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# A Multi-Phase Mathematical Programming Approach to Strategic Planning of an Efficient Reverse Supply Chain Network

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## Abstract

*Strategic planning of a supply chain network is one of the most challenging aspects of reverse logistics. To effectively satisfy drivers such as profitability, environmental regulations and asset recovery, only the most economical used-products must be reprocessed in only the recovery facilities that have the potential to efficiently reprocess them. Due to uncertainties in supply, quality and reprocessing times of used-products, the cost-benefit function in the literature that selects the most economical product to reprocess from a set of used-products is not appropriate for direct adoption. Moreover, due to the same uncertainties, any traditional forward supply chain approach to identify potential manufacturing facilities cannot be employed to identify potential recovery facilities. This paper proposes a three-phase mathematical programming approach, taking the above uncertainties into account, to completely design a reverse supply chain network. Application of the approach is detailed through an illustrative example in each phase.*

## INTRODUCTION

A reverse supply chain can be defined as a series of activities required to retrieve a used-product from a customer and either recover its left-over market value or dispose it of. Besides environmental regulations and asset recovery, an important driver for companies to engage in a reverse supply chain is that many used-products, especially electronic ones [4], represent a resource for recoverable value. Though direct reuse is infeasible in most cases, remanufacturing and recycling are the major recovery options applied in the reverse supply chain. While this process is common in European companies, it is still in its infancy in American companies. In the USA, cities and towns are responsible for retrieval of used-electronic-goods and properly disposing of the potentially environmentally dangerous and/or waste components (also called e-waste). Recently, there was a report [2] that in the state of Massachusetts, support is building for a re-filed bill that would require manufacturers of electronic goods to pay for retrieval and recycling of their equipment. If passed, the statewide take-back program would be the first of its kind in the nation and would relieve cities and towns, which are bracing for local aid cuts, from the costs associated with retrieving and disposing of the e-waste. The bill's supporters say that cities and towns in the USA spend between \$6 million and \$21 million a year on such endeavors.

Implementation of any reverse supply chain network (RSCN) requires at least three parties: collection centers where consumers return used-products, recovery facilities where reprocessing (remanufacturing or recycling) is performed, and demand centers where customers buy reprocessed products, viz., outgoing goods from recovery facilities. Figure 1 shows a generic reverse supply chain network.

While there are many strategic, tactical and operational aspects that are considered in designing and operating a RSCN, this paper concentrates on strategic planning that ideally should involve the following phases:

- I. Selection of the most economical product to reprocess, from a set of different used-products (this step in turn leads to the identification of potential collection centers and potential demand centers in the region where a RSCN is planned to be established)
- II. Identification of potential facilities in a set of candidate recovery facilities operating in the region
- III. Transportation of the right mix and quantities of goods across the RSCN

In this paper, we propose mathematical models for each of the above three phases.

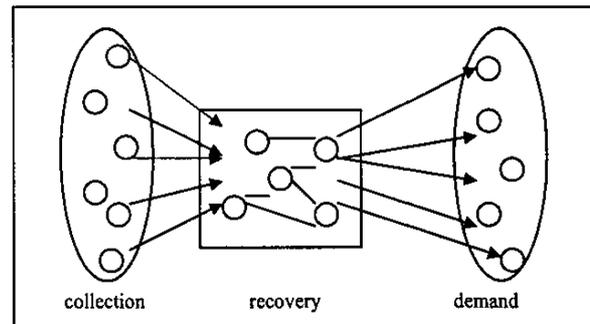


Figure 1. Generic reverse supply chain network

## LITERATURE REVIEW

In the literature for designing a RSCN, while many location models deal with the transportation issue (see [3] for a good review), no paper addresses the problem of either selecting the most economical product from a set of used-products or identifying the potential recovery facilities. In the case of discrete location models (for example, [6]), all the recovery facilities are assumed to be potential and in the case of continuous location models (for example, [5]), it is assumed that potential recovery facilities were already

established or can be established at the locations solved for. Also, each of these location models deals with a used-product that is given to be economical. Evidently, though every location model realizes the importance of reprocessing only an economical used-product in potential recovery facilities, it does not show how to either select that used-product from a set of many economical used-products or identify those potential recovery facilities.

