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Evaluation of observer structures with application to fault detection

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Evaluation of Observer Structures with Application to Fault Detection

A Thesis Presented by

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Abstract

As modern control systems become more and more complex and control algorithms more and more sophisticated, the issues of cost efficiency, reliability, availability, operating safety are of major importance. Since no system in practice can work perfectly all the time under various environmental changes and conditions, it is vital to be able to detect and identify the possible faults and failures in the system as early as possible, so that necessary measures can be taken to prevent significant performance degradation, damages to equipment or even lives of staff.

In the thesis, we studies the observer-based fault detection and isolation problem for linear time-invariant dynamic systems with an emphasis on robustness. After introducing some basic definitions, the problem of model-based fault detection is introduced. This is followed by a summary of different approaches in generating diagnostic residual signals. Observer-based methods are further studied. Five observer structures are discussed, including Luenberger Observer, Disturbance Observer, Unknown Input Observer, Proportional Integral Observer and Proportional Integral Fading Observer. Fault detection based on various observer structures is studied. The robustness issues are then defined in connection to fault detection. An important focus is to isolate the effect of disturbance on residual signals for robust fault detection. Illustrative examples as well as a practical one are presented to demonstrate the effectiveness of proposed robust fault detection schemes.

Chapter 1

Introduction

As modern control systems become more and more complex and control algorithms more and more sophisticated, the issues of cost efficiency, reliability, availability, operating safety are of major importance. Since no system in practice can work perfectly all the time under various environmental changes and conditions, it is vital to be able to detect and identify the possible faults and failures in the system as early as possible, so that necessary measures can be taken to prevent significant performance degradation, damages to equipment or even lives of staff. Consequently, research in fault diagnosis area has been gaining increasing consideration world-wide since the beginning of the 1970s. [1]-[4], [6]-[15], [19], [23]-[25].

A fault is defined as an unexpected change in a system, such as a component malfunction and variations in operating conditions, which tends to degrade overall system performance [1]. The purpose of fault detection is to determine whether a fault has occurred in the system, whereas fault isolation is used to determine the location of the fault, after detection. Fault detection and isolation is usually abbreviated as FDI in research community.

The most simple and frequently used method for fault detection is the limit checking of a directly measured signal $Y(t)$. Hereby, the measured signals of a process or a system are monitored and checked if their absolute values or trends exceed certain thresholds. A further possibility is to check their plausibility.

The widely studied and used fault detection method is model-based fault detection, which utilizes the mathematical model of the system to achieve detection. The model-based methodology has developed along two, so far distinct directions, namely *parameter estimation methods* and *analytical redundancy methods* [14].

The basic idea of fault detection using parameter estimation is that the system is repeatedly identified on-line and the results are compared with a reference model obtained initially under fault-free conditions. Any significant discrepancy indicates a potential fault in the system; by further analyze the discrepancy the faulty component may be isolated. Early publications on fault detection using parameter estimation methods are [4]-[7].

The basic idea behind the analytical redundancy approach is the comparison of the actual system behavior (outputs) with that expected on the basis of a reference model. It can be defined as the determination of faults of a system from the comparison of available system

measurements with a priori information represented by the system's mathematical model, through generation of residual quantities and their analysis [1]. A residual is a fault indicator or an accentuating signal which reflects the faulty situation of the monitored system. The residual is zero or almost zero when the system is normal and deviates significantly from zero when a fault occurs. Hence, the residual can be compared with a threshold for detection purposes.

Fault isolation, on the other hand, is more complicated than detection. One approach is to design a set of structured residual signals. Each residual is designed to be sensitive to a certain group of faults, whilst insensitive to others. The properties of sensitivity and insensitivity make isolation possible. Another way for isolation is to design a directional residual vector, which means to make the residual vector lie in a fixed and fault-specific direction in the residual space with respect to each fault. Then the fault isolation can be achieved by determining to which of the known fault signature directions the residual vector lies closest.

Residual generation is an essential problem in model-based fault detection and isolation. A rich variety of methods are available for residual generation, such as parity equations approach and observer-based approach.

Parity equations are input-output model equations, re-arranged and transformed to enhance their diagnostic ability. One of the early work was made by Chow and Willsky [8] who emphasized their derivation from the state-space model. Further publications have shown this method for different model structures, such as for transfer function relations developed by [9] and [10] and enhanced state-space models by [11].

The basic idea behind the observer-based approaches is to estimate the outputs of the system from the measurements by using a certain type of observer(s). Then, the (weighted) output estimation error is used as a residual. The residual is then examined for the likelihood of faults. Certain decision rules can be applied to determine if a fault has occurred.

Observer-based approaches as well as other model-based methods are able to detect faults correctly and reliably if the mathematical model is a very accurate representation of the system behavior. However, practical systems may never be free from unknown disturbances and modeling errors. Therefore it is essential to develop fault diagnosis algorithms to be robust against these effects so that fault detection can be done reliably. One of the most successful robust fault diagnosis approaches is to use the disturbance decoupling principle, in which the residual is designed to be insensitive to unknown disturbances, whilst still sensitive to faults. In the last decade, considerable attention has been drawn on the design of observers for systems with unknown inputs [3], [16]-[18]. By using these observers, a disturbance decoupled residual generator can be designed. Unknown input observer (UIO) is an estimator which is decoupled from the unknown input, such as certain disturbances, system uncertainties or faults that may be affecting the system and the measurements. This particular type of observers has been an important subject of study by researchers in fault detection and isolation (FDI) field during the last decade. Most of the existing UIO designs are based on the combination of disturbance decoupling and the classical proportional observer design. However, it has several drawbacks, such as a strict structural constraint

and existence conditions.

Another type of observers that have been widely studied in FDI area is disturbance observer (DO). The disturbance observer estimates state variables of a system augmented by a dynamic model of the disturbance signals. Hence the disturbance signals are a part of state variables of the augmented system. When the state variables of the augmented system are estimated, the disturbance signal is also estimated.

While most FDI approaches do not identify the exact shape of the fault, proportional integral observer (PIO) allows estimation of the states and faults affecting the system as well as the measurements [20]. PIOs are observers with an additional term, which is proportional to the integral of the output estimation error. The added integral term offers additional degrees of freedom for control or observer design purposes. As a matter of fact, PI observer has been used originally in loop transfer recovery (LTR) problem, in which time and frequency recoveries have been realized independently. Based on the LTR property of the PI observer, its connection to FDI problem has been recognized [21]. Because such an additional degree can be used, for instance, to decouple modeling errors or unknown disturbances.

In this paper, we propose five observer structures: Luenberger Observer, Disturbance Observer, Unknown Input Observer, Proportional Integral Observer and Proportional Integral Fading Observer as well as the relationships among these observer structures. Fault detection problem is defined and robust fault detection schemes based on DO, UIO and PIO are proposed. And simulation results are also given. The rest of the paper is organized as follows: Chapter 2 gives the system definition and the fault description. Information about different stages of fault diagnosis and different fault detection methods are also provided. Chapter 3 gives out the structures of the aforementioned five observers. Chapter 4 first considers fault detection based on the classical Luenberger observer. The issue of robustness is identified and robust fault detection and isolation schemes based on DO, UIO and PIO are discussed afterwards. Simulation results are given in Chapter 5.

Chapter 2

Fault Detection

2.1 System Definition

Consider a linear time invariant system described by

$$\dot{x} = Ax + Bu + Ed \tag{2.1}$$
$$y = Cx \tag{2.2}$$

where $x \in \mathbf{R^n}$ is the state vector; $y \in \mathbf{R^m}$ is the output vector; $u \in \mathbf{R^r}$ is the known input vector and $d \in \mathbf{R^q}$ is the unknown disturbance vector. A, B, C, and E are known matrices with appropriate dimensions.

Remarks:

(a) There is no loss of generality in assuming that the disturbance distribution matrix E is of full column rank. When this is not the case, the following rank decomposition can be applied to the matrix E, [1]:

$$Ed = E_1 E_2 d \tag{2.3}$$

where E_1 is a full column rank matrix and $E_2 d$ can now be considered as a new unknown input.

(b) The term Ed can be used to describe additive disturbance, as well as a number of different kinds of modeling uncertainties, for example, noise, interconnecting terms in large scale systems, nonlinear terms in system dynamics, terms arise from time-varying system dynamics, linearization and model reduction errors and parameter variations [1].

(c) It is a common practice to assume that E is known in robust FDI approaches. However, for real applications the following problems remain unsolved: How well can the term Ed characterize the real uncertainty? How can we determine the term E, even

approximately? A few publications have dealt with these problems. [23] used an approximation to obtain matrix E for real uncertain systems. A thermodynamic simulation model of a jet engine was utilized as an example to illustrate this approach in [23] as well. In the framework of this paper, we still assume E is known.

(d) In some cases, the system output y is also related to the control input u, i.e.

$$y = Cx + Du \tag{2.4}$$

As the control input u is known, a new output can be constructed as:

$$\bar{y} = y - Du = Cx \tag{2.5}$$

If the output y is replaced by $\bar{y}$, the problem will be equivalent to the one without the term Du.

(e) It is true that the disturbance term may also appear in the output equation, i.e.

$$y = Cx + E_y d \tag{2.6}$$

The disturbance term $E_y d$ in the output equation can be easily nulled by simply using a transformation of the output signal y, i.e.

$$\tilde{y} = T_y y = T_y Cx + T_y E_y d = T_y Cx \tag{2.7}$$

where $T_y E_y = 0$. If y and C are replaced by $\tilde{y}$ and $T_y C$, the problem will be equivalent to the one without output disturbances.

2.2 Fault Description

Model-based fault detection uses the relation among several measured variables to extract information on possible changes caused by faults. These relations are mostly analytical relations in form of system mathematical model. To detect a particular fault, the generated residual has to be sensitive to this fault. A system with possible sensor and actuator faults can be described as:

$$\dot{x} = Ax + Bu + Ed + Bf_a \tag{2.8}$$

$$y = Cx + f_s \tag{2.9}$$

where $f_a \in \mathbf{R^r}$ denotes the presence of actuator faults and $f_s \in \mathbf{R^m}$ denotes sensor faults .

2.3 Classification

The task of fault diagnosis is to detect faults and diagnose their location and significance in a system. Fault diagnosis normally consists of the following tasks:

- Fault detection: to make a binary decision – either that something has gone wrong or that everything is OK.
- Fault isolation: to determine the location of the fault, e.g., which sensor or actuator has gone wrong.
- Fault identification: to estimate the size and type or nature of the fault.

