8)$$

where

$$\Delta_a^{lin} = -\int \frac{k'}{2l} dl - \frac{1}{2} V. \quad (4.9)$$

Note that, in agreement with eq.(2.11), the duality transformation (2.9) gives $\Delta_a^{lin} = \text{Re } s$.

3) The axionic moduli couple to the chiral matter currents via the field-dependent fermion connections, as discussed in the previous section. There exists only one additional source of couplings involving gauge nonsinglet fermions: the interaction term (4.7) contains the coupling of the field-dependent vector \mathcal{V}_μ , eq.(4.6), to the chiral gaugino current $\lambda^a \sigma^\mu \bar{\lambda}_a$. The vector field \mathcal{V}_μ is, in the linear formalism, the analogue of $A'_\mu{}^G$ (3.6) whose coupling to the gaugino current is induced in the chiral formulation by the gauge kinetic term.

When discussing the couplings of axionic moduli to gauge bosons it is convenient to consider the three-point correlation function of the two gauge fields $A^{a\mu}$, $A^{b\nu}$ and one modulus t^I , with momenta p_1 , p_2 and p_3 , respectively:

$$\langle A^{a\mu}(p_1) A^{b\nu}(p_2) t^I(p_3) \rangle_{CP\text{ odd}} \equiv \delta^{ab} \epsilon^{\mu\nu\rho\lambda} p_{1\rho} p_{2\lambda} \Theta_a^I(\langle z \rangle, \langle \bar{z} \rangle). \quad (4.10)$$

In order to compute Θ_a^I in the effective field theory one expands the effective Lagrangian in the fluctuations t^I of moduli scalars about their vacuum expectation values. The term linear in the fluctuations contains then the contribution

$$\frac{1}{4} \sum_I (\Theta_a^I t^I + \Theta_a^{\bar{I}} \bar{t}^{\bar{I}}) F_{\mu\nu}^a \tilde{F}_a^{\mu\nu}. \quad (4.11)$$

As we have already mentioned before, the only contribution to this amplitude from \mathcal{L}_{lin} (2.4) comes from the Green-Schwarz term (4.1) and is given by

$$\Theta_a^I|_{GS} = -\frac{i}{2} \frac{\partial V}{\partial \langle t^I \rangle}. \quad (4.12)$$

For completeness, we write down the analogous contribution from \mathcal{L}_f (1.9):

$$\Theta_a^I|_f = \frac{i}{2} \frac{\partial f_a}{\partial \langle t^I \rangle}. \quad (4.13)$$

In a supersymmetric theory

$$\Theta_a^I = i \frac{\partial \Delta_a}{\partial \langle t^I \rangle}, \quad \Theta_a^{\bar{I}} = -i \frac{\partial \Delta_a}{\partial \langle \bar{t}^{\bar{I}} \rangle}, \quad (4.14)$$

therefore the couplings of axionic moduli are related to the moduli-dependence of the effective gauge coupling constants [5]. In fact, the simplest way to determine the moduli dependence of threshold corrections to gauge couplings in superstring theory is by computing the axionic amplitudes. This allows circumventing the problem of infrared divergences present in any direct computation of Δ .

We proceed now to the discussion of higher loop corrections to the couplings of axionic moduli to gauge bosons. The effective couplings induced by field-dependent fermion connections have already been computed in the previous section. The final result has been written in the form of the Lagrangian \mathcal{L}_{1-loop} of eq.(3.11). By expanding \mathcal{L}_{1-loop} in the fluctuations of the moduli fields about their vacuum expectation values (and keeping L fixed at its VEV) we obtain:

$$\Theta_a^I|_{loops} = -\frac{i}{2}(\omega + \beta_a^I)\langle t^I + \bar{t}^{\bar{I}}\rangle^{-1}; \quad \omega = -\frac{C}{8\pi^2}, \quad \beta_a^I = \frac{b_a^I}{8\pi^2}. \quad (4.15)$$

Here we assumed zero vacuum expectation values of all fields except for the dilaton and the moduli.