Although one paper [10] proposes a cost-benefit function that assesses the feasible combinations (set of components) of retrieval from the design of a used-product and compares the combination with the highest cost-benefit from one design against those from the others, the function assumes that every component selected for reuse will be in a reusable state after dismantling the product. It also assumes that all the components in the retrieved used-product are in their original multiplicities. It is inappropriate to adopt the cost-benefit function for selecting the most-economical product to reprocess from a set of used-products because neither of the above assumptions is universally valid in a reverse supply chain scenario.

Although identification of potential manufacturing facilities is addressed in a forward supply chain (the series of activities required to produce and distribute a new product to a customer), those approaches (for example, [1], [9]) are unsuitable for employment in a reverse supply chain. This is due to the problems associated with reprocessing, which include: (a) uncertainties in supply and timing of used-products, (b) unknown quality and quantity of components in used-products and (c) stochastic reprocessing times of used-products.

## METHODOLOGY

As mentioned earlier, we utilize a three-phase approach in our methodology. Phase I identifies the most economical product from a set of different types of used-products, using a mixed-integer mathematical programming model. Phase II implements the Analytic Hierarchy Process (AHP) [8] to identify potential facilities in a set of candidate recovery facilities. Phase III solves a discrete location model to achieve transportation of the right mix and quantities of goods (used as well as reprocessed) across the RSCN.

### Phase-I of RSCN Design

In the first phase of strategic planning, we modify the cost-benefit function [10] to incorporate the probability of breakage and the probability of missing components in the used-product. This function is used to formulate a mixed-integer linear programming model that in turn is implemented to the data of each used-product in the set, in order to select the most economical product to reprocess. The modified cost-benefit functional makes sure that a bad choice of the most-economical product is not made. We use the following notation to formulate the mixed-integer mathematical model:

$b_{ij}$  probability of breakage of component  $j$  in product  $i$ ;

$CD$  cost of reprocessing per unit time (\$/unit time);  
 $CF$  recycling revenue factor (\$/unit weight);  
 $DC_i$  total disposal cost of product  $i$  (\$);  
 $DI_{ij}$  disposal cost index of component  $j$  in product  $i$  (index scale 0 = lowest, 10 = highest);  
 $DF$  disposal cost factor (\$/unit weight);  
 $E_{ik}$  subassembly  $k$  in product  $i$ ;  
 $i$  product type;  
 $j$  component type;  
 $m_{ij}$  probability of missing component  $j$  in product  $i$ ;  
 $N_{ij}$  multiplicity of component  $j$  in product  $i$ ;  
 $PC_i$  total reprocessing cost of product  $i$  (\$);  
 $P_{ij}$  component  $j$  in product  $i$ ;  
 $RCP_{ij}$  percentage of recyclable contents by weight in component  $j$  of product  $i$ ;  
 $RCR_i$  total recycling revenue of product  $i$  (\$);  
 $RI_{ij}$  recycling revenue index of component  $j$  in product  $i$  (index scale 0 = lowest, 10 = highest);  
 $Root_i$  root node of product  $i$ ;  
 $RUR_i$  total reuse revenue of product  $i$  (\$);  
 $RV_{ij}$  resale value of component  $j$  in product  $i$ ;  
 $M_i$  total number of subassemblies in product  $i$ ;  
 $TC_i$  retrieval cost of product  $i$  (\$);  
 $T(Root_i)$  time to disassemble  $Root_i$ ;  
 $T(A_{ik})$  time to disassemble subassembly  $k$  in product  $i$ ;  
 $W_{ij}$  weight of component  $j$  in product  $i$ ;  
 $X_{ij}$  decision variable representing the selection of component  $j$  to be retrieved from product  $i$  for reuse ( $X_{ij} = 1$ ) or recycle and/or disposal ( $X_{ij} = 0$ ).

