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Beyond Sensor-Assisted Diagnosis of Used Products

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ABSTRACT

It is difficult to obtain information regarding compositions and remaining life periods of used products. Hence, they often undergo partial or complete disassembly for subsequent re-processing (remanufacturing and/or recycling). However, researchers are now studying sensor embedded products (SEPs), the composition and remaining life of which can be obtained at the end of their use from sensors. This paper addresses decision-making regarding the futurity of an SEP at its end of use: whether to disassemble it for subsequent recycling/remanufacturing or to repair it for subsequent sale on a second-hand market. We identify some important factors that must be considered before making a decision. Using a numerical example, we propose a simple approach that employs Bayesian updating and fuzzy set theory to aid the decision-making process.

Keywords: Second-hand Market, Reverse Supply Chain, Bayesian Updating, Fuzzy, Sensors.

1. INTRODUCTION

In general, collectors of used products seldom know when those products were bought and why were they discarded. In addition, the products do not indicate their remaining life periods. Hence, they often undergo partial or complete disassembly for subsequent re-processing (remanufacturing and/or recycling) [7]. However, it is interesting to note that researchers (see, for example, [4], [5], [6]) have now started to study sensor embedded products (SEPs), viz., products that contain smart sensors implanted in them to monitor critical components, predict component or product failures, and estimate remaining lives of the products at their end of use.

Figure 1 shows a framework that we are currently examining for transfer of SEPs across a reverse supply chain network (consisting of numerous facilities, such as repair/service center, disassembly center, remanufacturing center, material recycling center, disposal center, original equipment manufacturer (OEM), and retail distributor).

The remote product data warehouse (RPDW) in the framework receives and stores information (both static and dynamic) transmitted by smart sensors in the SEPs, and provides the information to different facilities in the network. The possible enablers of information transfer are wireless communication technologies, satellite links, and the Internet.

This paper is concerned with making a decision regarding the futurity of an SEP at its end of use: whether to disassemble it for subsequent recycling/remanufacturing or to repair it for subsequent sale on a second-hand market. Although one might think that the information (composition and remaining life of the SEP) provided by the smart sensors will make the job of the decision-maker easy, the authors are of the opinion that there are a lot of important issues that must be considered before making a decision regarding the futurity of the SEP. Some of those issues are as follows:

- i. How difficult is it to repair the product?
- ii. How much does it cost to repair the product?

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- iii. How saleable is the product? (Is the model almost obsolete?)
- iv. How bad are the cosmetic damages (scratches and dents that make the product less appealing)?
- v. Has the product met with an accident before?
- vi. Has the product been the subject of a safety recall in the past?
- vii. Does the product have all the relevant accessories (connecting cables, remote control, etc)? (Sometimes, replacing the missing parts may cost the buyer more than the discount he gets on the product)
- viii. Can the product be sold at a price that is substantially lower than that of a new product?
- ix. Can the product be sold with the same guarantee as the new product?
- x. Are spare parts easily available to the buyer if he needs them?
- xi. What is the attitude of the consumers towards the risk involved in buying the product?
- xii. How harmful are the product's components to the environment?
- xiii. How fast are the raw materials depleting that are used to make the components of the product?
- xiv. Are there enough products (of the same type) so that one can simultaneously run a leasing/renting business?

Using a numerical example, an approach is proposed that employs Bayesian updating [3] and fuzzy set theory [10] for the purpose of decision-making regarding the futurity of an SEP. It must be noted that this approach is an add-on to the sensor-assisted diagnosis rather than a substitute for it.

Brief introductions of Bayesian updating and the fuzzy set theory are presented in Sections 2 and 3 respectively. Section 4 presents the approach, and Section 5 gives some conclusions and future research information.