As already announced in the previous section, we have excluded from the Lagrangian (3.11) possible contributions of the composite vector field A_μ^G (3.6). In the linear formalism, this is equivalent to neglecting loop corrections to the interaction term (4.7) involving the moduli-dependent vector \mathcal{V}_μ (4.6). The *r.h.s.* of eq.(4.7) contains the divergence of the chiral gaugino current $\lambda^a\sigma^\mu\bar{\lambda}_a$. This current is not conserved due to anomalous loop corrections; its divergence contains a term proportional to $F_{\mu\nu}^a\tilde{F}_a^{\mu\nu}$, hence it induces an additional coupling of moduli to gauge bosons. However in a consistent supersymmetric theory this contribution must be cancelled by loop corrections to the divergence of the topological current⁵ $\epsilon^{\mu\nu\rho\lambda}\Omega_{\nu\rho\lambda}$. The reason is that the coupling under consideration

⁵The loop corrections to this divergence have been considered before in ref. [24], and are a subject of endless controversies. This is the reason why we do not enter into the complicated issue of actual computations, restricting our remarks to the consistency requirements only.

originates from the divergence

$$\partial_\mu \mathcal{H}^\mu = -\frac{1}{4}[F_{\mu\nu}^a \tilde{F}_a^{\mu\nu} - 2\partial_\mu(\lambda^a \sigma^\mu \bar{\lambda}_a)]. \quad (4.16)$$

which receives contributions from both the topological (first term) as well as the gaugino currents. This equation is a part of the linearity condition (2.2), therefore it must not be modified by loop corrections. In other words, there should be no radiative corrections to the *r.h.s.* of eq.(4.16). The same combination of the topological and gluino current divergences couples to $\text{Im}f$ in an f -type counterterm (1.9) therefore the corresponding couplings of axionic fields should also remain equal to their classical values. This statement allows rephrasing our arguments in the chiral formulation. The coupling of $\text{Im} s$ induced by the gauge kinetic term ($f = S$) remains equal to its classical value; therefore \mathcal{L}_{1-loop} does not depend on $A'_\mu{}^G$. It is worth mentioning that if this were not the case, then the variation (4.4) $s \rightarrow s - h$ would give additional contributions to the modular anomaly which could not be cancelled by any simple counterterm, *c.f.* eq.(3.6). We conclude that in a theory with Green-Schwarz and f -type counterterms the loop corrections to moduli couplings are due entirely to the anomalous interactions involving composite fermion connections, which justifies using the Lagrangian (3.11) to derive eq.(4.15). The final result in the effective field theory is:

$$\Theta_a^I|_{eff} = \Theta_a^I|_{GS} + \Theta_a^I|_f + \Theta_a^I|_{loops}, \quad (4.17)$$

with the corresponding contributions of eqs.(4.12, 4.13, 4.15).

The three-point amplitude of eq.(4.10) was computed directly in superstring theory [6], to all orders in the higher genus expansion. The result is

$$\Theta_a^I|_{string} = -\frac{i}{2}\beta_a^I \left\{ \langle t^I + \bar{t}^I \rangle^{-1} + \frac{d \ln \eta^2(i\langle t^i \rangle)}{d\langle t^I \rangle} \right\}, \quad (4.18)$$

where η is the Dedekind eta function. We can identify now the counterterms that are necessary in the effective field theory in order to reproduce the string-theoretical amplitudes, by requiring $\Theta_a^I|_{eff} = \Theta_a^I|_{string}$, *c.f.* eqs.(4.17) and (4.18). They are, up to fluctuations of

the matter fields about the vacuum,

$$V = \omega \sum_I g^I \quad \text{and} \quad f_a = - \sum_I \beta_a^I \ln \eta^2(iT^I), \quad (4.19)$$

modulo possible redefinitions $V \rightarrow V + h + \bar{h}$, $f_a \rightarrow f_a + h$, where h is an arbitrary analytic function [*c.f.* eqs.(4.12) and (4.13)]. Such a combination of counterterms also cancels the modular anomaly, as can be verified by using the transformation property $\eta^2 \rightarrow \eta^2 e^{F^I}$ and eqs.(1.8, 1.9, 4.3).