The following mixed-integer linear programming model assumes complete disassembly of the used-product of interest and maximizes its cost-benefit (*i.e.*, total revenue) to be obtained from reprocessing:

$$\text{Maximize } Z_i = RUR_i + RCR_i - PC_i - DC_i; \quad (1)$$

where

$$RUR_i = \sum_{j \in P_{ij} \in (Root_i)} (RV_{ij} \cdot N_{ij} \cdot (1 - b_{ij} - m_{ij}) \cdot X_{ij}) - TC_i; \quad (2)$$

$$RCR_i = \sum_{j \in P_{ij} \in (Root_i)} \left[ \frac{RI_{ij} \cdot W_{ij} \cdot RCP_{ij}}{\{N_{ij}(1 - m_{ij}) - N_{ij} \cdot (1 - b_{ij} - m_{ij}) \cdot X_{ij}\}} \right] \cdot CF; \quad (3)$$

$$PC_i = \left[ T(Root_i) + \sum_{k=1}^{M_i} T(E_{ik}) \right] \cdot CD; \quad (4)$$

$$DC_i = \sum_{j \in P_{ij} \in (Root_i)} \left[ \frac{(DI_{ij} \cdot W_{ij} \cdot (1 - RCP_{ij}))}{\{N_{ij}(1 - m_{ij}) - N_{ij} \cdot (1 - b_{ij} - m_{ij}) \cdot X_{ij}\}} \right] \cdot DF; \quad (5)$$

subject to

$$X_{ij} = 0 \text{ or } 1; \text{ for all } i \text{ and } j \quad (6)$$

The above formulation assesses the feasible combinations (set of components) of retrieval from a used-product and compares the combination with the highest cost-benefit from one product against those from the others.

### Illustrative Example

We take two different used-products whose product structures are shown in Figure 2 and Figure 3 respectively. The data necessary to implement the mathematical model for Product-1 and Product-2 are in Table 1 and Table 2 respec-

tively. Also,  $TC_1 = 20$  (\$/product);  $TC_2 = 13$  (\$/product);  $CF = 0.20$  (\$/lb);  $CD = 0.55$  (\$/min);  $DF = 0.1$  (\$/lb);  $T(Root_1) = 2$  (min);  $T(Root_2) = 3$  (min);  $T(E_{11}) = 9$  (min);  $T(E_{21}) = 12$  (min);  $T(E_{22}) = 4$  (min).

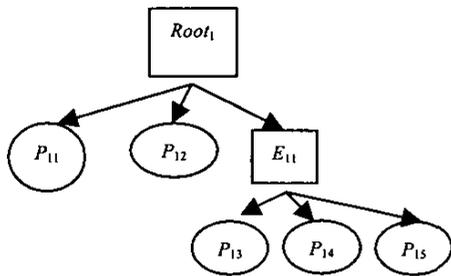


Figure 2. Structure of Product-1

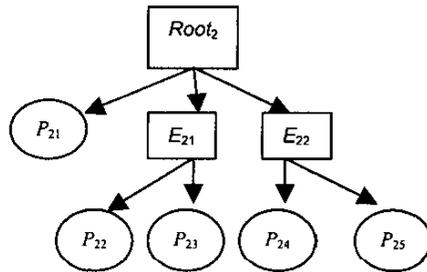


Figure 3. Structure of Product-2

Table 1. Data of Product-1

Part	$RV_{ij}$	$N_{ij}$	$W_{ij}$	$RI_{ij}$	$RCP_{ij}$	$DI_{ij}$	$b_{ij}$	$m_{ij}$
$P_{11}$	9.00	3	4.5	9.0	65%	3.0	0.1	0.0
$P_{12}$	0.50	9	9.0	4.0	40%	5.0	0.0	0.0
$P_{13}$	0.00	1	0.6	9.5	60%	5.5	0.0	0.4
$P_{14}$	2.10	1	2.5	5.0	55%	7.0	0.0	0.5
$P_{15}$	2.00	6	3.8	7.0	50%	1.0	0.0	0.0

Table 2. Data of Product-2

Part	$RV_{2j}$	$N_{2j}$	$W_{2j}$	$RI_{2j}$	$RCP_{2j}$	$DI_{2j}$	$b_{2j}$	$m_{2j}$
$P_{21}$	5.0	4	3.0	4.0	35%	3.0	0.0	0.3
$P_{22}$	0.8	7	5.0	2.0	50%	4.0	0.2	0.0
$P_{23}$	0.4	1	0.7	9.5	85%	5.5	0.0	0.0
$P_{24}$	1.1	5	3.5	5.0	45%	9.0	0.0	0.4
$P_{25}$	3.0	3	3.1	4.0	50%	3.0	0.0	0.0

Upon application of the model to the data of Product-1 and of Product-2 – using LINGO (v4), we get the optimal total revenue for Product-1 as \$20.52 and the optimal total revenue for Product-2 as \$8.40. Obviously, in this case, the decision-maker will proceed with Product-1.

