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Pricing of End-of-Life Items with Obsolescence

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***Abstract*— Variability in the inflow of end-of-life (EOL) products and fluctuating inventory levels often make the processing of EOL products an economically risky operation for product recovery facilities (PRFs). Choosing an appropriate pricing policy can enhance the performance of PRFs by methodically clearing their inventory and increasing profits. This work presents two pricing models to counter the prospect of product obsolescence that can happen either gradually or suddenly. Product obsolescence can cause demand drop and inventory pile up, both of which could dent the revenues of PRFs. In the first model, gradual obsolescence and environmental regulations that limit the disposal quantity in landfills are considered. In the second model, the case of sudden obsolescence is addressed. Examples are presented to illustrate the pricing strategies for each model.**

I. Introduction

Increasing amounts of discarded products and their environmental impact have led to legislations mandating the manufacturers to take-back such products. The increase in the quantity of discarded products can be attributed to their obsolescence accelerated by the faster introduction of products with newer and better technologies into markets. In addition, products that naturally reach their end-of-life (EOL) also add to the pool of discarded products. For example, computers which were once considered cutting edge become obsolete when newer computers with more advanced computing capabilities are introduced into markets.

In the literature, "EOL product", is used as an umbrella term that refers to products that are discarded because either they became obsolete or they naturally reached their EOL state. In comparison to the naturally aged products, obsolete products may still have some useful life left that could be reused. While the technological advances render products obsolete, their prolonged usage leads to natural aging. In case of electronic products obsolescence mainly occurs when they fail to meet the customer expectations in terms of (a) the trendiness or the look of the product, (b) the functionality of the product, (c) the reliability of components in the product, and (d) compatibility with the latest hardware/software. For example, iPods with their sleek and trendy look, huge storage capacity, and better features have nearly wiped out CD players from the personal music player market. In this case, the CD players discarded as obsolete may be in good working condition and may have enough useful life, whereas those discarded due to mechanical aging of components fall under the naturally aged category. This categorization of obsolete and naturally aged discarded products could save the effort spent in sorting them at their EOL processing stage.

The discarded products collected by manufacturers and third party firms either directly from customers or collection centers, are disassembled to recover reusable components and recyclable materials. The manufacturers can in turn sell the reusable components and recyclable materials in second-hand markets to sustain themselves. Such manufacturers and third party firms which are dedicated to product and material recovery from discarded products can be termed product recovery facilities (PRFs). For example IBM's Global Asset Recovery Services is a PRF which collects discarded electronic products such as desktop computers, laptops, and servers, and then disassembles them to recover and sell components and materials [1]. Some PRFs claim that processing EOL products is an economic burden because the sale of reusable components and recyclable materials in general yields low profits. The following are the key factors that contribute to low profit margins of PRFs.

1) The wages of skilled labor required to recover components and materials from EOL products is high.
2) The inconsistency in the inflow of EOL products makes it hard to plan for their materials, equipment, and human resource requirements.
3) Fluctuations in inventory levels is problematic. The ups and downs in inventory levels are caused by the disposal of products with no fixed predictable pattern and the irregular demand trends for reusable components and recyclable materials.
4) The holding cost of excess inventory levels as well as backorders resulting from depleted inventory levels can cause revenue losses to firms.
5) The cost of disposing unsold and obsolete inventory can also affect the profits. The firms have to spend extra money to dispose of hazardous and non-reusable materials. They may also have to pay penalties imposed by local governments whenever they fail to comply with an environmental legislation that specifies an upper limit on the disposal quantity.
6) Clearance sales such as discounts, promotions, and markdowns on list prices to control inventory levels may

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minify profits.