The function $V = \omega \sum_I g^I$ corresponds to a “minimal” Green-Schwarz counterterm in the sense that any function that gives $\omega \sum_I g^I$ in the zero limit of all gauge nonsinglet fields (and twisted moduli) *and* has the same $SL(2, \mathbf{Z})$ transformation properties, provides an acceptable counterterm. A string computation of axionic vertices in the presence of non-zero backgrounds of twisted moduli and matter fields is needed in order to impose any further restrictions on V . The same comment applies to the function f . Eq.(4.19) is the main result of this section. The agreement between string theory and the effective field theory is guaranteed to all orders in loop expansion by the Adler-Bardeen theorem.

To conclude this section we comment on the axionic amplitudes in the chiral formulation of the effective theory. We consider first the case of $\beta_a^I = 0$, *i.e.* when the moduli decouple from the gauge fields, *c.f.* eq.(4.18). In the linear formulation, the kinetic energy Lagrangian (4.5) does not contain the mixing of the universal axion represented by $b_{\mu\nu}$ with the pseudoscalars belonging to chiral supermultiplets. However when the universal axion is transformed to the chiral supermultiplet S , eq.(4.5) reads

$$\mathcal{L}_{kin} = -\partial_\mu t^I \partial^\mu \bar{t}^{\bar{J}} G_{I\bar{J}} (1 - \omega l) + l' \partial_\mu y \partial^\mu \bar{y} + \dots; \quad y = s + \omega G_I t^I, \quad (4.20)$$

with t^I and s denoting the fluctuations about their VEVs; therefore $\text{Im } s$ mixes with other pseudoscalars. The asymptotic states are y and the combinations of the moduli that diagonalize the metric $G_{I\bar{J}}$ evaluated at the vacuum. These are the moduli which decouple from the gauge fields. In the case of $\beta_a^I \neq 0$, they couple with the correct normalization, up to unknown modulus wave function renormalizations of order $g^2 \sim l$.

Note that our results do not depend on the form of (and eventual radiative corrections to) the linear part $k(L)$ of the Kähler potential. The classical limit $g^2 \rightarrow 0$ dictates though $k(L) \rightarrow \ln(L)$ as $L \rightarrow 0$.

5. Effective Cut-offs and Gauge Couplings

In this section we will use the results of Section 3 to identify the effective ultra-violet cut-offs for the low-energy theory. This is possible due to the condition (3.2) which is necessary for a manifestly supersymmetric calculation of the conformal and chiral anomalies. The ultra-violet cut-offs may be different for different sectors of the theory. We begin with the determination of the cut-off for the untwisted matter sector.

The untwisted matter sector consist of three charged 27 or $\overline{27}$ matter multiplets Φ_U^α , $\alpha = (A, a)$, $A = 1, 2, 3$. The relevant part of the Kähler potential can be obtained by dimensional reduction from ten-dimensional supergravity. It corresponds to the coefficients $q_A^I = \delta_A^I$. By comparing eqs.(3.2) and (3.10) we obtain

$$\Lambda_I^2 = e^{G-2g^I+k} ; \quad I = 1, 2, 3. \quad (5.1)$$

This and the following relations are written with the accuracy of order $O(g^2)$. In the symmetric case $T^1 = T^2 = T^3 = R^2 M_S^2 + \dots$, *i.e.* when the three complex tori have equal radii R , the cut-offs (5.1) are equal

$$\Lambda_U^2 = e^{G/3+k} = R^{-2} \equiv \Lambda^2, \quad (5.2)$$

where we used the relation $M_S = gM_P$ and set $M_P = 1$ as is appropriate with the normalization condition (2.6). The compactification scale Λ is often identified with the unification scale. It was first pointed out in [25] that in the case of orbifold compactifications the two scales are different; this fact will reemerge at the end of this section where we discuss the total sum of all moduli-dependent corrections to gauge coupling constants that include contributions from the matter and gauge loops as well as from the counterterms.