### Phase-II of RSCN Design

In the second phase of strategic planning, we implement the Analytic Hierarchy Process (AHP) [8] to identify potential facilities in a set of candidate recovery facilities.

#### Analytic Hierarchy Process

The Analytic Hierarchy Process (AHP) is a tool, supported by simple mathematics, that enables decision makers to explicitly weigh tangible and intangible criteria against each other for the purpose of resolving conflict or setting priorities. The process has been formalized by Saaty [8] and is used in a wide variety of problem areas (e.g., siting landfills, evaluating employee performance, ranking city livability).

In a large number of cases (for example [7]), the tangible and intangible criteria are considered as independent i.e., those criteria do not in turn depend upon sub-criteria and so on. The AHP in such cases is conducted in two steps: (1) Weigh independent criteria, each of which can compare two or more decision alternatives, using pair-wise judgments, (2) Compute the relative ranks of decision alternatives using pair-wise judgments with respect to each independent criterion.

1. **Computation of relative weights of criteria:** AHP enables a person to make pair-wise judgments of importance between independent criteria with respect to the scale shown in Table 3. The resulting matrix of comparative importance values is used to weigh the independent criteria by employing mathematical techniques like eigen value, mean transformation or row geometric mean. In our paper, we employ the eigen value technique for computing the relative weights of the criteria.
2. **Computation of the relative ranks:** Pair-wise judgments of importance using the scale shown in Table 3 are computed for the decision alternatives too. These judgments are obtained with respect to each independent criterion considered in step 1. The resulting matrix of comparative importance values is used to rank the decision alternatives by employing mathematical techniques like eigen value, mean transformation or row geometric mean. Here again, we employ the eigen value technique for computing the ranks of decision alternatives.

Table 3. Scale for pair-wise judgments

Comparative Importance	Definition
1	Equally important
3	Moderately more important
5	Strongly important
7	Very strongly more important
9	Extremely more important
2, 4, 6, 8	Intermediate judgment values

The degrees of consistency of pair-wise judgments in steps 1 and 2 are measured using an index called the Consistency Ratio (CR). Perfect consistency implies a value of zero for CR. However, perfect consistency cannot be demanded since, as human beings, we are often biased and inconsistent in our subjective judgments. Therefore, it is considered acceptable if CR is less than or equal to 0.1. For CR values greater than 0.1, the pair-wise judgments must be revised before the weights of criteria and the ranks of decision al-

ternatives are computed.  $CR$  is computed using the formula:

$$CR = \frac{(\lambda_{max} - n)}{(n-1)(R)} \quad (7)$$

where  $\lambda_{max}$  is the principal eigen value of the matrix of comparative importance values;  $n$  is the number of rows (or columns) in the matrix;  $R$  is the Random Index generated by Oak Ridge National Laboratory for each number of rows (or columns) starting from one [8].

The AHP is illustrated in the form of a hierarchy of three levels where the first level contains the primary objective, the second level contains the independent criteria and the last level contains the decision alternatives. Also, as mentioned earlier, an important feature of the AHP is that the tangible and intangible criteria in the second level must be chosen in such a way that they can somehow help the decision maker in comparing two or more decision alternatives.

#### Selection of potential recovery facilities using AHP

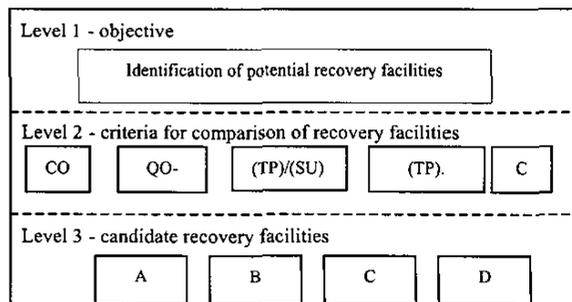
Here, the first level in the hierarchy contains our primary objective *i.e.*, selecting potential facilities from a set of candidate recovery facilities. The last level in the hierarchy contains the candidate recovery facilities. The level in the middle contains criteria that must somehow be useful in comparing the candidate recovery facilities. For example, fixed cost and average skill level of the employees are criteria that can compare the candidate facilities. Though the criteria to be considered in a reverse supply chain seem to be similar to those considered in a forward supply chain [1], there are three special factors in a reverse supply chain, which need to be incorporated in AHP in such a way that the hierarchy levels are not disturbed. The following are those special factors: Average quality of used-products; Average supply of used-products; Average disassembly time of used-products.