The relative importance of these three tasks is quite subjective, however the fault detection part is an absolute must for any practical system and isolation is also important. Fault identification, on the other hand, may not be crucial if no reconfiguration action is involved. Therefore, fault diagnosis is very often considered as fault detection and isolation (FDI) in the literature.

2.4 Methodologies

There are basically three types of methodologies for model-based fault detection. Parameter estimation methods compare the identified system parameters with a reference model obtained under fault-free conditions. Parity equations and observer-based methods generate a fault indicator-residual signal through analytical redundancy.

2.4.1 Parameter Estimation Methods for FDI

Mathematical system models describe the relationship between input signal u and output signals y are fundamental for model-based fault detection. However, in many cases the system models are not known at all or some parameters are unknown. Moreover, the models have to be rather precise in order to express deviations as result of system faults. Therefore, parameter estimation methods have to be applied frequently before applying any model-based fault detection method. But also the estimation method itself may be a source to gain information on, e.g. system parameters which change under the influence of faults. A considerable advantage of estimation methods is that with only one input and one output signal several parameters (up to about six) can be estimated [2], which give a detailed picture on internal system quantities. For linear systems, parameter estimation methods can generally be divided into two groups [2]: least squares and least squares unbiased with several variations in each group.

1. Least squares:
    - non-recursive

- recursive
- square root filtering (DSFI, DSFC)
- UD-factorization
- stochastic approximation

2. Least squares unbiased:

   - extended least squares
   - instrumental variables
   - maximum likelihood

2.4.2 Parity Equations Methods for FDI

Parity equations are derived directly from the input-output relationship. By a simple rearrangement, the "primary" parity equations are obtained. Transformations applied to the primary parity equations lead to parity equation sets with various isolation properties. To explain the method, an easy single-input single-output system is considered. The system is described by the transfer function

$$G_p(s) = \frac{y_p(s)}{u(s)} = \frac{B_p(s)}{A_p(s)} \tag{2.10}$$

and the reference model is given as

$$G_m(s) = \frac{y_m(s)}{u(s)} = \frac{B_m(s)}{A_m(s)} \tag{2.11}$$

The model is assumed to be known, fixed parameters, such that

$$G_p(s) = G_m(s) + \Delta G_m(s) \tag{2.12}$$

where $\Delta G_m(s)$ describes model errors.

Now suppose that there is a actuator fault $f_a(s)$ and a sensor fault $f_s(s)$ added to the actual system. The residuals r can then be constructed as the output error

$$\begin{aligned} r(s) &= y_p(s) - y_m(s) = y_p(s) - G_m(s)u(s) \\ &= G_p(s)[u(s) + f_a(s)] + f_s(s) - G_m(s)u(s) \\ &= \Delta G_m(s)u(s) + G_p(s)f_a(s) + f_s(s) \end{aligned} \tag{2.13}$$

The residual is zero for ideal matching of process and model, no actuator fault f_a and no sensor fault f_s. Usually, it shows deviations depending on the model error ΔG_m and the exciting input signal u. Eq.(2.13) is *parity equation*, and r is called *primary residual* [2].

The design of the residuals based on the state-space model of a linear multi-input multi-output system can be found in [8] as well as in [12].

2.4.3 Observer-Based Methods for FDI

Observer-based methods have been most widely considered for residual generation. State observers adjust the state variables according to initial conditions and to the time course of the measured input and output signals. As state observers use an output error between a measured system output and an adjustable model output, they are a further alternative for model-based fault detection. In Chapter 3, several observer structures are proposed and evaluated.

Chapter 3

Observer Structures

3.1 Luenberger Observer

A linear time invariant system without disturbance is described by

$$\dot{x} = Ax + Bu \tag{3.1}$$

$$y = Cx \tag{3.2}$$

where $x \in \mathbf{R^n}$ is the state vector; $y \in \mathbf{R^m}$ is the output vector and $u \in \mathbf{R^r}$ is the known input vector. A, B and C are known matrices with appropriate dimensions and it is assumed that the system is observable.

With the structure and the parameters of the system model are known, a state observer is used to reconstruct the unmeasurable state variables based on measured inputs and outputs

$$\dot{\hat{x}} = A\hat{x} + L(y - C\hat{x}) + Bu \tag{3.3}$$

where $\hat{x} \in \mathbf{R^n}$ is the estimated state vector and $L \in \mathbf{R^{n \times m}}$ is the observer gain. When the observer (3.3) is applied to the system (3.1) and (3.2), the estimation error ($e = x - \hat{x}$) is governed by the equation:

$$\dot{e} = (A - LC)e \tag{3.4}$$

If all eigenvalues of $A - LC$ are stable, e will approach zero asymptotically, e.g., $\hat{x} \longrightarrow x$. It can be reached by proper design of the observer gain matrix L, e.g. by pole placement.

If the disturbance is added to the system (3.1) and (3.2) as in (2.1) and (2.2), then the Luenberger observer is not capable of estimating the states correctly unless certain restriction is applied to the norm of the disturbance signal.

3.2 Disturbance Observer (DO)

First consider a system with constant disturbance

$$\dot{x} = Ax + Bu + Ed \tag{3.5}$$
$$\dot{d} = 0 \tag{3.6}$$
$$y = Cx \tag{3.7}$$

The system (3.5), (3.6) and (3.7) can be combined into an augmented system

$$\begin{bmatrix} \dot{x} \\ \dot{d} \end{bmatrix} = \begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ d \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u \tag{3.8}$$
$$y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ d \end{bmatrix} \tag{3.9}$$

If we denote

$$z = \begin{bmatrix} x \\ d \end{bmatrix}, \quad A_D = \begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix}, \quad B_D = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad C_D = \begin{bmatrix} C & 0 \end{bmatrix} \tag{3.10}$$

then (3.8) and (3.9) become

$$\dot{z} = A_D z + B_D u \tag{3.11}$$
$$y = C_D z \tag{3.12}$$

A full-order observer can be constructed as:

$$\dot{\hat{z}} = (A_D - L_D C_D)\hat{z} + L_D y + B_D u \tag{3.13}$$

Where L_D is the gain of the observer. By proper design of the gain L_D, (3.13) can estimate the state x and the disturbance d of the original system (3.5), (3.6) and (3.7).

Now consider consider a system with a disturbance model $\dot{d} = Md$, where matrix M with proper dimension describes the dynamics of the disturbance.

$$\dot{x} = Ax + Bu + Ed \tag{3.14}$$
$$\dot{d} = Md \tag{3.15}$$
$$y = Cx \tag{3.16}$$

Similarly, the system (3.14), (3.15) and (3.16) can be combined into an augmented system

$$\begin{bmatrix} \dot{x} \\ \dot{d} \end{bmatrix} = \begin{bmatrix} A & E \\ 0 & M \end{bmatrix} \begin{bmatrix} x \\ d \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u \tag{3.17}$$
$$y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ d \end{bmatrix} \tag{3.18}$$

If we again denote

$$z = \begin{bmatrix} x \\ d \end{bmatrix}, \quad A_{DM} = \begin{bmatrix} A & E \\ 0 & M \end{bmatrix}, \quad B_D = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad C_D = \begin{bmatrix} C & 0 \end{bmatrix} \tag{3.19}$$

then (3.17) and (3.18) become

$$\dot{z} = A_{DM} z + B_D u \tag{3.20}$$

$$y = C_D z \tag{3.21}$$

A full-order observer can be constructed as:

$$\dot{\hat{z}} = (A_{DM} - L_{DM} C_D)\hat{z} + L_{DM} y + B_D u \tag{3.22}$$

Where L_{DM} is the gain of the observer. By proper design of the gain L_{DM}, (3.22) can estimate the state x and the disturbance d of the original system (3.14), (3.15) and (3.16).

3.3 Unknown Input Observer (UIO)

3.3.1 Structure of UIO

Direct Derivation

From the system output equation (2.2), one can obtain

$$\dot{y} = C\dot{x} = CAx + CBu + CEd \tag{3.23}$$

A full-order observer is given as:

$$\dot{\hat{x}} = A\hat{x} + H(\dot{y} - CBu - CA\hat{x}) + K_1(y - C\hat{x}) + Bu \tag{3.24}$$

Let

$$z = \hat{x} - Hy \tag{3.25}$$

The observer can be written as:

$$\dot{z} = (A - HCA - K_1 C)z + [K_1 + (A - HCA - K_1 C)H]y + (I - HC)Bu \tag{3.26}$$

where $\hat{x} \in \mathbf{R^n}$ is the estimated state vector and $z \in \mathbf{R^n}$ is the state of this full-order observer, and H, K_1 are matrices to be designed for achieving unknown input de-coupling and other design requirements. Note that this observer reduces to a conventional full-order observer when $H = 0$.
Furthermore, if let

$$F = A - HCA - K_1 C \tag{3.27}$$

$$K_2 = FH \tag{3.28}$$

$$K = K_1 + K_2 \tag{3.29}$$

$$T = I - HC \tag{3.30}$$

(3.26) can be rewritten as:

$$\dot{z} = Fz + Ky + TBu \tag{3.31}$$

When (3.25) and the observer (3.31) are applied to the system (2.1) and (2.2), the estimation error ($e = x - \hat{x}$) is governed by the equation:

$$\begin{aligned}\dot{e} &= (A - HCA - K_1C)e + (A - HCA - K_1C - F)z \\ &+ [(A - HCA - K_1C)H - K_2]y + [(I - HC) - T]Bu + (I - HC)Ed\end{aligned} \tag{3.32}$$

Which leads to

$$\dot{e} = Fe + (I - HC)Ed \tag{3.33}$$

If the following relation is true:

$$(I - HC)E = 0 \tag{3.34}$$

The state estimation error will then be:

$$\dot{e} = Fe \tag{3.35}$$

If all eigenvalues of F are stable, e will approach zero asymptotically, e.g., $\hat{x} \longrightarrow x$. This means that the observer (3.26) is an unknown input observer for the system (2.1) and (2.2). The design of the UIO is to solve Eq.(3.34) and make all eigenvalues of the system matrix F stable.

