

January 01, 2007

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Ann W. Morgenthaler

He Zhan

Carey M. Rappaport
Northeastern University

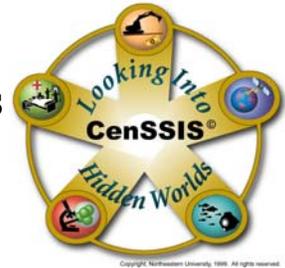
Recommended Citation

Morgenthaler, Ann W.; Zhan, He; and Rappaport, Carey M., "The semi-analytic mode matching (SAMM) algorithm for efficient computation of nearfield scattering in lossy ground from borehole sources" (2007). *Research Thrust R1 Presentations*. Paper 20.
<http://hdl.handle.net/2047/d10010130>

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The Semi-Analytic Mode Matching (SAMM) Algorithm for Efficient Computation of Nearfield Scattering in Lossy Ground from Borehole Sources



Ann W. Morgenthaler, He Zhan and Carey M. Rappaport

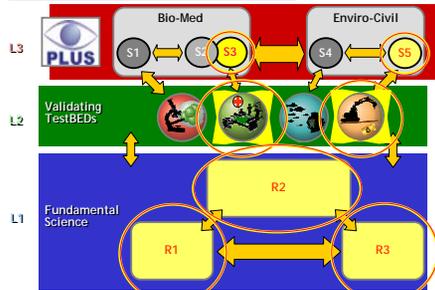
Contact: rappaport@ece.neu.edu

This work is supported by Gordon-CenSSIS, the Gordon Center for Subsurface Sensing and Imaging Systems under the ERC Program of the NSF (Award number EEC-9986821).

Abstract

The **three dimensional** semi-analytic mode matching (SAMM) algorithm is used to determine nearfield scattering from underground targets in lossy soil, where the source is a dipole placed within a borehole in the ground. Scattering is described by moderately low-order superpositions of spherical modes placed at multiple user-specified coordinate scattering centers (CSCs); the mode coefficients are found numerically by least-squares fitting all boundary conditions at discrete points along the relevant interfaces while at the same time obeying radiation conditions. SAMM results are compared with a completely different method: the Half-Space Born Approximation (HSBA). Good agreement between methods serves to validate both algorithms. Unlike HSBA, SAMM is not a perturbative algorithm and does not require small dielectric or volumetric perturbations of the half-space geometry. In general, SAMM is a faster method and for inverse problems, key scattering features can often be determined with low-order modes, sacrificing precise field details in favor of computational speed. By contrast, the HSBA method allows scatterers to be continuously distributed, and one can "reuse" HSBA results once the time-consuming problem-specific Green's function is calculated for the underlying geometry, needing only a fast matrix multiplication for the particular scatterer(s).

Value Added to CenSSIS



SAMM Algorithm: Basic Principles

- Identify 2D (cylindrical) / 3D (spherical) modes which satisfy wave equation and radiation conditions. The appropriate (spherical) Bessel functions are chosen so that fields remain finite away from sources and radiation conditions hold in the far field.
- Place points on the surface of all interfaces (e.g., air/ground and ground/target) for which boundary conditions on all three (cylindrical) six (spherical) Cartesian field components are to be matched, with point spacing and number chosen to maximize algorithm speed and minimize computational storage.
- Choose locations and orientations of coordinate scattering centers (CSCs) from which the waves will emerge with as yet undetermined coefficients; CSCs are generally placed anywhere scattered waves appear to originate.
- Construct a dense matrix **F** linking the undetermined mode coefficients (**c**) with the field mismatches (**b**) which arise in **E**, **H** at each point on the material interfaces.
- Minimize the matrix equation $\mathbf{F} \cdot \mathbf{c} = \mathbf{b}$ using singular value decomposition, selecting the (small) group of mode coefficients which best fit the (large) number of boundary conditions; the size of built-in error function $\|\mathbf{F} \cdot \mathbf{c} - \mathbf{b}\|$ indicates how successfully SAMM has modeled scattering. The SAMM solution will be optimal in the nearfield region where it is properly constrained and inaccurate in the extrapolated region away from the fitting points.