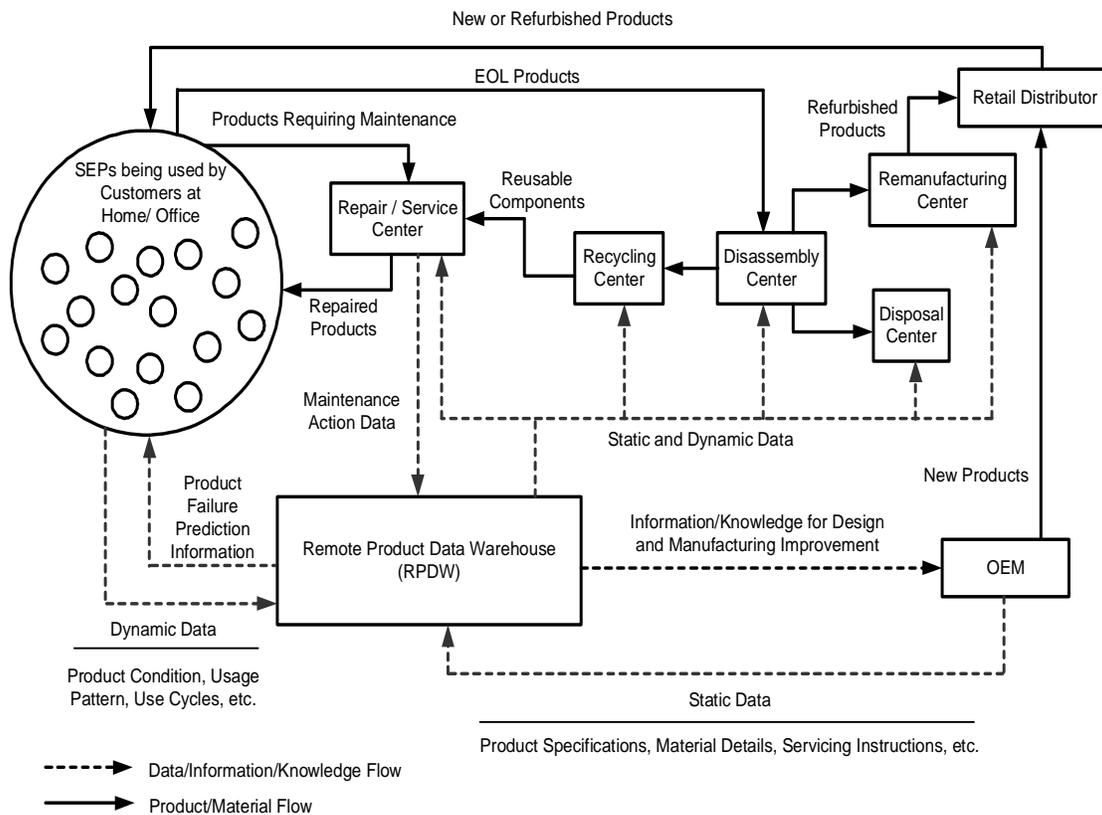


Figure 1. Framework for Transfer of SEPs across a Reverse Supply Chain Network

2. BAYESIAN UPDATING

Bayesian updating [3] is an uncertainty modeling technique that assumes that it is possible for an expert in a domain to guess a probability of every hypothesis or assertion in that domain, and that this probability can be updated in light of evidence for or against the hypothesis or assertion.

Suppose the probability of a hypothesis H is P(H). Then, the formula for the odds of that hypothesis, O(H), is given by:

$$O(H) = \frac{P(H)}{1-P(H)} \quad (1)$$

A hypothesis that is absolutely certain, i.e., has a probability of 1, has infinite odds. In practice, limits are often set on odds so that, for example, if $O(H) > 1000$ then H is true, and if $O(H) < 0.01$ then H is false.

2.1. Updating Probabilities with Supporting Evidence

The standard formula for updating the odds of hypothesis H, given that evidence E is observed, is:

$$O(H|E) = (A).O(H) \quad (2)$$

where $O(H|E)$ is the odds of H, given the presence of evidence E, and A is the *affirms* weight of E. The definition of A is:

$$A = \frac{P(E|H)}{P(E|\sim H)} \quad (3)$$

where $P(E|H)$ is the probability of E, given that H is true and $P(E|\sim H)$ is the probability of E, not given that H is true.

2.2. Updating Probabilities with Opposing Evidence

Bayesian updating assumes that the absence of supporting evidence is equivalent to the presence of opposing evidence. The standard formula for updating the odds of a hypothesis H, given that the evidence E is absent, is:

$$O(H|\sim E) = (D).O(H) \quad (4)$$

where $O(H|\sim E)$ is the odds of H, given the absence of evidence E, and D is the *denies* weight of E. The definition of D is:

$$D = \frac{P(\sim E|H)}{P(\sim E|\sim H)} = \frac{1-P(E|H)}{1-P(E|\sim H)} \quad (5)$$

If a given piece of evidence E has an *affirms* weight A which is greater than 1, then its *denies* weight must be less than 1, and vice versa. Also, if $A > 1$ and $D < 1$, then the presence of evidence E is supportive of hypothesis H. Similarly, if $A < 1$ and $D > 1$, then the absence of E is supportive of H.