In many ways original equipment manufacturers (OEMs) and PRFs are similar. The demand uncertainty exists for the output of OEMs as well as PRFs. Also both OEMs and PRFs may have product obsolescence looming over them. Inventory control is therefore equally important for both OEM and PRF to achieve financial success. However the availability of raw/starting material is more certain or predictable for OEMs than it is for PRFs. Since the unpredictability in customer disposal patterns is beyond the authority of PRFs, they could instead focus on controlling the inventory levels by selling reusable items at appropriate prices. The regulation of price can simultaneously serve two purposes: control inventory levels and increase profits. In this work the effect of obsolescence on prices of reusable items is investigated.

Obsolescence which is usually associated with products in primary markets can be applicable for products in second-hand markets as well. The introduction of new products in primary markets may encourage customers to dispose of the existing products prematurely. When PRFs introduce in second-hand markets newer versions of reusable products/components, they may render the current set of products/components in secondary markets obsolete. For example, refurbished hard-drives can become obsolete if technologically better ones are introduced into the primary or secondary markets. The introduction of new and better products can affect obsolescence and the decline in demand for the existing products either gradually or suddenly. Gradual obsolescence occurs over a longer period of time. In contrast, sudden obsolescence can occur in a much shorter period of time, such as overnight, a week, a fortnight, or a month. In case of electronic products, gradual obsolescence occurs when their trendiness and compatibility wean over a period of time. Sudden obsolescence of electronic products occurs when products with enhanced reliability and functionality are introduced in second-hand markets.

On the PRF's front, the introduction of new "reusable products/components" in secondary markets renders their existing inventories either partially or completely obsolete. The demand drop due to obsolescence can increase the inventory levels of PRFs. Such a rapid build up of inventory may shoot up the holding costs thus forcing the PRFs to adopt inventory clearance strategies such as selling at cheap prices, donating to charitable or non-profit organizations, or salvaging recyclable materials from their components and disposing the worthless components. These choices are not economically attractive to PRFs — they only dent their revenues. Hence it is important for PRFs to proactively adopt appropriate pricing strategies to sell reusable items taking into account their obsolescence. The main research objective of this work is to develop two pricing models: one that considers sudden obsolescence and the other subject to gradual obsolescence and a limit on the disposal quantity.

II. Previous Work

The research work pertaining to pricing of remanufactured/refurbished items is still in its infancy. Among the prominent ones are Guide *et al.* [2] who studied the pricing of remanufactured products keeping the firms profits in perspective. They present an economic model to determine the optimal product acquisition price and selling price for a class of remanufactured products that are ranked according to their quality. Majumder and Groenevelt [3] and Ferrer and Swaminathan [4] modeled the competition between an OEM and a remanufacturer. Ferguson and Toktay [5] studied the effect of competition between new and remanufactured products produced by an OEM. Vorasayan and Ryan [6] presented analytical models to determine the optimal price and quantities of refurbished products. Debo *et al.* [7] focused on the optimal pricing of remanufactured products and the choice of production technology necessary for remanufacturing in the context of monopolistic markets, where customers distinguish between new and remanufactured products produced by a firm. Mitra [8] studied the second-hand markets for cellular phones in India and developed analytical models to determine prices of remanufactured and refurbished products by considering quality levels and availability of discarded products. Most of the pricing models presented so far are suitable for large to medium scale OEMs who process discarded products. These OEMs can afford green practices in their manufacturing operations, work under environmental regulations, and can bear any loss in sale of remanufactured/refurbished products. However the small scale OEMs and third party firms often struggle to achieve profits under the same environmental laws and market conditions. The work of Vadde *et al.* [9] focuses on small scale firms involved purely in product recovery under strict environmental regulations. In the product obsolescence domain, inventory decisions [10] and optimal selling prices [11] in light of obsolescence have been studied. However, there is no paper which considers pricing decisions for reusable items that can become obsolete during the selling horizon. The consideration of obsolescence in determining prices of reusable items, makes this work unique in its contribution to the fast growing body of work on pricing for secondary markets.