SAMM Equations

The scalar Debye potential is a solution of the Helmholtz equation and forms the starting point for the SAMM algorithm, with 3D solutions to the Debye potential given by spherical modes:

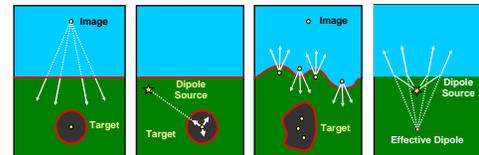
$$\Pi^{e,m}(\mathbf{r}) = \sum_{n=0}^{\infty} \sum_{m=-n}^n \sum_{p=1}^{\min(n,M)} c_{nm,p}^e f_n(kr_p) P_n^m(\cos\theta_p) e^{im\phi_p}$$

Electric and magnetic fields, found by differential operations on the Debye potential, are thus also superpositions of spherical modes. Here, $f_n(kr_p)$ is a spherical Bessel or spherical Hankel function of the first kind, and $P_n^m(\cos\theta_p)$ is an associated Legendre polynomial. The coordinates \mathbf{r} and $\mathbf{r}_p = (r_p, \theta_p, \phi_p)$ are related by $\mathbf{r}_p = \mathbf{r} - \mathbf{c}_p$ where \mathbf{c}_p is the location of the p th coordinate scattering center. As well as having a spatial location, each CSC may also have a user-specified orientation. The series expansion is truncated by choosing N and M , the maximum radial and angular mode indices, such that $n \leq N$ and $|m| \leq M \leq n$. Well-chosen CSC locations and/or orientations can reduce N and M and still achieve convergence: in successful SAMM simulations, N is approximately 7-15 and M is on the order of 1-5 for target size and burial depths which are on the order of a few wavelengths.

Half-Space Born Approximation (HSBA)

The 3D half-space Born approximation (HSBA) algorithm uses plane wave decomposition techniques to construct spherical waves from the superposition of all possible plane wave modes in half-space geometries containing well-characterized dispersive media, where fields are found from the vector Helmholtz equation using the dyadic Green's function. Classical Fresnel theory is used to calculate the resulting reflected plane waves at the planar interface, and the first-order Born approximation (weaker scatterer assumption) is applied to establish a linear relationship between the field and the object characteristic function. SAMM and HSBA are two very different methods for generating fields from pre-specified geometries but should yield similar results for identical geometries and can be used for cross-validation.

Choosing CSCs

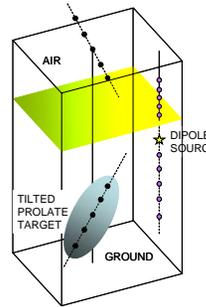


- Add a CSC at the target image to describe back scattering
- Expand modes about rotated CSC axis if preferred axis exists
- Add CSCs in rough ground surface and irregular target
- Add a CSC at the target location to describe refraction

Two-Step Solution for Dipole & Scatterer

- Ignore the scatterer (replacing the scatterer's dielectric constant with that of the ground) and find the electric and magnetic fields that would exist were the dipole source located within infinite uniform ground. The resulting analytic field solution is now a partial solution to the inhomogeneous Helmholtz wave equation *solely* in the ground region of the half-space problem. **No azimuthal modes needed because of symmetry! CSCs along the source axis only!**
- Reintroduce the correct dielectric constant within the target region, and "zero out" fields within that region because they no longer obey the Helmholtz equation there. The discontinuities in fields across the target-ground interface now act as sources for additional homogeneous, scattered half-space field solutions, which must exist within the scatterer, ground, and air regions. **CSCs only within scatterer and image!**

Choosing CSCs for Underground Target with Borehole Dipole Source



Step 1 - No Target:

Use purple CSCs (no need for azimuthal modes) to generate fields for buried dipole source configuration with no target

Step 2 - Add Target:

Use black CSCs within target and its image to generate total scattered fields, using Step 1 fields as the source