Indirect Derivation

The structure for a full-order observer can be described as:

$$\begin{cases} \dot{z} &= Fz + Ky + TBu \\ \hat{x} &= z + Hy \end{cases} \tag{3.36}$$

where $\hat{x} \in \mathbf{R^n}$ is the estimated state vector and $z \in \mathbf{R^n}$ is the state of this full-order observer, and F, K, T, H are matrices to be designed for achieving unknown input decoupling and other design requirements. When the observer (3.36) is applied to the system (2.1) and (2.2), the estimation error e can be computed as:

$$\begin{aligned}\dot{e} &= (A - HCA - K_1C)e + [F - (A - HCA - K_1C)]z \\ &+ [K_2 - (A - HCA - K_1C)H]y + [T - (I - HC)]Bu + (HC - I)Ed\end{aligned} \tag{3.37}$$

where

$$K = K_1 + K_2 \tag{3.38}$$

If the following relations can be hold true:

$$(HC - I)E = 0 \tag{3.39}$$

$$T = I - HC \tag{3.40}$$

$$F = A - HCA - K_1C \tag{3.41}$$

$$K_2 = FH \tag{3.42}$$

The state estimation error will then be:

$$\dot{e} = Fe \tag{3.43}$$

If all eigenvalues of F are stable, e will also approach zero asymptotically. This means that the observer (3.36) is an unknown input observer for the system (2.1) and (2.2). Compare observers derived from two methods, we find these two observers have the same structure. In addition, Eq.(3.34) and Eq.(3.39) are essentially the same.

3.3.2 Existence Condition

Lemma 3.3.1. *Eq.*(3.34) *or Eq.*(3.39) *is solvable iff:*

$$rank(CE) = rank(E) \tag{3.44}$$

Proof: *Necessity*, When Eq.(3.34) or Eq.(3.39) has a solution H, it has $HCE = E$ or $(CE)^T H^T = E^T$. E^T belongs to the range space of the matrix $(CE)^T$ and this leads to:

$$rank(E^T) \leq rank((CE)^T)$$

or

$$rank(E) \leq rank(CE)$$

However,

$$rank(CE) \leq \min\{rank(C), rank(E)\} \leq rank(E)$$

Suppose the number of measurements is greater or equal than the number of disturbances. Therefore, $rank(CE) = rank(E)$ and the necessary condition is proved.

Sufficiency: When $rank(CE) = rank(E)$ is true, CE is a full column rank matrix (because E is assumed to be of full column rank), and a left inverse of CE exists:

$$(CE)^+ = [(CE)^T CE]^{-1}(CE)^T \tag{3.45}$$

Clearly, $H = E(CE)^+$ is a solution to Eq.(3.34) or Eq.(3.39).

3.3.3 Design Procedure

A direct design procedure for UIO is to utilize (3.45) and substitute $H = E(CE)^+$ into (3.41). Denote $A - HCA$ as A_1. Then any eigenvalue assignment methods can be applied to stabilize $F = A_1 - K_1 C$.

A reliable algorithm which is designed from computational aspect is introduced as follows. Notice Eq.(3.34) or Eq.(3.39) is equivalent to $(CE)^T H^T = E^T$. First, find the singular value decomposition (SVD) of CE, e.g.

$$U_1(CE){V_1}^T = \Sigma = \left[\begin{array}{ccc|c} \sigma_1 & \dots & 0 & \\ \vdots & \ddots & \vdots & 0 \\ 0 & \dots & \sigma_q & \\ \hline & 0 & & 0 \end{array}\right] \tag{3.46}$$

where $U_1 \in \mathbf{R^{m \times m}}$, $V_1 \in \mathbf{R^{q \times q}}$, $q < m$ and $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_q > 0$.
As CE is given by

$$CE = {U_1}^T \Sigma V_1 \tag{3.47}$$

After substituting these quantities in Eq.(3.34) or Eq.(3.39) and using the orthogonality property of V_1 matrix, we have

$$\Sigma U_1 H^T = V_1 E^T \tag{3.48}$$

Now define

$$U_1 H^T = \widetilde{H} \tag{3.49}$$

$$V_1 E^T = \widetilde{E} \tag{3.50}$$

Eq.(3.48) can be rewritten as

$$\Sigma \widetilde{H} = \widetilde{E} \tag{3.51}$$

Note that H can be recovered from $\widetilde{H}$ by first premultiplying by ${U_1}^T$, and then transpose the product. This yields

$$H = ({U_1}^T \widetilde{H})^T \tag{3.52}$$

Substituting (3.46) into (3.51), we immediately see that $\widetilde{H}$ has the following structure

$$\widetilde{H} = \left[\begin{array}{c} \square \\ \hline * \end{array}\right] \tag{3.53}$$

where $\square$ is an $q \times n$ block whose elements are completely specified by the parameters of the constraint Eq.(3.34) or Eq.(3.39), e.g.

$$(\widetilde{H})_{ij} = \frac{(\widetilde{E})_{ij}}{\sigma_i}, \quad i = 1, \ldots, q \tag{3.54}$$

On the other hand, the elements of the $*$ block can be chosen freely.
Now let us consider Eq.(3.27). Note it can be written in the augmented form

$$F = A - \widetilde{K}\widetilde{C} \tag{3.55}$$

where

$$\widetilde{K} = \begin{bmatrix} H & K_1 \end{bmatrix} \qquad \widetilde{C} = \begin{bmatrix} CA \\ C \end{bmatrix} \tag{3.56}$$

In the above augmented form, any eigenvalue assignment method can be used to obtain the gain $\widetilde{K}$ by making all eigenvalues of the matrix F stable.

3.4 Proportional Integral Observer (PIO)

A PI observer for the system (2.1) and (2.2) is defined as follows

$$\dot{\hat{x}} = A\hat{x} + K_P(y - C\hat{x}) + Bu + E\hat{d} \tag{3.57}$$

$$\dot{\hat{d}} = K_I(y - C\hat{x}) \tag{3.58}$$

where K_P and K_I are proportional and integral gains respectively. Note that the PI observer reduces to a conventional full-order observer when $K_I = 0$. The above general setting allows one to write (3.57) and (3.58) in the following augmented form

$$\dot{\hat{z}} = A_Z\hat{z} + B_Z u + K_Z y \tag{3.59}$$

where

$$\hat{z} = \begin{bmatrix} \hat{x} \\ \hat{d} \end{bmatrix}, A_Z = \begin{bmatrix} A - K_P C & E \\ -K_I C & 0 \end{bmatrix}, B_Z = \begin{bmatrix} B \\ 0 \end{bmatrix}, K_Z = \begin{bmatrix} K_P \\ K_I \end{bmatrix} \tag{3.60}$$

In the above augmented form, any eigenvalue assignment method can be applied to obtain the gain K_Z from $A_Z = A_X - K_Z C_Z$ by making all eigenvalues of the matrix A_Z stable. Where

$$A_X = \begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix} \qquad C_Z = \begin{bmatrix} C & 0 \end{bmatrix} \tag{3.61}$$

When the PIO (3.57) and (3.58) is applied to the system described in (2.1) and (2.2), the state estimation error ($e = x - \hat{x}$) and the disturbance estimation error ($\varepsilon = d - \hat{d}$) will be (suppose disturbance d is constant $\dot{d} = 0$):

$$\begin{bmatrix} \dot{e} \\ \dot{\varepsilon} \end{bmatrix} = \begin{bmatrix} A - K_P C & E \\ -K_I C & 0 \end{bmatrix} \begin{bmatrix} e \\ \varepsilon \end{bmatrix} \tag{3.62}$$

Let

$$\xi = \begin{bmatrix} e \\ \varepsilon \end{bmatrix} \tag{3.63}$$

(3.62) can be rewritten as

$$\dot{\xi} = A_Z \xi \tag{3.64}$$

If all eigenvalues of A_Z are stable, e will approach zero asymptotically, e.g. $\hat{x} \longrightarrow x$. This means that the observer (3.59) is a PI observer for the system (2.1) and (2.2). The design of the PIO is to make all eigenvalues of the matrix A_Z stable.

3.5 Proportional Integral Fading Observer (PIF)

Suppose the disturbance model is known as $\dot{d} = Md$. Then one can use the following modified PI observer (PIF) to achieve both states and disturbance estimation.

$$\dot{\hat{x}} = A\hat{x} + K_P(y - C\hat{x}) + Bu + N\hat{d} \tag{3.65}$$

$$\dot{\hat{d}} = K_I(y - C\hat{x}) + M\hat{d} \tag{3.66}$$

Where N is an arbitrary matrix with proper dimension and M describes the dynamics of the disturbance ($\dot{d} = Md$). The above general setting allows one to write (3.65) and (3.66) into the following augmented format

$$\dot{\hat{z}} = A_{ZF}z + B_Z u + K_Z y \tag{3.67}$$

where

$$\hat{z} = \begin{bmatrix} \hat{x} \\ \hat{d} \end{bmatrix}, A_{ZF} = \begin{bmatrix} A - K_P C & N \\ -K_I C & M \end{bmatrix}, B_Z = \begin{bmatrix} B \\ 0 \end{bmatrix}, K_Z = \begin{bmatrix} K_P \\ K_I \end{bmatrix} \tag{3.68}$$

In the above formulation the parameter N is chosen freely first and any eigenvalue assignment method can then be applied to obtain the gain K_Z from $A_{ZF} = A_{XF} - K_Z C_Z$ where

$$A_{XF} = \begin{bmatrix} A & N \\ 0 & M \end{bmatrix} \qquad C_Z = \begin{bmatrix} C & 0 \end{bmatrix} \tag{3.69}$$

3.6 Comparisons of Different Observer Structures

There are interesting connections among the aforementioned observer structures. The disturbance observer (DO) is essentially a Luenberger full-state observer for an augmented system, the original system together with the disturbance model.

On the other hand, the PI observer is a special case of the PIF observer if $M = 0$ and $N = E$. The fading of the integral term discounts the integral term over time. Decaying the integral term allows the rejection of transient that would otherwise cause large offsets in the estimates of the disturbance and plant perturbation. Under certain conditions, the new structure makes it possible to handle time varying disturbances (e.g. $\dot{d} = Md$) with the additional advantage of estimating directly the disturbance due to its special structure.