The cut-offs for the twisted matter fields can be determined in exactly the same way as in the untwisted case. They depend on the weights q_A^I which are known from the results of [5]. For instance, in an $N = 1$ twisted sector, $q_A^I = q_A$ and

$$\Lambda_T^2(q_A) = e^{G(1-2q_A)+k}. \quad (5.3)$$

In general, the following property holds for all matter fields Φ^A . The ultra-violet cut-off corresponds to the Pauli-Villars regulator mass that one would obtain by supplementing the theory with the mirror fields Φ'_A , in the gauge group representations conjugate to Φ^A , and adding the superpotential

$$W_{PV} = \sum_A \Phi^A \Phi'_A. \quad (5.4)$$

Indeed,

$$m_A^2 = (K^{A\bar{A}})^2 e^K = \exp(G - 2 \sum_I q_A^I g^I + k), \quad (5.5)$$

therefore the mass m_A is equal to the cut-off as determined from eqs.(3.2) and (3.10). In fact it has been shown [26] for the simple model [13] that such a supersymmetric regularization gives consistent results for quadratically divergent one-loop correction to the full effective Lagrangian in the limit of a large number of gauge nonsinglet chiral supermultiplets.

Finally, we use eqs.(3.2) and (3.5) to obtain the ultra-violet cut-off for the Yang-Mills sector. The result is

$$\Lambda_G^2 = e^{K/3} = g^{-4/3} \Lambda^2, \quad (5.6)$$

which is in agreement with previous results [21], [14] in a sense that will be made explicit below.

We conclude by discussing the effective gauge coupling constants and their moduli-dependence. The cut-off dependent gauge and matter loops (3.11) combine with the

contributions of counterterms (4.19) to give a manifestly modular-invariant result⁶

$$\frac{1}{g_a^2} = \Delta_a = \delta_a(l) - \frac{1}{2} \sum_I \beta_a^I \ln \left[\eta^2(it^I) \overline{\eta^2(it^I)} (t + \bar{t})^I \right], \quad (5.7)$$

where

$$\delta_a(l) = - \int \frac{k'}{2l} dl - \frac{1}{16\pi^2} (C_G^a - C_M^a) k(l) - \frac{2}{16\pi^2} \sum_A C_A^a \ln(1 - p_A l). \quad (5.8)$$

Here, the *r.h.s.* are evaluated at the vacuum expectation values of the scalar fields. The effective gauge coupling constants exhibit nontrivial dependence [5, 6] on the moduli only if the orbifold contains an $N = 2$ supersymmetric twisted sector, *i.e.* when some $\beta_a^I \neq 0$. In particular, these couplings do *not* depend on the compactification scale in the case of Z_3 orbifold, in agreement with the general arguments of [25].

We close this section with a simple observation which may be important for the future discussions of the unification scales. In a supersymmetric gauge theory with massless matter, the following quantity [24] is invariant under the renormalization group transformations, at least in the two leading orders of the perturbative expansion in the renormalized coupling constants $g_a(\mu)$:

$$\delta_a[g_a(\mu), \mu] = \frac{1}{g_a^2(\mu)} - \frac{1}{16\pi^2} (3C_G^a - C_M^a) \ln \mu^2 + \frac{2C_G^a}{16\pi^2} \ln g_a^2(\mu) + \frac{2}{16\pi^2} \sum_A C_A^a \ln Z_A^a(\mu), \quad (5.9)$$

where Z_A^a are the renormalization factors for the matter fields. The “renormalized coupling constants” $g_a^{-2} = \Delta_a$ in (5.7) are scale-independent quantities, determined by the effective cut-offs and the tree couplings, that are related to the running coupling constants $g_a^{-2}(\mu)$ by additional terms involving the scale μ which serves as an infrared regulator. It is very intriguing that the moduli-independent part (5.8) of Δ_a can be written as

$$\delta_a(l) = \delta_a[g_a(\mu), \mu], \quad (5.10)$$

if we impose as a boundary condition the relations $M_s = g_a(M_s)$ in Planck mass units

⁶We write down the result for a general form of V (3.9) with $p_A \neq 0$.