III. Pricing Strategies With Obsolescence

This section presents two pricing models for reusable items which incorporate obsolescence. The first model addresses pricing with gradual obsolescence by considering the environmental regulation that limits the quantity of material disposed into landfills. In this model, a parameter quantifying obsolescence and a non-increasing demand are used. The second model considers pricing with sudden obsolescence and decreasing demand. Sudden obsolescence is modeled as a probability density function. These two pricing models provide PRF managers with an insight into pricing of remanufactured/refurbished items that are vulnerable to obsolescence. Both models are generic in their treatment of demand functions and allow "what-if" analysis of pricing for various demand functions. The models can serve as a guide for management to plan their production control operations by providing them the quantity of EOL products to acquire and the number of items to refurbish/remanufacture. Following are the general assumptions for both models:

- The PRF in the models is operating in a monopolistic market.
- The cost of acquiring EOL products from customers/collection centers is beyond the boundaries of the models.
- The production cost involved in disassembly, refurbishing, and remanufacturing operations is also beyond the focus of the models.
- Customers do not exhibit strategic behavior and they purchase items without price reservations.
- The demand for reusable items is deterministic in the selling horizon.
- Only discrete items of a single type of reusable product/component are sold by the PRF.
- The inventory is not replenished during the selling horizon and the excess demand is not backlogged.

A. Pricing with Gradual Obsolescence

This model considers the effect of gradual obsolescence on demand in determining the price. The demand for EOL items is assumed to be dependent on price and obsolescence rate. The obsolescence rate signifies the extent to which a product becomes obsolete with respect to time starting from the instant new "reusable items" are introduced in the second-hand market. Let, $\theta(t) \in [0,1]$, be a continuous function that represents the obsolescence rate at time t. For an item $\theta(t) = 0$ implies that it is not obsolete; this is usually associated with new "reusable items" just introduced in the market. For a completely obsolete item $\theta(t) = 1$. It is assumed that $\theta(t)$ increases with respect to time t. Let n be the number of items in inventory at the onset of the selling horizon. Let p be the price and $\lambda(p, \theta(t))$ be the demand intensity function which gives the number of items in demand per unit time. Assume $\lambda(p, \theta(t))$ to be continuous, differentiable, and strictly decreasing with increasing p. Typically the market demand decreases as obsolescence rate increases, so $\lambda(p, \theta(t))$ is assumed to be decreasing in $\theta(t)$. When an item becomes completely obsolete, its demand may not completely wither because for whatever reasons there can still be customers who want to purchase the obsolete items. Since the possibility of stockouts is not considered in this model, $\lambda(p, \theta(t)) \leq n$.

It is also assumed that at the end of the selling horizon, the left over inventory has no salvage value and a portion of it is sent to disposal. Let α be the portion of unsold inventory that is sent for disposal and $c > 0$ be the pre-specified upper limit on the disposal quantity, usually determined by the local environmental regulations. In a continuous time setting, the objective is to find the price function that maximizes the revenue over the selling horizon H. This can be mathematically stated as,

$$\max_p \int_0^H p\lambda(p, \theta(t))dt$$

where

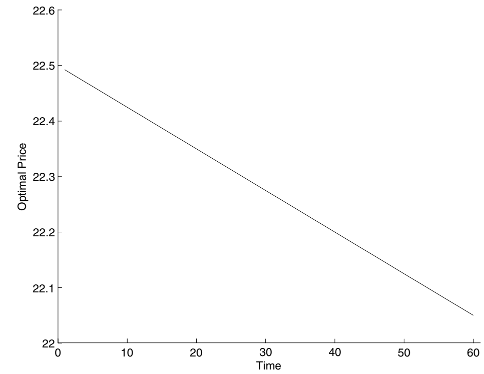

Fig. 1. Price variation with time

$$\int_0^H \lambda(p, \theta(t))dt \leq n \tag{1}$$

$$\alpha \left(n - \int_0^H \lambda(p, \theta(t))dt \right) \leq c \tag{2}$$

Using the Euler-Lagrange equations [12], the optimal price policy is,

$$p^* = \eta_1 - \eta_2 - \frac{\lambda(p^*, \theta(t))}{\frac{\partial}{\partial p}\lambda(p^*, \theta(t))} \tag{3}$$

where $\frac{\partial}{\partial p}$ represents the partial derivative with respect to p. In equation (3), η_1 and η_2 are the Lagrangean multipliers of equations 1 and 2 respectively; η_1 and η_2 can be determined if the price at the start and the end of the selling horizon is known.