The PI observer also has a direct relationship with disturbance observer (DO). The disturbance observer (3.13) is equivalent to

$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{d}} \end{bmatrix} = \left(\begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \begin{bmatrix} C & 0 \end{bmatrix} \right) \begin{bmatrix} \hat{x} \\ \hat{d} \end{bmatrix} + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} y + \begin{bmatrix} B \\ 0 \end{bmatrix} u \tag{3.70}$$

Where

$$L_D = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \tag{3.71}$$

While using (3.60) and (3.61), PIO (3.59) can be rewritten as:

$$\begin{bmatrix} \dot{\hat{x}} \\ \dot{\hat{d}} \end{bmatrix} = \left(\begin{bmatrix} A & E \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} K_P \\ K_I \end{bmatrix} \begin{bmatrix} C & 0 \end{bmatrix} \right) \begin{bmatrix} \hat{x} \\ \hat{d} \end{bmatrix} + \begin{bmatrix} K_P \\ K_I \end{bmatrix} y + \begin{bmatrix} B \\ 0 \end{bmatrix} u \tag{3.72}$$

Clearly, the disturbance observer (DO) is the same as the PIO with constant disturbance ($\dot{d} = 0$) and the gains of two observers are related:

$$L_1 = K_P \tag{3.73}$$

$$L_2 = K_I \tag{3.74}$$

However, the structure of PI observer is a priori defined and the augmented state estimates are obtained directly, including the estimates of both the system states x and the disturbance d. The DO is only valid under the condition that the disturbance is constant. Therefore, DO is a special type of PIO.

As for UIO, disturbance is decoupled in the estimation of the states, so that the states estimates can be obtained directly. However, the disturbance estimate has to be attained afterwards. From (3.23) and *Lemma* 3.3.1, the disturbance estimate $\hat{d}$ can be obtained:

$$\hat{d} = (CE)^{+}[\dot{\hat{y}} - CA\hat{x} - CBu] \tag{3.75}$$

Chapter 4

Fault Detection Using Observers

4.1 Fault Detection

A linear time invariant system without disturbance, but with actuator faults f_a and sensor faults f_s is considered:

$$\dot{x} = Ax + Bu + Bf_a \tag{4.1}$$
$$y = Cx + f_s \tag{4.2}$$

where, as described in Section 2.2, $f_a \in \mathbf{R^r}$ denotes the presence of actuator faults and $f_s \in \mathbf{R^m}$ denotes sensor faults. When the state estimation is available, the residual r can be generated as:

$$r = y - C\hat{x} \tag{4.3}$$

When a Luenberger observer based residual generator is applied to the system described in (4.1) and (4.2), the residual and the state estimation error ($e = x - \hat{x}$) will be:

$$\dot{e} = (A - LC)e + Bf_a - Lf_s \tag{4.4}$$
$$r = Ce + f_s \tag{4.5}$$

Clearly, the residual is sensitive to both actuator faults and sensor faults, which means that both actuator faults and sensor faults can be successfully detected. A simple threshold logic can be applied:

$$\begin{cases} \|r\| < \text{Threshold} & \text{for fault-free case} \\ \|r\| \geq \text{Threshold} & \text{for faulty case} \end{cases} \tag{4.6}$$

However, when the system is corrupted with disturbance, as described in (2.8) and (2.9), Luenberger observer is no longer appropriate for robust fault detection application. As we apply Luenberger observer based residual generator to (2.8) and (2.9), the state estimation error and residual will be:

$$\dot{e} = (A - LC)e + Ed + Bf_a - Lf_s \tag{4.7}$$
$$r = Ce + f_s \tag{4.8}$$

Though the residual is still sensitive to both actuator faults and sensor faults, it is sensitive to disturbance term Ed as well, which means the residual is unable to differentiate between faults and disturbance.

As stated in Chapter 1, robustness is essential in fault detection application, since practical systems may never be free from unknown disturbances and modeling errors. The generated residual should be robust against the system uncertainty and disturbance whilst still sensitive to the faults that to be detected. The following section considers robust fault detection using other types of observers.

4.2 Robust Fault Detection and Isolation Schemes

4.2.1 Robust Fault Detection Schemes Based on DOs

For simplicity, let us assume the disturbance is constant $\dot{d} = 0$. When a DO-based residual generator is applied to the system described in (2.8) and (2.9), the residual, the state estimation error ($e = x - \hat{x}$) and the disturbance estimation error ($\varepsilon = d - \hat{d}$) will be:

$$\begin{bmatrix} \dot{e} \\ \dot{\varepsilon} \end{bmatrix} = \begin{bmatrix} A - L_1 C & E \\ -L_2 C & 0 \end{bmatrix} \begin{bmatrix} e \\ \varepsilon \end{bmatrix} + B_D f_a - L_D f_s \tag{4.9}$$

$$r = C_D \begin{bmatrix} e \\ \varepsilon \end{bmatrix} + f_s \tag{4.10}$$

Let

$$\xi = \begin{bmatrix} e \\ \varepsilon \end{bmatrix} \quad A_F = \begin{bmatrix} A - L_1 C & E \\ -L_2 C & 0 \end{bmatrix} \tag{4.11}$$

We have

$$\dot{\xi} = A_F \xi + B_D f_a - L_D f_s \tag{4.12}$$

$$r = C_D \xi + f_s \tag{4.13}$$

From (4.12) and (4.13), it is clear that the disturbance effects have been de-coupled from the residual. The faults can be detected by comparing the robust residual with a threshold (4.6).

4.2.2 Robust Fault Detection Schemes Based on UIOs

When a UIO is used, the residual can be further described as:

$$r = (I - CH)y - Cz \tag{4.14}$$

If this UIO-based residual generator is applied to the faulty system (2.8) and (2.9), the residual and the state estimation error ($e = x - \hat{x}$) will be:

$$\dot{e} = Fe + (I - HC)Bf_a - K_1 f_s - H\dot{f}_s \tag{4.15}$$

$$r = Ce + f_s \tag{4.16}$$

From and (4.16), it shows that the disturbance d has been de-coupled from the residual. To detect actuator faults, one has to make:

$$(I - HC)B \neq 0 \tag{4.17}$$

Similarly, the residual has to be made sensitive to f_s if sensor faults are to be detected. This condition is normally satisfied, as the sensor fault vector f_s has a direct effect on the residual r. The faults can also be detected by comparing the robust residual with a simple threshold (4.6).

4.2.3 Robust Fault Detection Schemes Based on PIOs

When a PIO-based residual generator applied to the system described in (2.8) and (2.9), the residual, the state estimation error ($e = x - \hat{x}$) and the disturbance estimation error ($\varepsilon = d - \hat{d}$) will be (suppose disturbance d is constant $\dot{d} = 0$):

$$\begin{bmatrix} \dot{e} \\ \dot{\varepsilon} \end{bmatrix} = \begin{bmatrix} A - K_P C & E \\ -K_I C & 0 \end{bmatrix} \begin{bmatrix} e \\ \varepsilon \end{bmatrix} + \begin{bmatrix} -K_P \\ -K_I \end{bmatrix} f_s + \begin{bmatrix} B \\ 0 \end{bmatrix} f_a \tag{4.18}$$

$$r = Ce + f_s \tag{4.19}$$

Similarly, let

$$\xi = \begin{bmatrix} e \\ \varepsilon \end{bmatrix} \tag{4.20}$$

We have

$$\dot{\xi} = A_Z \xi - K_Z f_s + B_Z f_a \tag{4.21}$$

$$r = C_Z \xi + f_s \tag{4.22}$$

From (4.21) and (4.22), it can be seen that the disturbance effects have also been de-coupled from the residual. And the residual is sensitive to either actuator faults or sensor faults. Again, the faults can be detected by comparing the robust residual with a threshold (4.6).

4.2.4 Robust Fault Isolation Schemes

The fault isolation problem is to locate the fault. For example, to determine in which sensor (or actuator) the fault has occurred. One approach, as pointed out in the Introduction, is to design a *structured* residual set. Each residual in the set is formed through an observer, i.e. using a bank of observers. Examples of such banks of observers are the *dedicated observer scheme* (DOS) and the *generalized observe scheme* (GOS) [24]. The term "structured" means that each residual is designed to be sensitive to a particular group of faults while insensitive to others. The sensitivity and insensitivity of a set of such residuals make isolation possible. However, this ideal situation is usually difficult to achieve. Even when the ideal situation can be obtained, the design freedom will be used up and no freedom will be left for robustness

purposes. In order to exploit the maximum design freedom for robustness, a commonly accepted scheme in fault isolation is to make each residual to be sensitive to faults in all but one sensors (or actuators).

Chapter 5

Simulation Results

5.1 An Illustrative Example

5.1.1 Illustration of Observers

A simple linear time invariant system is provided to illustrate DO, UIO, PIO and PIF in states and disturbance estimation.

$$\dot{x} = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} d \tag{5.1}$$

$$y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} x \tag{5.2}$$

First DO, UIO and PIO are tested under a constant disturbance. Input u, system initial states $x(0)$ and disturbance d are

$$u = 1 \qquad x(0) = \begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \end{bmatrix} \qquad d = 0.5$$

The initial states of all observers are set to be 0.