For example, while controlling a power station boiler, a rule - "IF (temperature is high) and NOT (water level is low) THEN (pressure is high)" can also be written as "IF (temperature is high - AFFIRMS A_1 , DENIES D_1) AND (water level is low - AFFIRMS A_2 , DENIES D_2) THEN (pressure is high)". Here,

$$A_1 = \frac{P(\text{Temperature is high} | \text{Pressure is high})}{P(\text{Temperature is high} | \sim \text{Pressure is high})}; \quad D_1 = \frac{P(\sim \text{Temperature is high} | \text{Pressure is high})}{P(\sim \text{Temperature is high} | \sim \text{Pressure is high})};$$

$$A_2 = \frac{P(\text{Water level is low} | \text{Pressure is high})}{P(\text{Water level is low} | \sim \text{Pressure is high})}; \quad D_2 = \frac{P(\sim \text{Temperature is high} | \text{Pressure is high})}{P(\sim \text{Temperature is high} | \sim \text{Pressure is high})};$$

2.3. Dealing with Uncertain Evidence

Sometimes, an evidence is neither definitely present nor definitely absent. For example, if one is diagnosing a TV set that is not functioning properly, it is not definite if this is due to a malfunctioning picture tube or not. In such a case, depending on the value of the probability of the evidence P(E), the affirms and denies weights are modified using the following formulae:

$$A' = [2.(A-1).P(E)]+2-A \quad (6)$$

$$D' = [2.(1-D).P(E)]+D \quad (7)$$

When P(E) is greater than 0.5, the *affirms* weight is used to calculate O(H|E), and when P(E) is less than 0.5, the *denies* weight is used.

2.4. Combining Evidence

If n statistically independent pieces of evidence are found that support or oppose a hypothesis H, then the updating equations are given by:

$$O(H|E_1 \& E_2 \& E_3 \dots E_n) = (A_1).(A_2).(A_3) \dots (A_n).O(H) \quad (8)$$

and
$$O(H|\sim E_1 \& \sim E_2 \& \sim E_3 \dots \sim E_n) = (D_1).(D_2).(D_3) \dots (D_n).O(H) \quad (9)$$

A_i and D_i are given by Equations 10 and 11 respectively.

$$A_i = \frac{P(E_i|H)}{P(E_i|\sim H)} \quad (10)$$

$$D_i = \frac{P(\sim E_i|H)}{P(\sim E_i|\sim H)} \quad (11)$$

3. FUZZY SET THEORY

Expressions such as “not very clear”, “probably so” and “very likely” can be heard very often in daily life. The commonality in such terms is that they are all tainted with imprecision. This imprecision or vagueness of human decision-making is called “fuzziness” in the literature. With different decision-making problems of diverse intensity, the results can be misleading if fuzziness is not taken into account. However, since Zadeh [10] first proposed fuzzy set theory, an increasing number of studies have dealt with imprecision (fuzziness) in problems by applying the fuzzy set theory. The concepts of the fuzzy set theory, which are utilized in this paper, are as follows:

3.1. Linguistic Values and Fuzzy Sets

When dealing with imprecision, decision-makers may be provided with information characterized by vague language such as: high risk, low profit and good customer service. By using *linguistic* values like “high”, “low”, “good”, “medium”, “cheap”, etc., people are usually attempting to describe factors with uncertain or imprecise values. For example, weight of an object may be a factor with an uncertain or imprecise value and so, its linguistic value can be “very low”, “low”, “medium”, “high”, “very high”, etc. The fuzzy set theory is primarily concerned with quantifying the vagueness in human thoughts and perceptions.

To deal with quantifying vagueness, Zadeh [10] proposed a membership function which associates with each quantified linguistic value a grade of membership belonging to the interval [0, 1]. Thus, a fuzzy set is defined as:

$$\forall x \in X, \mu_A(x) \in [0, 1]$$

where $\mu_A(x)$ is the degree of membership, ranging from 0 to 1, of a quantity x of the linguistic value, A, over the universe of quantified linguistic values, X. X is essentially a set of real numbers. The more x fits A, the larger the degree of membership of x . If a quantity has a degree of membership equal to 1, this reflects a complete fitness between the quantity and the vague description (linguistic value). On the other hand, if the degree of membership of a quantity is 0, then that quantity does not belong to the vague description.