For this price policy a lower bound is obtained by substituting $\eta_1 = \eta_2 = 0$ in equation (3),

$$p^* = -\frac{\lambda(p^*, \theta(t))}{\frac{\partial}{\partial p}\lambda(p^*, \theta(t))} \tag{4}$$

In equation (3), if $\eta_1 - \eta_2 \geq 0$ then the optimal price is,

$$p^* \leq -\frac{\lambda(p^*, \theta(t))}{\frac{\partial}{\partial p}\lambda(p^*, t, \theta(t))}$$

In case the fraction of unsold inventory at H exceeds c, the surplus is disposed of at a penalty cost. In that case replacing equation (2) with $\alpha \left(n - \int_0^H \lambda(p, \theta(t))dt \right) > c$, and solving for p gives,

$$p^* = \eta_1 + \eta_2 - \frac{\lambda(p^*, \theta(t))}{\frac{\partial}{\partial p}\lambda(p^*, \theta(t))}$$

Case Example: Let $\lambda(p, \theta(t)) = a_1 - a_2 p - a_3 t$ with $\theta(t) = a_3 t$, then $\frac{\partial}{\partial p}\lambda(p, \theta(t)) = -a_2$. From equation (4), the lower bound on the price is,

$$p^* = \frac{a_1 - a_3 t}{2a_2}$$

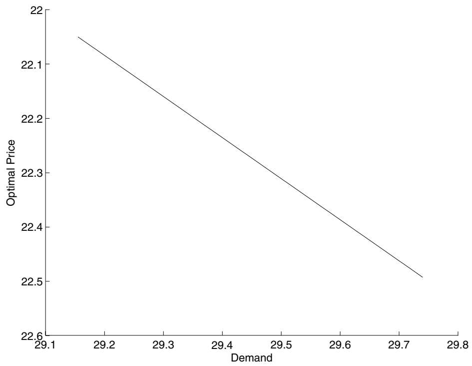

Fig. 2. Price variation with demand

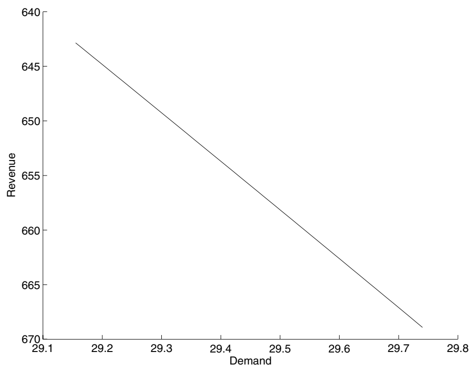

Fig. 3. Revenue variation with demand

Let $H = 60$ days, $a_1 = 50$, $a_2 = 0.2$, and let complete obsolescence occur at H. So $a_3 = 1/H$. For this set of numerical values, the lower bound on the price is, $p^* = 125 - 0.04167t$. From this equation it can be observed that as the obsolescence rate increases the price decreases in the selling horizon (see figure 1). Usually the demand grows as the prices fall. On the contrary in this model the demand decreases with lowering price (see figure 2). This can be explained in terms of the two factors that are at play — the effect of price drop and increasing obsolescence rate on demand. Of the two factors, the increasing obsolescence rate dominates the price advantage to cause the overall demand to drop. Similar trend is observed with revenue, it decreases with price and time (see figure 3).