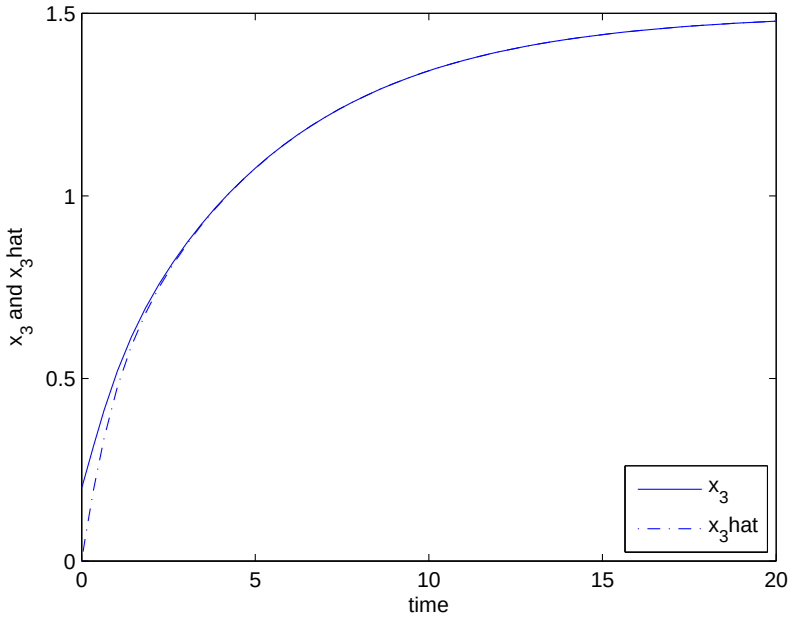

Figure 5.1: DO x_3 estimation

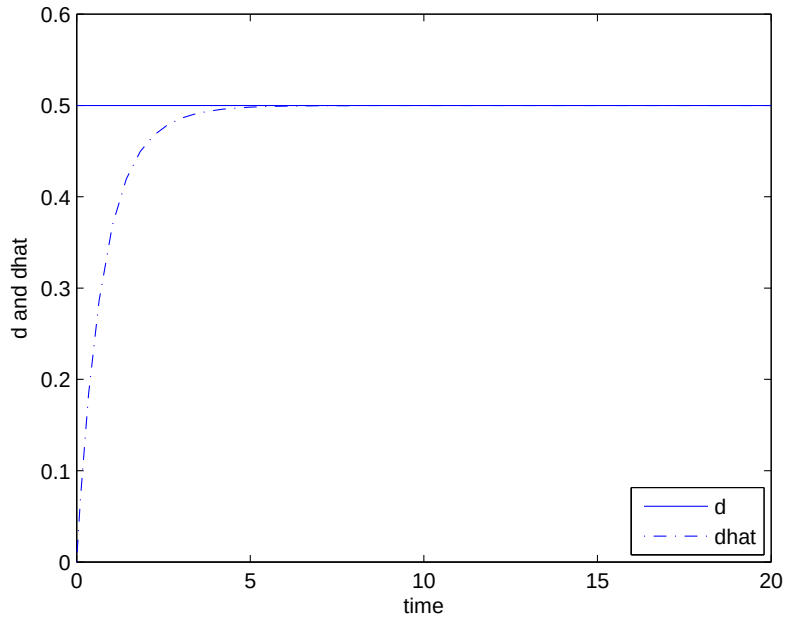

Figure 5.2: DO disturbance d estimation

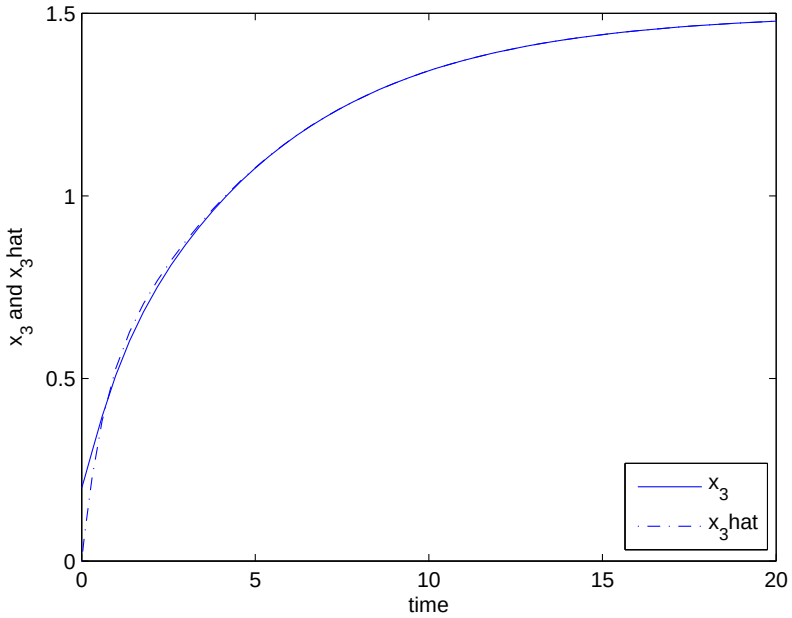

Figure 5.3: UIO x_3 estimation

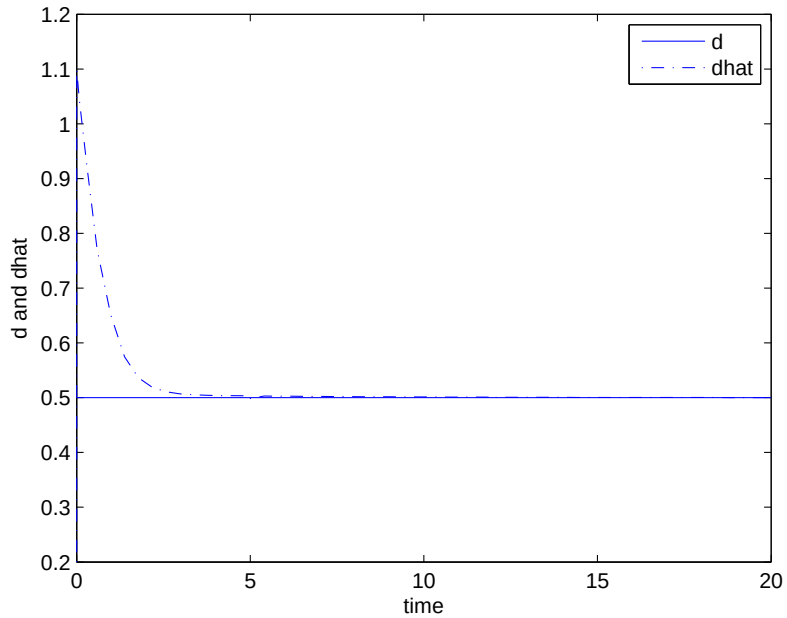

Figure 5.4: UIO disturbance d estimation

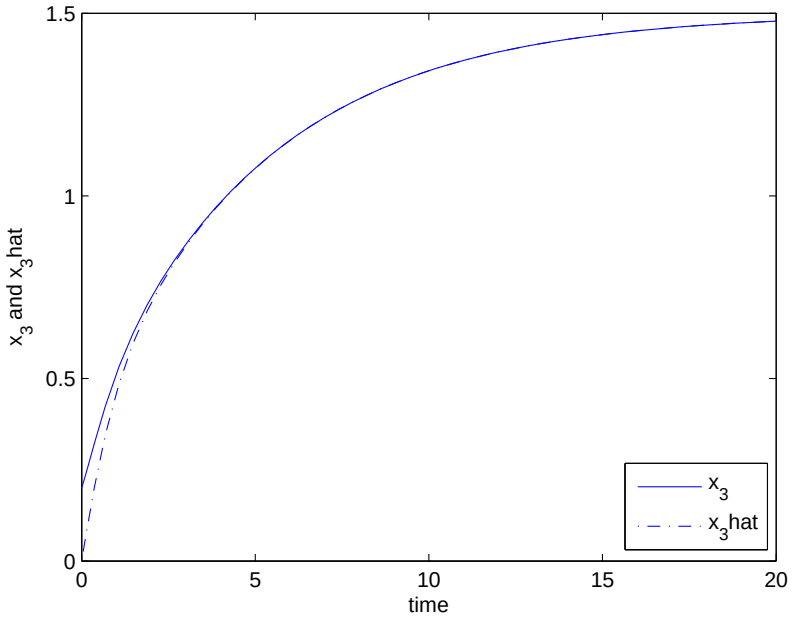

Figure 5.5: PIO x_3 estimation

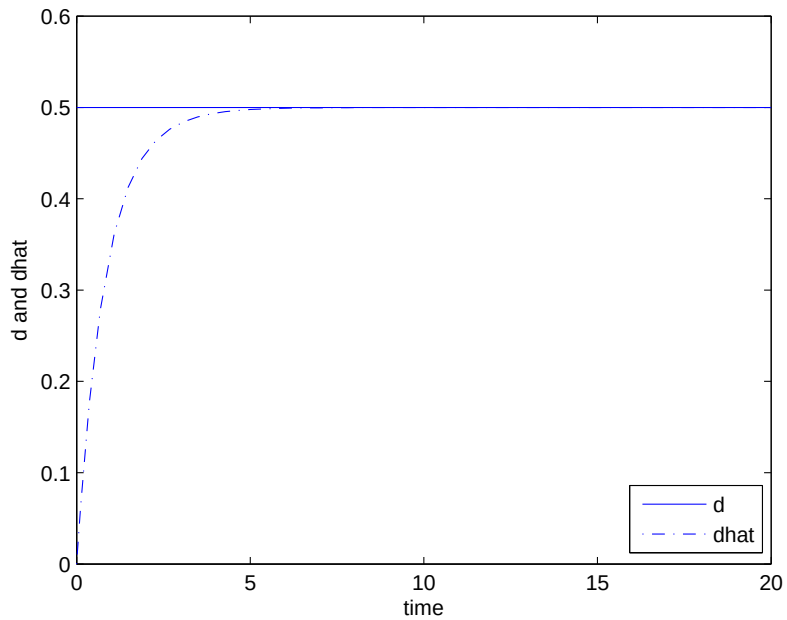

Figure 5.6: PIO disturbance d estimation

Then PIF is used under a time-varying disturbance. Input u, system initial states $x(0)$

and disturbance d are

$$u = 1 \qquad x(0) = \begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \end{bmatrix} \qquad d(t) = e^{-t} \text{ with } d(0) = 0.5 \ (\dot{d} = -d)$$

The initial states of PIF are also set to be 0.

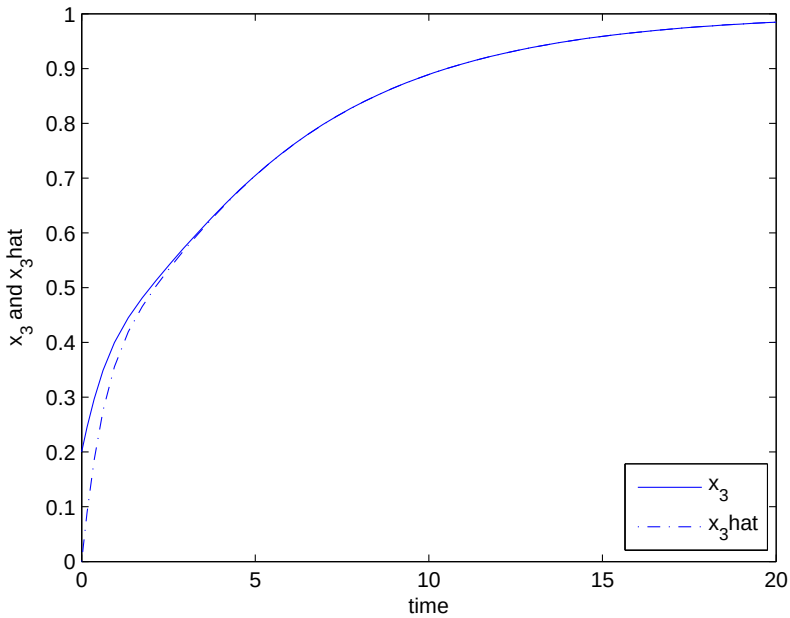

Figure 5.7: PIF x_3 estimation

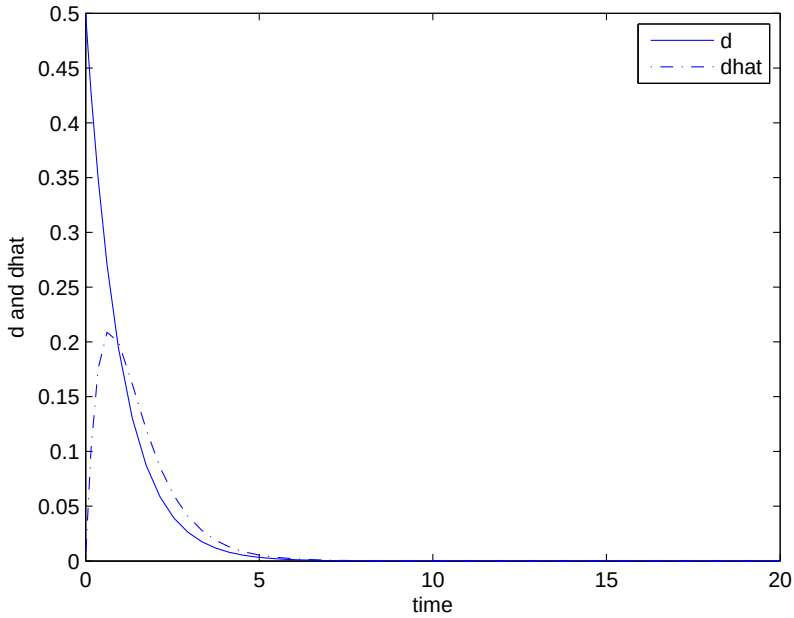

Figure 5.8: PIF disturbance d estimation

The results are shown in Figure 5.1 to 5.8. All observers are able to estimate states and disturbance successfully.