$M_p = 1$. Then eq.(5.10) holds provided that we identify

$$g_a(M_S) = 2l \ , \quad Z_A^a(M_S) = (1 - p_A l)^{-1} \quad (5.11)$$

and $k(l) = \ln(l)$, which is consistent with the results of previous sections.⁷ We should mention that a similar relation between the one-loop anomalous lagrangians and the higher order renormalization group equations has been pointed out before in [21] and [14], in the case of a pure E_8 Yang-Mills sector with $C_G = C$ and $C_A = 0$. Evaluating eq.(5.10) for $a = E_8$ and $\mu = \Lambda$, *i.e.*, at the compactification scale defined in (5.2), gives

$$\frac{1}{g_{E_8}^2(\Lambda)} = \text{Re } s + \frac{C}{16\pi^2} \left\{ \ln[g_{E_8}^2(\Lambda)] - k \right\} \ , \quad (5.12)$$

which agrees with the boundary condition for the two-loop β -function found in [21] and [14].⁸ In this case, eq.(5.12) follows immediately from the g -dependence of the gaugino cut-off (5.6).

The present formalism, which incorporates the constraints of both modular invariance and results from higher genus string theory, has a natural interpretation with the string scale M_S as the scale of two-loop unification [*c.f.* eqs.(5.10, 5.11)], up to the possible additional, moduli-dependent threshold corrections in (5.7) for certain orbifolds. Determining whether or not this interpretation is fully consistent requires an understanding of the renormalization of the matter fields.

6. Conclusions

The linear formalism for the dilaton supermultiplet provides a natural framework for incorporating a Green-Schwarz counterterm that cancels the modular anomaly induced by quantum corrections in the effective low energy field theory. The modular anomaly is

⁷Note that such wave function renormalizations restore the tree-level form of the kinetic energy terms for the matter fields, *c.f.* eq.(4.5).

⁸In these references, $k = -\ln(S + \bar{S})$ was used for the dilaton Kähler potential.

related to the axial and conformal anomalies of ordinary field theory through the nontrivial transformation properties of the axial currents and the cut-offs of the effective theory under modular transformations. There are *a priori* three types of axial currents coupled to fermions in a general supergravity theory: the axial $U(1)$ current associated with Kähler invariance, the Kähler connection for matter associated with invariance under field redefinitions, and an additional axial current coupled to gauginos that arises from the noncanonical form of the gauge supermultiplet kinetic energy. We have argued that the contribution of this last current to the one-loop chiral anomaly must be cancelled by other contributions involving gauge-sector loops. The remaining axial currents are determined once the Kähler potential is specified. In effective field theories from orbifold compactifications of superstring theory the Kähler potential is known at the classical level to lowest order in fluctuations about the vacuum; we have studied the induced anomalous couplings in the same approximation.

Results from string theory imply that the fully quantum corrected theory is modular invariant to all loop orders. This implies that the effective low energy theory must be modified to include anomaly cancelling counterterms. The anomalous terms induced at one loop of the unmodified effective field theory include couplings of the Yang-Mills field to the axionic moduli; these couplings are changed once the counterterms are included, and the full $\text{Im } t F \tilde{F}$ vertex depends on the precise choice of counterterm. As discussed in the text, there are different types of counterterms that can be used to cancel the modular anomaly. We have used the results of string loop calculations of the axion-Yang-Mills vertex to determine the correct choice of counterterm. For those cases in which this coupling has been shown to vanish, implementation of anomaly cancellation is particularly straightforward within the linear formalism, because, in contrast with the chiral dilaton multiplet S , the linear dilaton multiplet L is modular invariant and unmixed with the moduli multiplets to all orders in the gauge coupling constant.

The forms of the chiral and conformal anomalies are related by supersymmetry. We used this constraint to determine the field dependence of the cut-offs of the effective low

energy theory which appear explicitly in the component field expression for the conformal anomaly. The results for matter fields coincide with the regulator masses obtained by a supersymmetric (but not modular invariant) Pauli-Villars regularization via the introduction of a bilinear superpotential for heavy regulator fields, and the result for the gauge sector agrees with results from earlier studies of effective lagrangians for gaugino condensation. Our final result for the renormalized gauge coupling constants has a simple interpretation in terms of the renormalization group invariants, with two-loop unification occurring at the string scale (up to possible additional moduli-dependent threshold effects that can be present in orbifold compactifications with an $N = 2$ twisted sector).