- Average quality of used-products: Unlike in a forward supply chain, components of incoming goods (used-products) of even the same type in a recovery facility are likely to be of varied quality (worn-out, low-performing, etc). Though the average quality of reprocessed goods (QO) is a criterion that can compare two or more candidate facilities, it is not justified to use QO as an independent criterion for comparison because QO depends on average quality of incoming products (QI). However, QI must not be taken as an independent criterion too because it cannot compare the candidate facilities. So, the idea is to take the difference between QO and QI as a criterion in the hierarchy.
- Average supply of used-products: The only driver to design a forward supply chain network is the demand for new products and so if there is low demand for new products, there is practically no forward supply chain. However, this is not the case in some RSCNs where even if there is low supply of used-products

(SU), reverse supply chain must be administered due to the possible drivers like environmental regulations and asset recovery. In supply-driven cases like these, it is unfair to judge a recovery facility without considering SU in the hierarchy. Though throughput (TP) is a criterion that can compare two or more candidate recovery facilities, it is not justified to use TP as an independent criterion because TP depends on SU. However, SU must not be taken as an independent criterion too because it cannot compare the candidate facilities. So, the idea is to take (TP)/(SU) as a criterion in the hierarchy.

- Average disassembly time of used-products: Average disassembly time (DT) is not exactly the inverse of TP because TP takes into account the whole reprocessing (disassembly plus recovery) time. Unlike in a forward supply chain, components of incoming goods (*viz.*, used-products) in a recovery facility are likely to be deformed and/or broken and/or different in number even for the same type of products. Hence, incoming products of the same type might have different reprocessing times, unlike in a forward supply chain where manufacturing time and assembly time are predetermined and equal for products of the same type. Since TP of a recovery facility depends upon the DT, it is unfair to not consider DT in the hierarchy. However, DT must not be taken as an independent criterion because it cannot compare the candidate facilities. So, the idea is to take (TP)(DT) as a criterion in the hierarchy.

The intangible criterion that we consider in our approach is customer service (CS). CS basically gives an idea about how well a recovery facility is utilizing the incentives provided by the government, by what extent it is meeting the environmental regulations, what kind of incentives it is giving the collection centers supplying the used-products and what kind of incentives it is giving the customers buying the reprocessed goods. We are using the term 'customer service' here because in our opinion, any beneficiary is a customer, be it the government or the collection center or the actual customer buying the reprocessed goods. In addition to the above criteria, we consider the fixed cost of the facility (CO) too in the hierarchy. Figure 4 illustrates the three-level hierarchy in our approach to implement the AHP for identifying potential facilities.



**Figure 4. Three-level hierarchy**

*Illustrative Example*

Table 4 shows comparative importance values given to the criteria in the second level of hierarchy in this example. It also gives the normalized eigen vector of the comparative importance value matrix. This vector represents the relative weights given by the decision maker to the independent criteria.

**Table 4. Comparative importance values at second level**

	CO	QO-QI	(TP)/(SU)	(TP).(DT)	CS	Norm. eigen vector
CO	1	1/5	3	3	1/5	0.13
QO-QI	5	1	7	7	5	0.54
(TP)/(SU)	1/3	1/7	1	1	1/3	0.06
(TP).(DT)	1/3	1/7	1	1	1/3	0.06
CS	5	1/7	3	3	1	0.21

Tables 5, 6, 7, 8 and 9 show comparative importance values of the decision alternatives viz., recovery facilities A, B, C and D with respect to the criteria viz., CO, (QO-QI), (TP)/(SU), (TP)(DT) and CS respectively. They also show the normalized eigen vectors of the respective comparative importance value matrices.

**Table 5. Comparative importance values of recovery facilities with respect to CO**

CO	A	B	C	D	Norm. eigen vector
A	1	3	6	2	0.460
B	1/3	1	7	3	0.310
C	1/6	1/7	1	1/4	0.050
D	1/2	1/3	4	1	0.180

**Table 6. Comparative importance values of recovery facilities with respect to (QO-QI)**

QO-QI	A	B	C	D	Norm. eigen vector
A	1	1	7	4	0.380
B	1	1	7	7	0.445
C	1/7	1/7	1	1/5	0.050
D	1/4	1/7	5	1	0.125

Each of the matrices in Tables 4, 5, 6, 7, 8 and 9 has a CR whose value is less than 0.1. Table 10 shows the aggregate matrix of rankings of recovery facilities with respect to each criterion in the second level of hierarchy. This matrix

is nothing but the aggregate of the eigen vectors obtained in Tables 5, 6, 7, 8 and 9.