5.1.2 Observer-Based Robust Fault Detection and Isolation

The above system examples (5.1) and (5.2) are used in order to illustrate the applicability of DO, UIO and PIO in robust FDI.

Several types of faults are introduced to the system at $t = 5$. The list of the simulation is:

(a) No fault occurs. Only the disturbance is introduced to the system.

(b) An actuator fault occurs when $t > 5$.

(c) A sensor fault occurs in y_1 when $t > 5$.

Similarly, input u, system initial states $x(0)$ and disturbance d are

$$u = 1 \qquad x(0) = \begin{bmatrix} 0.2 \\ 0.2 \\ 0.2 \end{bmatrix} \qquad d = 0.5$$

The initial states of all observers are set to be 0.

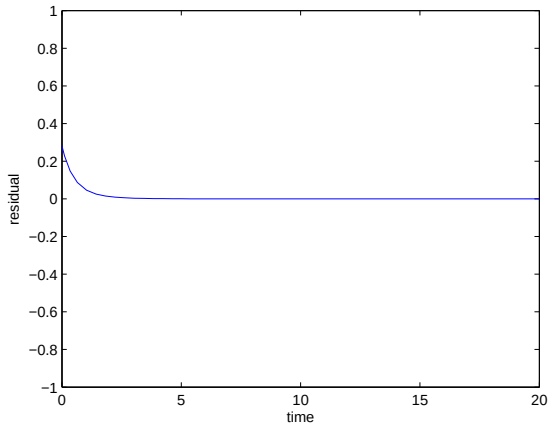

Figure 5.9: DO residual when there is only the disturbance

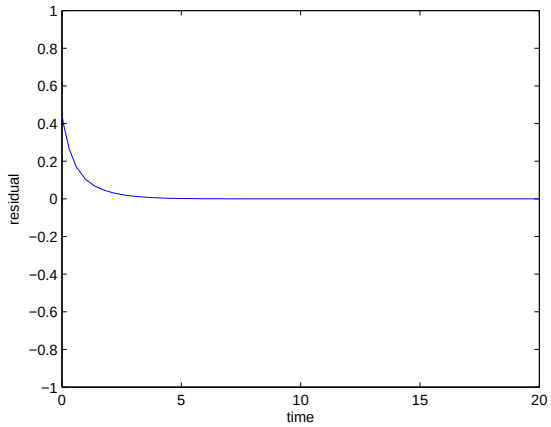

Figure 5.10: UIO residual when there is only the disturbance

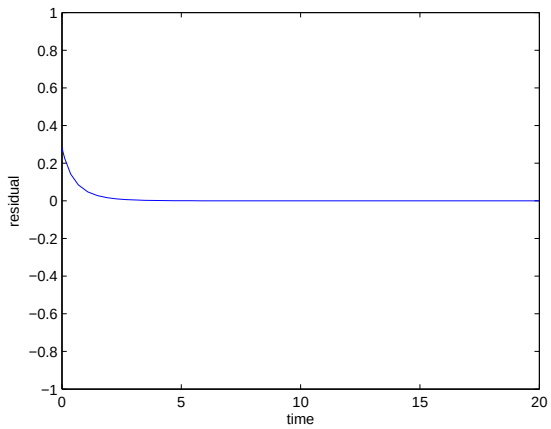

Figure 5.11: PIO residual when there is only the disturbance

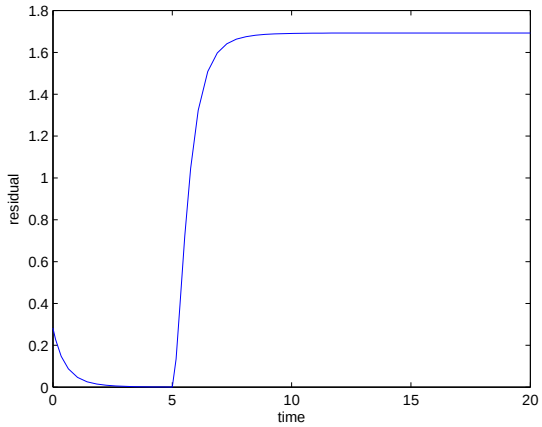

Figure 5.12: DO residual when an actuator fault occurs at $t = 5$

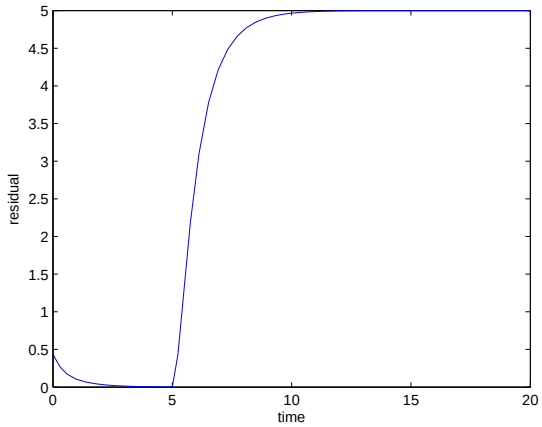

Figure 5.13: UIO residual when an actuator fault occurs at $t = 5$

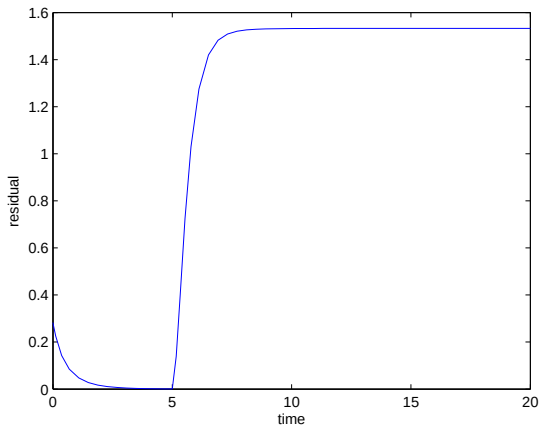

Figure 5.14: PIO residual when an actuator fault occurs at $t = 5$

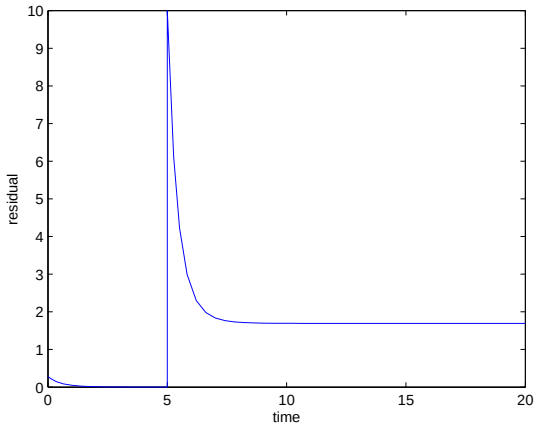

Figure 5.15: DO residual when a sensor fault occurs in y_1 at $t = 5$

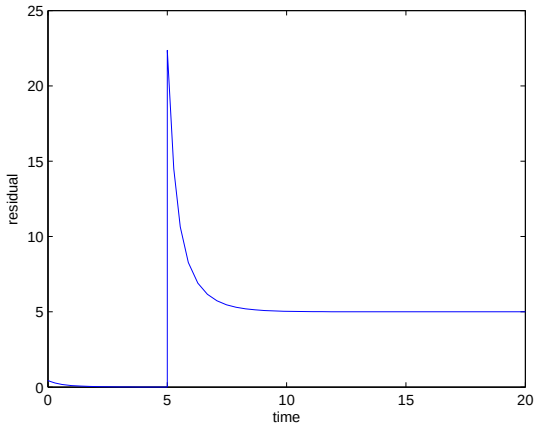

Figure 5.16: UIO residual when a sensor fault occurs in y_1 at $t = 5$

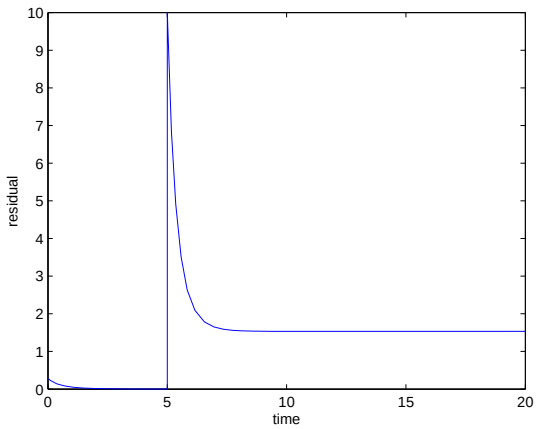

Figure 5.17: PIO residual when a sensor fault occurs in y_1 at $t = 5$

The results are shown in Figure 5.9 to 5.17. All three observers are able to properly estimate the system states under the presence of external disturbance as well as the disturbance itself. The residuals are zero when there is no fault, despite the disturbance, and rise in magnitude considerably when faults are introduced at $t = 5$.

Furthermore, a sensor fault isolation scheme can be used to determine in which sensor the fault has occurred. PIO case is used as an example here. Two PIOs are designed. The first PIO is driven by y_1 and the second PIO is driven by y_2. Both PIOs are robust to the disturbance d. DO-based and UIO-based fault isolation schemes can be designed similarly.