3.2. Triangular Fuzzy Numbers

A triangular fuzzy number (TFN) is a fuzzy set with three parameters, each representing a quantity of a linguistic value associated with a degree of membership of either 0 or 1. It is graphically depicted in Figure 2. The parameters a , b and c

respectively denote the smallest possible quantity, the most promising quantity and the largest possible quantity that describe the linguistic value.

Each TFN, P , has linear representations on its left and right side such that its membership function can be defined as:

$$\mu_P = 0, \quad x < a \quad (12)$$

$$= (x-a) / (b-a) \quad a \leq x \leq b \quad (13)$$

$$= (c-x) / (c-b) \quad b \leq x \leq c \quad (14)$$

$$= 0 \quad x \geq c. \quad (15)$$

For each quantity x increasing from a to b , its corresponding degree of membership linearly increases from 0 to 1. When x increases from b to c , its corresponding degree of membership linearly decreases from 1 to 0. The membership function is a mapping from any given x to its corresponding degree of membership.

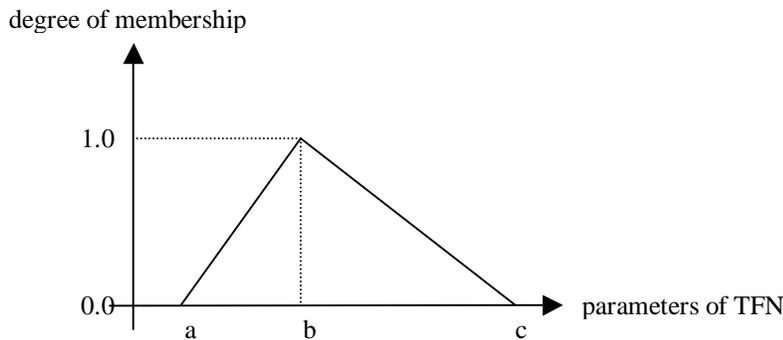


Figure 2. Triangular Fuzzy Number

The TFN is mathematically easy to implement, and more importantly, it represents the rational basis for quantifying the vague knowledge in most decision-making problems.

The basic operations on triangular fuzzy numbers are as follows [1], [2], [9]:

For example, $P_1 = (a, b, c)$ and $P_2 = (d, e, f)$.

$$P_1 + P_2 = (a+d, b+e, c+f) \quad \text{addition;} \quad (16)$$

$$P_1 - P_2 = (a-f, b-e, c-d) \quad \text{subtraction} \quad (17)$$

$$P_1 * P_2 = (a*d, b*e, c*f) \text{ where } a \geq 0 \text{ and } d \geq 0 \quad \text{multiplication} \quad (18)$$

$$P_1 / P_2 = (a/f, b/e, c/d) \text{ where } a \geq 0 \text{ and } d > 0 \quad \text{division.} \quad (19)$$

3.3. Defuzzification

Defuzzification is a technique to convert a fuzzy number into a crisp real number. There are several methods to serve this purpose [8]. For example, the Centre-of-Area method [11] converts a fuzzy number $P = (a, b, c)$ into a crisp real number Q where

$$Q = \frac{(c-a) + (b-a)}{3} + a \quad (20)$$

4. DECISION-MAKING REGARDING FUTURITY OF SEP

Consider an SEP at its end of use (it is assumed here that the SEP functions improperly; it is obviously sensible to sell a properly functioning SEP on a second-hand market). Table 1 shows the probability values that are used in the example to implement the Bayesian updating process.

H: it is sensible to repair product

$P(H) = 0.60$ (given by an expert). Hence, $O(H) = 1.50$.

4.1. Usage of Fuzzy Set Theory

Since it is difficult for an expert to guess the probabilities in Table 1, they are calculated using the fuzzy set theory as follows:

- I. Ask the expert to assign a linguistic value to $P(E)$, $P(E|H)$ and $P(E|\sim H)$ for each evidence.
- II. Use the data in Table 2 to convert the linguistic values into TFNs.
- III. If the linguistic values are given by multiple experts, calculate the average fuzzy probability values for each evidence.
- IV. Defuzzify the average probability values for each evidence.