Multiplying the matrix in Table 10 with the normalized eigen vector obtained in Table 4, we get the following normalized ranks for the facilities: Rank<sub>A</sub> = 0.28; Rank<sub>B</sub> = 0.41; Rank<sub>C</sub> = 0.09; Rank<sub>D</sub> = 0.22. If the decision maker wishes to choose only those recovery facilities whose ranks are at least 25 percent as the potential recovery facilities, he will choose recovery facilities A and B.

**Table 7. Comparative importance values of recovery facilities with respect to (TP)/(SU)**

(TP)/(SU)	A	B	C	D	Norm. eigen vector
A	1	1/7	1/3	1/2	0.072
B	7	1	2	7	0.574
C	3	1/2	1	1	0.212
D	2	1/7	1	1	0.142

**Table 8. Comparative importance values of recovery facilities with respect to (TP)(DT)**

(TP)(DT)	A	B	C	D	Norm. eigen vector
A	1	1/7	1/3	1/2	0.072
B	7	1	2	7	0.574
C	3	1/2	1	1	0.212
D	2	1/7	1	1	0.142

**Table 9. Comparative importance values of recovery facilities with respect to CS**

CS	A	B	C	D	Norm. eigen vector
A	1	1/6	1/3	1/7	0.053
B	6	1	5	1/3	0.298
C	3	1/5	1	1/6	0.101
D	7	3	6	1	0.548

**Table 10. Aggregate of rankings of recovery facilities with respect to each decision factor**

RF	A	B	C	D
CO	0.460	0.310	0.050	0.180
QO - QI	0.380	0.445	0.050	0.125
(TP)/(SU)	0.072	0.574	0.212	0.142
(TP).(DT)	0.072	0.574	0.212	0.142
CS	0.053	0.298	0.101	0.548

**Phase-III of RSCN Design**

In the third phase of strategic planning, we formulate a single time-period discrete location model to achieve transportation of the right mix and quantities of goods (used as well as reprocessed) across the RSCN. For simplicity, re-

manufacturing is the only recovery option that is considered in the formulation. The following notation is used in the model:

- $a_1$  space occupied by one unit of reprocessed product (square units/product);
- $a_2$  space occupied by one unit of used-product (square units/product);
- $C_u$  cost per product retrieved at collection center  $u$  (\$/product);
- $d_w$  demand of reprocessed products at demand center  $w$ ;
- $F_v$  fixed cost of recovery facility  $v$  (\$);
- $I_{uv}$  decision variable representing the number of products to be transported from collection center  $u$  to recovery facility  $v$ ;
- $O_{vw}$  decision variable representing the number of products to be transported from recovery facility  $v$  to demand center  $w$ ;
- $R_v$  cost of reprocessing per product at recovery facility  $v$  (\$/product);
- $S_{1v}$  storage capacity of recovery facility  $v$  for reprocessed-products (square units);
- $S_{2v}$  storage capacity of recovery facility  $v$  for used-products (square units);
- $S_u$  storage capacity of collection center  $u$  for used-products (square units);
- $TI_{uv}$  cost of transporting one product from collection center  $u$  to recovery facility  $v$  (\$/product);
- $TO_{vw}$  cost of transporting one product from recovery facility  $v$  to demand facility  $w$  (\$/product);
- $Y_v$  decision variable representing selection of recovery facility  $v$  ( $Y_v = 1$ , if  $v$  is selected and 0, if not selected).