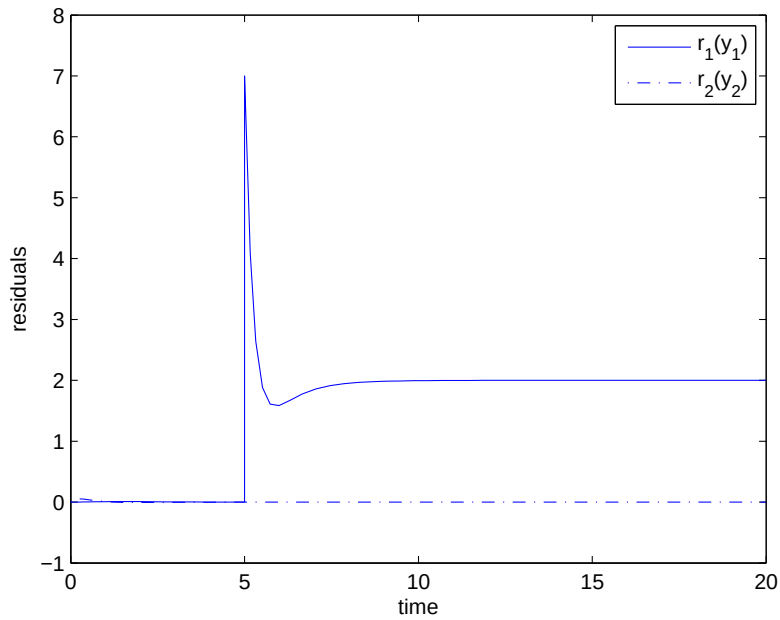

Figure 5.18: PIO residuals when a sensor fault occurs in y_1

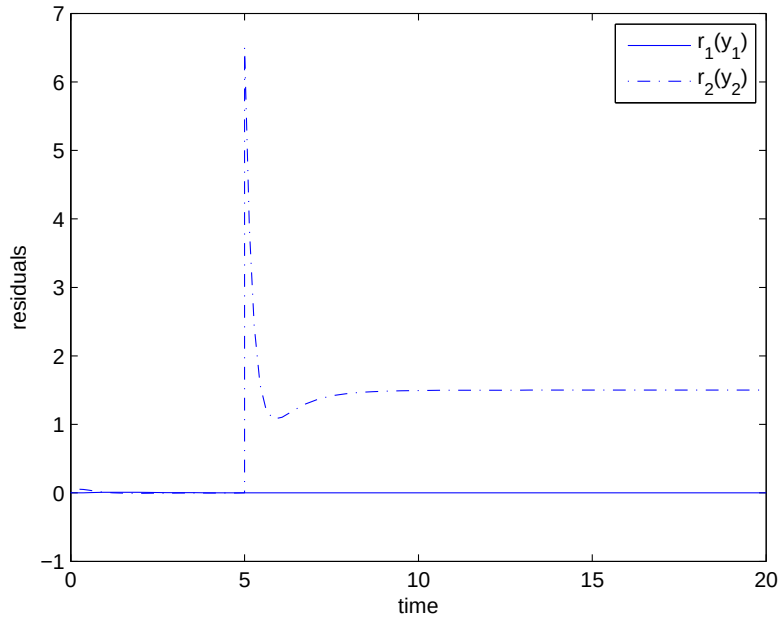

Figure 5.19: PIO residuals when a sensor fault occurs in y_2

The simulation results for fault isolation are shown in Figure 5.18 and 5.19. The residuals are zero throughout the simulation run for fault-free situation. The residual of the respective PIO increases in magnitude significantly when sensor faults occur at $t = 5$. The faults can be easily isolated using the information provided by residuals.

5.2 A Practical Example

[25] studied the sensor fault detection problem for a well-stirred chemical reactor with heat exchanger. The system was also studied in [1]. This system is used here to demonstrate the robust actuator fault detection and isolation scheme.

5.2.1 System Representation

The state, input and output vectors for the considered chemical reactor are:

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} C_o \\ T_o \\ T_w \\ T_m \end{bmatrix} \quad u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 3.6C_i \\ 3.6T_i \\ 36T_{wi} \end{bmatrix} \quad y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} C_o \\ T_o \\ T_w \end{bmatrix}$$

where

$$
\begin{array}{rcl}
C_o & \longmapsto & \text{concentration of the chemcial product} \\
T_o & \longmapsto & \text{temperature of the product} \\
T_w & \longmapsto & \text{temperature of jacket water of heat exchanger} \\
T_m & \longmapsto & \text{coolant temperature} \\
C_i & \longmapsto & \text{inlet concentration of reactant} \\
T_i & \longmapsto & \text{inlet temperature} \\
T_{wi} & \longmapsto & \text{coolant water inlet temperature}
\end{array}
$$

According to [25], the system is modeld as:

$$
\begin{aligned}
\dot{x} &= Ax + Bu + Ed \\
y &= Cx
\end{aligned}
$$

where the term Ed is used to represent the nonlinearity in the system, and

$$
d = 3.012 \times 10^{12} exp\{-\frac{1.2515 \times 10^7}{T_o}\} = 3.012 \times 10^{12} exp\{-\frac{1.2515 \times 10^7}{x_2}\}
$$

The system matrices are:

$$
A = \begin{bmatrix} -3.6 & 0 & 0 & 0 \\ 0 & -3.6702 & 0 & 0.0702 \\ 0 & 0 & -36.2588 & 0.2588 \\ 0 & 0.6344 & 0.7781 & -1.4125 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}
$$

$$
C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad E = \begin{bmatrix} 1 \\ 20.758 \\ 0 \\ 0 \end{bmatrix}
$$

5.2.2 UIOs Design and Residuals Generation

Both control inputs u_1 (C_i) and u_2 (T_i) are related to the inlet chemicals and any fault in u_1 or u_2 will result a similar consequence. Hence it is not necessary to differentiate faults between u_1 and u_2. Two UIOs are designed here with the first UIO driven by u_1 and u_2 and the second UIO driven by u_3. These two UIOs are robust to the nonlinear factor in d.

UIO 1: The first UIO is:

$$
\begin{aligned}
\dot{z}_1 &= F_1 z_1 + K_1 y + T_1 \begin{bmatrix} b_1 & b_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \\
\hat{x} &= z_1 + H_1 y
\end{aligned}
$$

where b_1 and b_2 are the first two columns of B, and the parameter matrices for the first UIO are:

$$H_1 = \begin{bmatrix} 21.758 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2075.8 & 100 & 0 \end{bmatrix} \quad T_1 = \begin{bmatrix} -20.8 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2075.8 & -100 & 0 & 1 \end{bmatrix}$$

$$F_1 = \begin{bmatrix} -10 & 0 & 0 & 0.0702 \\ 0 & -11 & 0 & 0 \\ 0 & 0 & -12 & 0 \\ 0 & 0 & 0 & -8.4325 \end{bmatrix} \quad K_1 = \begin{bmatrix} -278.5724 & 13.3496 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 10031.304 & -475.5956 & 0.7781 \end{bmatrix}$$

The residual is generated by:

$$r_1 = y_1 - \hat{y}_1 = y_1 - \hat{x}_1$$

UIO 2: The second UIO is:

$$\begin{aligned} \dot{z}_2 &= F_2 z_2 + K_2 y + T_2 b_3 u_3 \\ \hat{x} &= z_2 + H_2 y \end{aligned}$$

where b_3 is the third column of B, and the parameter matrices for the second UIO are:

$$H_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 40 \end{bmatrix} \quad T_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -40 & 1 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} -12 & 0 & 0 & 0 \\ 0 & -9 & 0 & 0 \\ 0 & 0 & -9.2355 & 0.2588 \\ 0 & 0 & -5.2124 & 11.7645 \end{bmatrix} \quad K_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -16.6713 \\ 0 & 0.6344 & 985.7625 \end{bmatrix}$$

The residual is generated by:

$$r_2 = y_3 - \hat{y}_3 = y_3 - \hat{x}_3$$

5.2.3 Simulation

The above two UIOs are applied to the chemical reactor process to detect and isolate actuator faults. The system input and the initial state vectors are:

$$u = \begin{bmatrix} 34.632 \\ 1641.6 \\ 29980 \end{bmatrix} \qquad x(0) = \begin{bmatrix} 0.3412 \\ 525.7 \\ 472.2 \\ 496.2 \end{bmatrix}$$

The initial values for UIOs are:

$$z_1(0) = \begin{bmatrix} 518.6174 \\ 0 \\ 0 \\ -51365.537 \end{bmatrix} \qquad z_2(0) = \begin{bmatrix} 0 \\ 0 \\ 472.2 \\ -18391.8 \end{bmatrix}$$

The sampling interval is set as 0.05 hours, and the simulation is carried out for $t = 10$ hours. Various types of faults are introduced to the system at $t = 4$ hours. The list of the simulated faults is provided:

(a) A fault occurs in the inlet reactant when $t > 4$ hour. The fault signal in the first input is $20\% u_1$.

(b) A fault occurs in the inlet reactant when $t > 4$ hour. The fault signal in the second input is $20\% \sin(2t - 8) u_2$.

(c) A fault occurs in the coolant circular when $t > 4$ hour. The fault signal in the third input is $-2\% u_3$.