Our results should have interesting implications for applications of the renormalization group equations to the low energy theory, such as threshold corrections to coupling constant unification and effective lagrangians for hidden gaugino condensation.

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References

- [1] L.J. Dixon, V.S. Kaplunovsky and J. Louis, Nucl. Phys. B 329 (1990) 27.
- [2] K. Kikkawa and M. Yamasaki, Phys. Lett. 149B (1984) 357;
N. Sakai and I. Senda, Prog. Theor. Phys. 75 (1986) 692.
- [3] A. Shapere and F. Wilczek, Nucl. Phys. B 320 (1989) 669.
- [4] S. Ferrara, D. Lüst, A. Shapere and S. Theisen, Phys. Lett. B 225 (1989) 363;
S. Ferrara, D. Lüst and S. Theisen, Phys. Lett. B 233 (1989) 147.
- [5] L.J. Dixon, V.S. Kaplunovsky and J. Louis, Nucl. Phys. B 355 (1991) 649.
- [6] I. Antoniadis, K.S. Narain and T.R. Taylor, Phys. Lett. B 267 (1991) 37.
- [7] J. Louis, preprint SLAC-PUB-5527 (1991), to appear in Proceedings of the PASCOS '91 Symposium;
V. Kaplunovsky and J. Louis, to appear.
- [8] J.-P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, preprint CERN-TH.6004/91-REV, LPTENS 92-4 (1991).
- [9] G.L. Cardoso and B.A. Ovrut, University of Pennsylvania preprints UPR-0464T (1991) and UPR-0481T (1991), to be published in Proc. Strings and Symmetries 1991, Stony Brook, May 20-25, 1991.
- [10] A. Giveon, N. Malkin and E. Rabinovici, Phys. Lett. B 220 (1989) 551;
E. Alvarez and M. Osorio, Phys. Rev. D 40 (1989) 1150.
- [11] M.B. Green and J.H. Schwarz, Phys. Lett. B 149 (1984) 117.
- [12] S. Ferrara, J. Wess and B. Zumino, Phys. Lett. B 51 (1974) 239;
W. Siegel, Phys. Lett. B 85 (1979) 333.
- [13] E. Witten, Phys. Lett. 155B (1985) 151.

- [14] S. Ferrara, N. Magnoli, T.R. Taylor and G. Veneziano, Phys. Lett. B 245 (1990) 409.
- [15] S. Ferrara and M. Villasante, Phys. Lett. B 186 (1987) 85;
S. Cecotti, S. Ferrara and M. Villasante, Int. J. of Mod. Phys. A 2 (1987) 1839.
- [16] P. Binétruy, G. Girardi, R. Grimm and M. Muller, Phys. Lett. B 195 (1987) 389;
P. Binétruy, G. Girardi and R. Grimm, Phys. Lett. B 265 (1991) 111.
- [17] P. Binétruy, G. Girardi, R. Grimm and M. Müller, Phys. Lett. B 189 (1987) 83;
P. Binétruy, G. Girardi and R. Grimm, Annecy preprint LAPP-TH-275/90 (1990).
- [18] E. Cremmer, S. Ferrara, L. Girardello, and A. Van Proeyen, Nucl. Phys. B 212 (1983) 413.
- [19] T.R. Taylor, Phys. Lett. B 164 (1985) 43.
- [20] P. Binétruy and M.K. Gaillard, Phys. Lett. B 232 (1989) 83.
- [21] P. Binétruy and M.K. Gaillard, Nucl. Phys. B 358 (1991) 121.
- [22] J. Burton, M.K. Gaillard and V. Jain, Phys. Rev. D 41 (1990) 3118.
- [23] S.J. Gates, M. Grisaru, M. Roček and W. Siegel, *Superspace* (Benjamin/Cummings, 1983).
- [24] M.A. Shifman and A.I. Vainshtein, Nucl. Phys. B 277 (1986) 456.
- [25] T.R. Taylor and G. Veneziano, Phys. Lett. B 212 (1988) 147.
- [26] P. Binétruy and M.K. Gaillard, Phys. Lett. B 220 (1989) 68.