Table 1. Probability Values Used in Bayesian Updating

No.	E	P(E)	P(E H)	P(E ~H)	A	D
i	Easy to repair product	0.60	0.70	0.20	0.29	2.67
ii	Repair is cheap	0.65	0.60	0.30	0.50	1.75
iii	Product is saleable	0.55	0.45	0.60	1.33	0.73
iv	Not many cosmetic damages	0.20	0.10	0.75	7.50	0.28
v	No history of major accidents	0.45	0.85	0.40	0.47	4.00
vi	No history of safety recall	0.10	0.25	0.70	2.80	0.40
vii	Product has all relevant accessories	0.10	0.70	0.40	0.57	2.00
viii	Product can be sold at substantially lower price	0.35	0.90	0.35	0.39	6.50
ix	Customers can be given good guarantee	0.70	0.25	0.65	2.60	0.47
x	Spare parts are easily available	0.55	0.65	0.20	0.31	2.29
xi	Attitude of customers towards risk involved is favorable to sale	0.40	0.35	0.70	2.00	0.46
xii	Product's components are harmful to environment	0.30	0.40	0.55	1.38	0.75
xiii	Raw materials used to make product's components are depleting fast	0.65	0.80	0.15	0.19	4.25
xiv	There are enough used products to run leasing/renting business	0.60	0.20	0.75	3.75	0.31

In order to save space from tedious calculations, it assumed here that the values shown in Table 1 are the defuzzified average probabilities obtained *after* performing steps I, II, III and IV. For the sake of clarity, however, a numerical example to show the calculation procedure is presented below:

Suppose that we wish to calculate the $P(E|H)$ value (numerical) for a particular evidence. The four steps are implemented as follows:

- I. Five experts assign linguistic values to a probability for a particular evidence, as “very high”, “high”, “medium”, “medium”, and “low” respectively.
- II. Using Table 2, we convert the linguistic values into TFNs.

- III. The average fuzzy probability value is equal to $\left(\frac{0.7+0.5+0.3+0.3+0.1}{5}, \frac{0.9+0.7+0.5+0.5+0.3}{5}, \frac{1.0+0.9+0.7+0.7+0.5}{5} \right)$, or (0.38, 0.58, 0.76).
- IV. Defuzzifying the average fuzzy probability value using Equation 20, we get $\frac{(0.76 - 0.38) + (0.58 - 0.38)}{3} + 0.38 = 0.57$.

Table 2. Conversion Table for Linguistic Ratings in Bayesian Updating

Linguistic rating	Triangular fuzzy number
Very high (VH)	(0.7, 0.9, 1.0)
High (H)	(0.5, 0.7, 0.9)
Medium (M)	(0.3, 0.5, 0.7)
Low (L)	(0.1, 0.3, 0.5)
Very low (VL)	(0.0, 0.1, 0.3)

4.2. Rule Used in Bayesian Updating

IF (it is easy to repair product) (AFFIRMS: 0.29; DENIES: 2.67)

AND (repair is cheap) (AFFIRMS: 0.50; DENIES: 1.75)

AND (product is saleable) (AFFIRMS: 1.33; DENIES: 0.73)

AND (there are not many cosmetic damages) (AFFIRMS: 7.50; DENIES: 0.28)

AND (no history of major accidents) (AFFIRMS: 0.47; DENIES: 4.00)

AND (no history of safety recall) (AFFIRMS: 2.80; DENIES: 0.40)

AND (product has all relevant accessories) (AFFIRMS: 0.57; DENIES: 2.00)

AND (product can be sold at substantially lower price) (AFFIRMS: 0.39; DENIES: 6.50) AND (customers can be given good guarantee) (AFFIRMS: 2.60; DENIES: 0.47)

AND (spare parts are easily available) (AFFIRMS: 0.31; DENIES: 2.29)

AND (attitude of customers towards risk involved is favorable to sale) (AFFIRMS: 2.00; DENIES 0.46)

AND (product's components are harmful to environment) (AFFIRMS: 1.38; DENIES: 0.75)

AND (raw materials used to make product's components are depleting fast) (AFFIRMS: 0.19; DENIES: 4.25)

AND (there are enough used products to run leasing/renting business) (AFFIRMS: 3.75; DENIES: 0.31)

THEN it is sensible to repair the product.