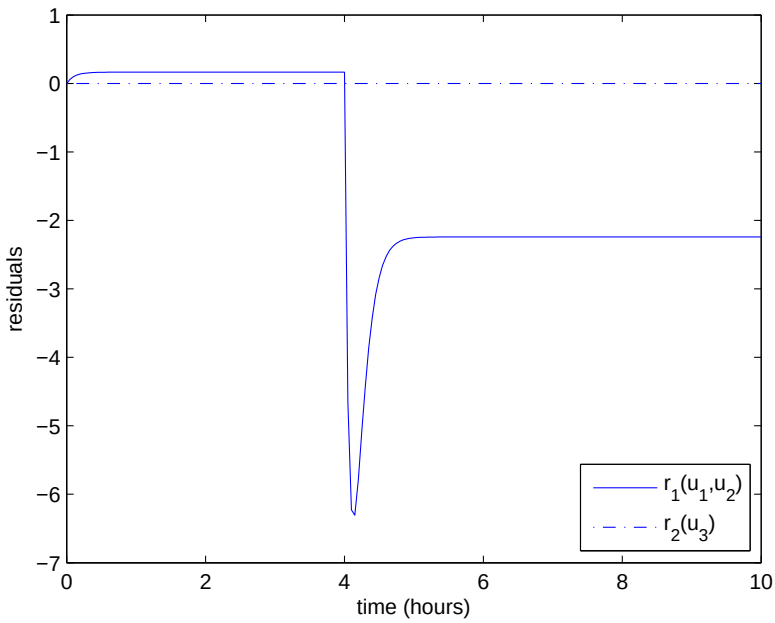

Figure 5.20: UIO residuals when a fault occurs in u_1 at $t = 4$

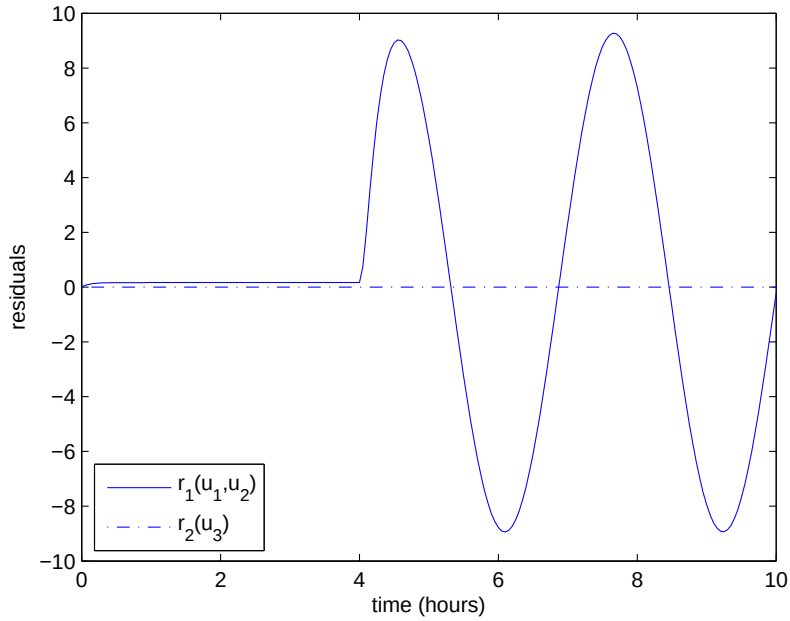

Figure 5.21: UIO residuals when a fault occurs in u_2 at $t = 4$

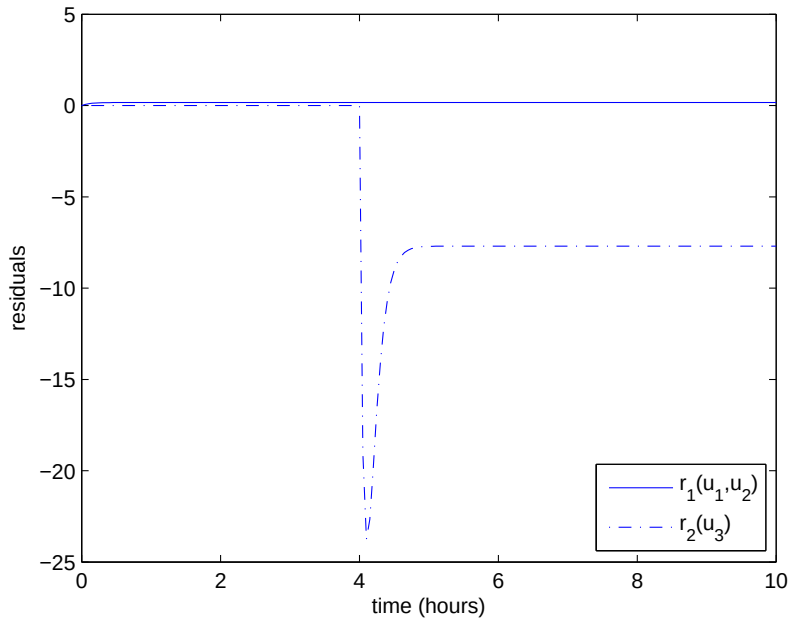

Figure 5.22: UIO residuals when a fault occurs in u_3 at $t = 4$

The simulation results are shown in Figure 5.20, Figure 5.21 and Figure 5.22. The residual is almost zero throughout the ten hours simulation run for fault-free situation. The residuals of the respective UIO increase in magnitude significantly when actuator faults occur at $t = 4$

hour. The faults can be easily isolated using the information provided by residuals.

Chapter 6

Conclusion

The purpose of this thesis is to evaluate various types of observers with application to fault detection. A brief introduction of fault detection and isolation (FDI) is presented at the beginning. The system and faults are identified. Five observer structures are considered and the connection among these observers have been explored. Observer-based fault detection schemes are developed and the importance of robustness is addressed. The effectiveness of robust fault detection based on DO, UIO and PIO are demonstrated by simulation examples. Possible fault isolation schemes have also been developed and tested.

Robust FDI based on various types of observers have been studied for many years. However, the number of reported applications is very limited. The main issue is that the unknown input distribution matrix, namely the E matrix, required for designing DO, UIO and PIO, is actually unknown for most practical systems. Further study on determination or approximation of the unknown input distribution matrix is needed for future possible real industrial applications.

Bibliography

[1] J. Chen and R. J. Patton, *Robust Model-Based Fault Diagnosis for Dynamic Systems*, Kluwer Academic Publishers, Boston, 1999.

[2] R. Isermann, *Fault-Diagnosis Systems: An In troduction from Fault Detection to Fault Tolerance*, Springer, 2006.

[3] J. Chen, R. J. Patton, and H. Zhang, "Design of unknown input observers and robust fault detection filters", *Int. J. Control*, Vol. 63, No. 1, pp. 85-105, 1996.

[4] C. Bakiotis, J. Raymond and A. Rault, "Parameter and Discrimininat Analysis for Jet Engine Mechanical State Diagnosis", *Proc. of The 1979 IEEE Conf. on Decision & Control*, Fort Lauderdale, USA, 1979.

[5] R. Isermann, "Parameter-Adaptive Control Algorithms - a Tutorial", *Automatica*, Vol. 18, No. 5, pp. 513-528, 1982.

[6] R. Isermann, "Process Fault Detection on Modeling and Estimation Methods - a Survey", *Automatica*, Vol. 20, No. 4, pp. 387-404, 1984.

[7] D. Filbert, "Fault Diagnosis in Nonlinear Electromechanical Systems by Continuous-Time Parameter Estimation", *ISA Trans.*, Vol. 24, No. 3, pp. 23-27, 1985.

[8] E. Chow and A. Willsky, "Analytical Redundancy and the Design of Robust Failure Detection Systems", *IEEE Trans. Automat. Contr.*, Vol. 29, No. 7, pp. 603-614, July 1984.

[9] J. Gertler and D. Singer, "Augmented Models for Statistical Fault Isolation in Complex Dynamic Systems", *Proceeds of the American Control Conference*, Vol 1, pp. 317-322, Boston, MA, 1985.

[10] J. Gertler and D. Singer, "A New Structural Framework for Parity Equation-Based Failure Detection and Isolation", *Automatic*, Vol. 26, No. 2, pp. 381-388, 1990.

[11] R. Patton and J. Chen, "A Review of Parity Space Approaches to Fault Diagnosis", In *Prepr. IFAC Symposium on Fault Detection, Supervision and Safety for Technical Processes (SAFEPROCESS)*, Vol. 1, pp. 239-255, Baden-Baden, Germany, September 1991.

[12] J. Gertler, "Analytical Redundancy Methods in Failure Detection and Isolation - Survey and Synthesis", In *Prepr. IFAC Symposium on Fault Detection, Supervision and Safety for Technical Processes (SAFEPROCESS)*, Vol. 1, pp. 9-21, Baden-Baden, Germany, September 1991.

[13] R. Isermann, "Model Based Fault Detection and Diagnosis Methods", *Proceedings of the American Control Conference*, Seattle, WA, pp. 1605-1609, June 1995.

[14] J. Gertler, "Diagnosing Parametric Faults: From Parameter Estimation to Parity Relations", *Proceedings of the American Control Conference*, Seattle, WA, pp. 1615-1620, June 1995.

[15] G. Duan and R. Patton, "Robust Fault Detection in Linear Systems Using Luenberger Observers", *UKACC International Conference on Control '98*, pp. 1468-1473, September 1-4, 1998.

[16] Y. Guan and M. Saif, "A Novel Approach to the Design of Unknown Input Observers", *IEEE Trans. Automat. Contr.*, Vol. 36, No. 5, pp. 632-635, May 1991.

[17] M. Hou and P. C. Muller, "Design of Observers for Linear Systems with unknown Inputs", *IEEE Trans. Automat. Contr.*, Vol. 37, No. 6, pp. 871-875, June 1992.

[18] F. Yang and R. W. Wilde, "Observers for Linear Systems with Unknown Inputs", *IEEE Trans. Automat. Contr.*, Vol. 33, No. 7, pp. 677-681, July 1988.

[19] B. Shafai, C. T. Pi, O. Bas, S. Nork and S. P. Linder, "A General Purpose Observer Architecture with Application to Failure Detection and Isolation", *Proceedings of the American Control Conference*, Arlington, VA, pp. 1133-1138, 2001.

[20] B. Shafai, C. T. Pi, S. Nork and S. P. Linder, "Proportional Integral Adaptive Observer for Parameter and Disturbance Estimations", *Proceedings of the 41st IEEE Conference on Decision and Control*, Las Vegas, NV, pp. 4694-4699, 2002.

[21] S. P. Linder, B. Shafai and M. Saif, "Estimating and Accommodating Unknown Actuator Faults with PI Observers", *Proceedings of the 1998 IEEE International Conference on Control Applications*, Trieste, Italy, pp. 461-465, 1998.

[22] H. Oloomi and B. Shafai, "Constrained Stabilization Problem and Transient Mismatch Phenomenon in Singularly Perturbed Systems", *Int. J. Control*, Vol.67, No.3, pp. 435-454, 1997.

[23] R. J. Patton, H. Y. Zhang, J. Chen, "Modelling of Uncertainties for Robust Fault Diagnosis", *Proceedings of the 31st Conference on Decision and Control*, Tucson, Arizona, pp. 921-926, 1992.

[24] E. Alcorta Garcia and P. M. Frank, "A Novel Design of Structured Observer-Based Residuals for FDI", *Proceedings of the American Control Conference*, San Diego, CA, pp. 1341-1345, 1999.

[25] K. Watanabe and D. M. Himmelblau, "Instrument Fault Detection in Systems with Uncertainties", *Int. J. Sys. Sci.* Vol. 13, No. 2, pp. 137-158, 1982.