B. Pricing with Sudden Obsolescence

Pricing with the prospect of sudden obsolescence in the selling horizon is presented in this model. When a reusable item suddenly becomes obsolete, its demand falls sharply — in some cases almost instantaneously.

Let the selling horizon of length H be divided into T equal periods. Consider that the demand function in the selling horizon, $\lambda(p)$, is dependent only on the price p and is continuous and strictly decreasing with increasing p. Since customers are unaware of sudden obsolescence apriori, it has no effect on demand until it actually occurs. The price p is selected from a discrete set to make the modeling of the problem amenable. However this may not be too restrictive in practice. Also discrete prices are convenient and easy for the customers to understand [13]. For example, it is common practice to offer prices close to whole dollar amounts such as \$9.99 or \$14.99. Let $\{p_1, p_2, ..., p_N\}$ be the set of prices, chosen such that $p_1 < p_2 < ... < p_N$ and $\lambda(p_1) > \lambda(p_2) > ... > \lambda(p_N)$. These two conditions guarantee that the revenue is monotonously decreasing, $p_1\lambda(p_1) > p_2\lambda(p_2) > ... > p_N\lambda(p_N)$.

Similar to what is reported by Arcelus *et al.* [11], obsolescence in this paper is modeled as a probability density function and it is assumed that obsolescence occurs at the end of one of the periods in the selling horizon. Let f_j be the probability of the occurrence of obsolescence at t_j, the end of period j, where $t_j = j\frac{H}{T}$, and $t_j > 0, \forall j = 1, 2, ..., T$. The assumption that obsolescence occurs at the end of a period makes it convenient to model the problem. The obsolescence may even occur beyond the selling horizon; therefore it implies that $\sum_{j=1}^{T} f_j < 1$. This is a useful assumption to avoid sharp demand drops due to the occurrence of obsolescence in the selling horizon and its subsequent impact on the PRF's revenues. If obsolescence does occur somewhere during a period, the selling horizon can be terminated at the point of occurrence. Let $F(j)$ be the cumulative probability distribution of the occurrence of obsolescence at the end of period j and let $\overline{F}(j) = 1 - F(j)$. Let $\overline{F}(j|j-1) = \frac{\overline{F}(j)}{\overline{F}(j-1)}$ represent the conditional probability that items survive obsolescence in period j given that it did not occur at the end of period $(j-1)$. For $j = 1$, $\overline{F}(j|j-1) = \overline{F}(j) = \overline{F}(1)$.

With this problem backdrop, the PRF has to post a price that is selected from the discrete set, for each of the T periods. The problem is formulated as a dynamic programming model [14]. The T periods represent the T stages, the inventory level of the jth period represents the state variable of the jth stage, and price is the decision variable at each stage. At stage j, let s_j be the inventory level, p_j be the price, and $r_j(s_j, p_j) = p_j\lambda(p_j)$ be the revenue subject to $\lambda(p_j) \leq s_j$. If $s_j = 0$, the PRF posts an exorbitant price to drive away any demand.

The PRF reviews its inventory level at the beginning of each stage and chooses the price that maximizes the expected revenue from the beginning of that stage to the end of the selling horizon, i.e., cumulative revenue from stage j through stage T. The objective function at stage j is the total expected revenue from stage j to the end of the selling horizon:

$$V_j(s_j, p_j) = r_j(s_j, p_j) + \overline{F}(j|j-1)V^*_{j+1}(s_{j+1}, p^*_{j+1}) \qquad (5)$$

In equation (5), the inventory level at the beginning of stage $j+1$, is computed as $s_{j+1} = s_j - \lambda(p_j)$. The optimal expected revenue, $V^*_j(s_j, p^*_j)$, is obtained as,

$$V^*_j(s_j, p^*_j) = \max\{V_j(s_j, p_i)\}_{i=1}^{N}, j = 1, 2, ..., T \qquad (6)$$

and $V^*_{T+1}(s_{T+1}, p^*_{T+1}) = 0$. The price, p^*_j, corresponding to $V^*_j(s_j, p^*_j)$ is the optimal price for inventory level s_j at stage j.