Comparison of SAMM and HSBA

SAMM	HSBA
Scatterer(s) are not necessarily small dielectric or volumetric perturbations of the half-space geometry	Scatterers may be distributed, or many discrete scatterers may exist in the geometry
Computational speed is faster - a Matlab implementation of SAMM running on a laptop takes about 1/10 the computational time of HSBA run on an Alpha workstation.	
For inverse problems, key scattering features can often be determined with low-order modes, sacrificing details for speed	Can "reuse" results once the time-consuming problem-specific Green's function is calculated for the underlying geometry, needing only a fast matrix multiplication for the particular scatterer(s)

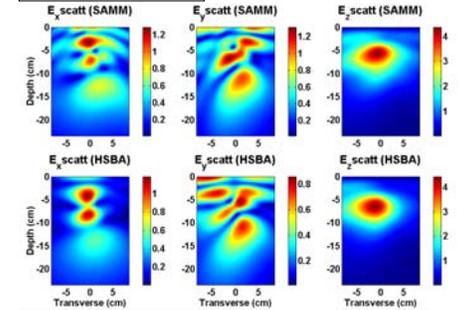
3D DNAPL Tilted Prolate Spheroid Buried in Wet Sand with a Dipole Source

- Frequency: 1.4 GHz, Grid Step Size $h = \lambda/10 = 0.0047$ m
- z -directed dipole source located at $(-17h, 17h, -10h)$
- Permittivity, wet sand: $\epsilon = (20.7 + 1.4i) \epsilon_0$, DNAPL: $\epsilon = (2.63 + 0.016i) \epsilon_0$
- Prolate spheroidal scatterer located at $(0, 0, -15h)$;
- Dimensions: $R_1 = R_2 = 2h$ and $R_3 = 5h$, where $(x/R_1)^2 + (y/R_2)^2 + (z/R_3)^2 = 1$
- Spheroid major axis tilted 30°
- SAMM: 841 points placed over planar surface $85h \times 85h$ in size centered over the prolate spheroid; 786 points placed over spheroid at equi-angular spacing

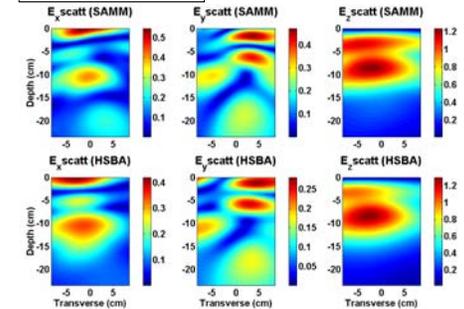
CSC locations for SAMM:

- Step 1 - Half-space with no target:** 5 CSCs are located within the ground at $(-17h, 17h, -10h \pm z_k)$ where $z_k = (1 + k/4)$, $k = -2, -1, 0, 1, 2$ and 5 image CSCs are located within the air at $(-17h, 17h, +10h \pm z_k)$ where n_{img} is the index of refraction of wet sand
- Step 2 - Target sourced by half-space fields from Step 1:** 5 CSCs are located within the prolate spheroid along its semi-major axis at $z_k = (-\frac{1}{2}R_3 - \frac{1}{2}R_3, 0, \frac{1}{2}R_3, \frac{1}{2}R_3)$ with respect to its center and 5 image CSCs are located within the air where the z coordinate of the CSCs has been reflected about the ground plane; CSCs are rotated to align with semi-major axis

y - z slice at $x = 8h$:



y - z slice at $x = 24h$:



SAMM (Matlab laptop) takes about 1/10 the computational time of HSBA (Alpha workstation)

Summary

The SAMM algorithm is a useful nearfield algorithm, orders of magnitude faster than FDFD and requiring substantially less computational overhead. SAMM has wide applicability and may be used for irregularly shaped boundaries and for both metallic and dielectric objects, multiple scatterers and rough surfaces. By choosing CSC locations/orientations properly and treating background half-space scattering separately, a full 3D SAMM simulation may be quickly obtained of underground (borehole) dipole sources with quasi-2D computational size. SAMM and HSBA, two very different computational methods, give excellent agreement in a wide variety of cases.

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