The following is the discrete location model formulation that is implemented to achieve transportation of the right mix and quantities of goods across the network (in the formulation, we assume that the inventory cost of a used-product is 25 percent of its retrieval cost,  $C_u$ , and that of a reprocessed product is 25 percent of its reprocessing cost,  $R_v$ ):

Minimize

$$\begin{aligned} & \sum_u \sum_v C_u I_{uv} + && \text{Retrieval costs} \\ & \sum_u \sum_v TI_{uv} I_{uv} + \sum_v \sum_w TO_{vw} O_{vw} + && \text{Transportation costs} \quad (8) \\ & \sum_v \sum_w R_v O_{vw} + && \text{Reprocessing costs} \\ & \sum_u \sum_v (C_u / 4) \cdot I_{uv} + \sum_v \sum_w (R_v / 4) \cdot O_{vw} + && \text{Inventory costs} \\ & \sum_v F_v Y_v; && \text{Fixed costs} \end{aligned}$$

subject to

$$\sum_v O_{vw} = d_w \forall w; \quad (9)$$

$$\sum_v \sum_w O_{vw} \leq \sum_u \sum_v I_{uv}; \quad (10)$$

$$\sum_w a_1 \cdot O_{vw} \leq S_{1v} Y_v; \forall v \quad (11)$$

$$\sum_v a_2 \cdot I_{uv} \leq S_u; \forall u \quad (12)$$

$$\sum_v a_2 \cdot I_{uv} \leq S_{2v} Y_v; \forall v \quad (13)$$

$$I_{uv} \geq 0 \forall u, v; \quad (14)$$

$$O_{vw} \geq 0 \forall v, w; \quad (15)$$

$$Y_v \in \{0,1\} \forall v; \quad (16)$$

### Illustrative Example

Besides three collection centers and three demand centers, we consider the product type chosen in phase-I and the two potential recovery facilities  $A$  and  $B$  chosen in phase-II. The necessary data for implementation of the location model are as follows:  $C_1 = 29$ ;  $C_2 = 25$ ;  $C_3 = 37$ ;  $TI_{1A} = 3$ ;  $TI_{2A} = 4$ ;  $TI_{3A} = 5$ ;  $TI_{1B} = 1.1$ ;  $TI_{2B} = 2$ ;  $TI_{3B} = 3$ ;  $TO_{A1} = 2.6$ ;  $TO_{B1} = 1.2$ ;  $TO_{A2} = 3.4$ ;  $TO_{B2} = 2.9$ ;  $TO_{A3} = 1.6$ ;  $TO_{B3} = 4.7$ ;  $R_A = 4$ ;  $R_B = 8$ ;  $F_A = 10000$ ;  $F_B = 15000$ ;  $d_1 = 800$ ;  $d_2 = 600$ ;  $d_3 = 500$ ;  $a_1 = a_2 = 0.5$ ;  $S_{1A} = 550$ ;  $S_{1B} = 550$ ;  $S_{2A} = 550$ ;  $S_{2B} = 550$ ;  $S_1 = 550$ ;  $S_2 = 550$ ;  $S_3 = 550$ .

Upon application of the above data to the discrete location model - using LINGO (v4), we get the following optimal solution:

- $Y_A = 1$ , i.e., recovery facility A is open;
- $Y_B = 0$ , i.e., recovery facility B is closed;
- $I_{1A} = I_{1B} = 0$ , i.e., no products are to be transported from collection center - 1 to the recovery facilities;
- $I_{2A} = 440$ , i.e., 440 products are to be transported from collection center - 2 to recovery facility A;
- $I_{2B} = 0$ , i.e., no products are to be transported from collection center - 2 to recovery facility B;
- $I_{3A} = I_{3B} = 0$ , i.e., no products are to be transported from collection center - 3 to the recovery facilities;
- $O_{A1} = 200$ , i.e., 200 products are to be transported from recovery facility A to demand center - 1;
- $O_{A2} = 150$ , i.e., 150 products are to be transported from recovery facility A to demand center - 2;
- $O_{A3} = 90$ , i.e., 90 products are to be transported from recovery facility A to demand center - 3;
- $O_{B1} = O_{B2} = O_{B3} = 0$ , i.e., no products are to be transported from recovery facility B to the demand centers.

### CONCLUSIONS

We utilized a three-phase mathematical programming approach in our methodology to effectively design an efficient reverse supply chain network. Phase I selected the most economical product to reprocess from a set of different types of used-products, using a mixed-integer mathematical programming model. Phase II implemented the Analytic Hierarchy Process (AHP) to identify potential facilities in a set of candidate recovery facilities. Phase III solved a single time-period discrete location model to achieve transportation of the right mix and quantities of goods (used as well as reprocessed) across the RSCN.

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