TABLE I
OPTIMAL PRICING STRATEGY

Period (j)	s_j	$\lambda(p_j^*)$	$F(j\|j-1)$	$V_j^*(s_j, p_j^*)$	p_j^*
1	20	9	0.94	237.52	12
2	11	9	0.82	137.79	12
3	2	1	0.73	36.39	21
4	1	1	0.64	21.00	21

Given n–item inventory at the start of the selling horizon, the optimal pricing strategy is computed using the following dynamic programming algorithm.

Dynamic Programming Algorithm

Step 1 : Going back from $j = T$ to $j = 1$ (backward induction), $\forall s_j \in \{0, 1, ..., n\}$, compute $V_j^*(s_j, p_j^*)$ from equation (6), and record the corresponding p_j^*.

Step 2 : For $j = 1, 2, ..., T$ select the optimal price, p_j^*, recorded in Step 1 for the corresponding inventory level s_j. This determines the pricing policy for the selling horizon.

The above algorithm can also be applied to the case where demand functions are different for each period.

Let the probability density function of product obsolescence be the Weibull distribution with shape parameter, ϕ, and scale parameter, β. So, $F(j) = 1 - e^{-(t_j/\beta)^\phi}$.

Lemma 1: *$\overline{F}(j|j-1)$ decreases with advancing t_j in the selling horizon.*

Proof: $\overline{F}(j|j-1) = \frac{e^{(-t_j/\beta)^\phi}}{e^{(-t_{j-1}/\beta)^\phi}}$. Substituting $t_{j-1} = t_j - \frac{H}{T}$ and simplifying, $\overline{F}(j|j-1) = e^{-t_j}(e^{H/T} - 1)$. Since e^{-t_j} is decreasing with increasing t_j, $\overline{F}(j|j-1)$ also follows the same decreasing trend as e^{-t_j}. $\diamond$

Lemma 1 states that the conditional probability of product surviving obsolescence decreases as the time progresses in the selling horizon from one stage to the next. Intuitively, as the probability of obsolescence increases, the maximum expected revenue in each stage decreases with depleting on-hand inventory.

Case Example

Let $\{\$12, \$15, \$18, \$21\}$ be the set of prices. Let the demand function for all periods be $\lambda(p) = 20 - 0.9p$. Let $\phi = 2$, $\beta = 2$, $H = 2$ months, $T = 4$ periods, and $n = 20$ items. Applying the dynamic programming algorithm, one gets the pricing strategy tabulated in Table 1. As the probability of items surviving each period decreases, the expected revenue also decreases, demand drops, and the price of the item increases.

IV. CONCLUSIONS AND FUTURE WORK

Inventory control is extremely important for the financial success of PRFs. Introduction of "new" reusable items in second-hand markets can render items in PRF's inventories obsolete. This can decrease demand, increase inventory levels, and adversely affect the profits. Adopting an appropriate pricing policy in the face of a possible obsolescence is therefore important to control inventory levels and enhance profits. Two pricing models were presented in this work. The first model considered gradual obsolescence with environmental regulation that prescribes a limit on quantity of material disposed to landfills. A parameter that quantifies the rate of obsolescence influences the demand which is also price dependent. A case example illustrated that price, demand, and revenue fall as the obsolescence parameter increases in the selling horizon. In the second model sudden obsolescence is modeled as a probability density function with the price restricted to a discrete set. An example showed that the optimal price increases as the selling horizon progresses, whereas the demand and revenue flows down.

As part of the future work, the authors are currently studying the pricing strategy in a stochastic demand environment in the case of sudden obsolescence. Another enhancement to the work could consider the disposal-limit regulation in the second pricing model.

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