4.3 Bayesian Updating

H = it is sensible to repair product; O(H) = 1.50;

E1 = it is easy to repair product; $P(E1) = 0.60$; $D1 = 2.67$; $D1' = [2.(1-D1).P(E1)] + D1 = 1.50$;

E2 = repair is cheap; $P(E2) = 0.65$; $D2 = 1.75$; $D2' = [2.(1-D2).P(E2)] + D2 = 1.38$;

E3 = product is saleable; $P(E3) = 0.55$; $A3 = 1.33$; $A3' = [2.(A3-1).P(E3)] + 2 - A3 = 1.03$;

E4 = there are not many cosmetic damages; $P(E4) = 0.20$; $A4 = 7.50$; $A4' = [2.(A4-1).P(E4)] + 2 - A4 = 6.2$;

E5 = no history of major accidents; $P(E5) = 0.45$; $D5 = 4.00$; $D5' = [2.(1-D5).P(E5)] + D5 = 1.3$;

E6 = no history of safety recall; $P(E6) = 0.10$; $A6 = 2.80$; $A6' = [2.(A6-1).P(E6)] + 2 - A6 = 1.54$;

E7 = product has all relevant accessories; $P(E7) = 0.10$; $D7 = 2.00$; $D7' = [2.(1-D7).P(E7)] + D7 = 1.8$;

E8 = product can be sold at substantially lower price; $P(E8) = 0.35$; $D8 = 6.50$; $D8' = [2.(1-D8).P(E8)] + D8 = 2.65$;

E9 = customers can be given good guarantee; $P(E9) = 0.70$; $A9 = 2.60$; $A9' = [2.(A9-1).P(E9)] + 2 - A9 = 1.64$;

E10 = spare parts are easily available; $P(E10) = 0.55$; $A10 = 0.31$; $A10' = [2.(A10-1).P(E10)] + 2 - A10 = 0.93$;

E11 = attitude of customers towards risk involved is favorable to sale; $P(E11) = 0.40$; $D11 = 0.46$; $D11' = [2.(1-D11).P(E11)] + D11 = 0.89$;

E12 = product's components are harmful to environment; $P(E12) = 0.30$; $D12 = 0.75$; $D12' = [2.(1-D12).P(E12)] + D12 = 0.90$;

E13 = raw materials used to make product's components are depleting fast; $P(E13) = 0.65$; $A13 = 0.19$; $A13' = [2.(A13-1).P(E13)] + 2 - A13 = 0.76$;

E14 = there are enough used products to run leasing/renting business; $P(E14) = 0.60$; $A14 = 3.75$; $A14' = [2.(A14-1).P(E14)] + 2 - A14 = 1.55$;

$O(H|E1\&E2\&E3\&E4\&E5\&E6\&E7\&E8\&E9\&E10\&E11\&E12\&E13\&E14) =$

$O(H).(D1').(D2').(A3').(A4').(D5').(A6').(D7').(D8').(A9').(A10').(D11').(D12').(A13').(A14') = 2.9767$;

$P(H|E1\&E2\&E3\&E4\&E5\&E6\&E7\&E8\&E9\&E10\&E11\&E12\&E13\&E14) = (2.9767)/(3.9767) = 0.75$.

That is, $P(\text{sensible to repair the product} | \text{evidences}) = 0.75$. If the cut-off value as decided the by the decision-maker is say, 0.70, he will decide to send the used product for repair and for subsequent sale on a second-hand market.

5. CONCLUSIONS AND FUTURE RESEARCH

Some important factors must be considered before making a decision regarding the futurity of an SEP, even though information regarding its remaining life and composition can be obtained from smart sensors implanted in it. In this paper, we identified some of those factors, and using a numerical example, proposed an approach that employs Bayesian updating and fuzzy set theory to aid the decision-making process.

For their future research, the authors plan to work on the following:

- Consideration of all evidences (information which may be obtained from smart sensors) in the decision-making process.

- Examination of any other rules that must precede the rule used in Section 4.2 (Bayesian updating normally is implemented on *multiple* rules).
- Investigation of difficulties faced in obtaining linguistic probability values from multiple experts.
- Study of feasibility of building an expert system for decision-making.

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