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## Superconducting windings in a power transformer, an old question with a new answer

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SUPERCONDUCTING WINDINGS IN A POWER TRANSFORMER, AN OLD  
QUESTION WITH A NEW ANSWER

Part II: Some of the practical problems

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ABSTRACT

This is the second of two papers examining the practicality of superconducting windings for large power transformers. The issues examined in this paper include elements of the behavior of superconducting tapes that would be important in power transformers, the refrigeration requirements for a step-up transformer of practical size, a coil structure appropriate to a superconducting winding and its cryogenic environment, current crowding in the end turns, and an estimate of the energy savings that a practical superconducting transformer could be expected to realize.

A. INTRODUCTION

In Part I of these papers (1) we examined the possibility of using superconducting windings for a very large step-up transformer suitable for sustained power system service. We showed that the state-of-the-art in superconducting materials is now sufficiently advanced to make such a transformer possible, and we gave some broad guidelines on what such a transformer would be like. In this paper, we go into greater depth on five subjects that arise naturally out of the discussions of the transformer itself. These comprise:

1. AC losses in a superconducting tape, diamagnetism, and the inutility of transposition;
2. refrigeration requirements, including some elementary estimates of heat leaks and the costs of refrigeration apparatus;
3. building a coil to withstand electrical and mechanical stress in a cryogenic environment;
4. current crowding in the turns at the ends of the coils; and
5. estimates of the energy to be saved by employing superconducting windings.

Our objective in each of these discussions is to bring the issues into sharp focus in a framework that is recognizable to Power Systems Engineers. Since much of the material included in this paper is derived from relatively distant disciplines, we include tutorial discussions so that a designer of conventional transformers can better appreciate the special properties and problems of superconducting windings that we discuss both in this paper and in part I.

B. AC CURRENT DISTRIBUTION AND AC LOSSES IN SUPERCONDUCTING TAPES

1. An Introduction to Superconductivity

Were the Red Queen writing this section, she would begin by stating that: "Superconductors are not what you think they are!" There are, in fact, two types of superconductivity, and neither of them resembles the fictional "perfect conductor" that perme-

ates the texts on field theory. As long as the superconducting state is maintained, all superconductors share the property of:

Persistence of a current.

Once established, a current will continue to flow until the superconducting state is destroyed. The net flow of current can be halted, increased or reversed by inducing appropriate additional currents, but as long as the superconducting state persists, so will the current.

The second property of superconductors that is shared in varying measure is:

Diamagnetism

In a type-I superconductor, such as Pb or Sn or Al, the diamagnetism is absolute -- all magnetic flux is excluded from the interior volume of the superconductor. [Just as with statements such as "the charge resides on the surface of a metal", closer examination reveals that "surface properties" vanish exponentially as you proceed inward toward the bulk material. The depth of penetration for flux into the surface of a type-I superconductor is of the order of a few hundred Angstroms.] This absolute exclusion of flux from the bulk, called the Meissner effect, is a dynamic property. If you establish magnetic flux in a potentially superconducting metal, such as Pb, while it is still in its normal metallic state, and then lower the temperature till the lead goes superconducting, currents will be established on the surface of the superconductor that cancel -- or exclude, if you prefer -- the flux that had been there in the normal state.

In a type-II superconductor, such as a work-hardened type-I element or an alloy such as  $Nb_3Sn$  or  $V_3Ge$ , the diamagnetism is not so absolute. All such materials show some of the absolute diamagnetism of the type-I materials, but if the field at the surface gets above a critical value, the flux begins to penetrate. An excellent phenomenological view of what is going on is to think of the type-II superconductor as a random network of superconducting filaments embedded in a matrix of normal metal. This is a model due to Bean (2) that proves most useful in predicting the AC behavior of type-II superconductors. What permits the flux to enter the bulk region is the inability of the superconducting currents to get out of their individual filaments. They are said to be pinned in place.

All superconductors have a maximum temperature,  $T_c$ , at which they can maintain their superconducting properties. Below that temperature, there exists for each superconductor a maximum current  $I_c$  and a maximum magnetic field,  $H_c$ , at which they can still maintain their superconducting state. Exceeding either the critical field or the critical current will cause the material to switch back quite abruptly to its normal, lossy, metallic state.  $H_c$  and  $I_c$  are both functions of the sample's temperature.

The highest critical fields, currents and temperatures are all found in alloyed type-II superconductors. In fact, the critical parameters for the purely type-I superconductors are generally too low to make them of much interest to designers of high field or high power devices. Accordingly, we will deal exclusively with type-II superconductors, in particular  $Nb_3Sn$ , a material with excellent critical parameters whose difficult metallurgical properties have been tamed to the point of making it the material of choice for most high power AC applications.

At the outset it is important to recognize that combining the concepts of persistent currents and the bulk distribution of these currents, as in Bean's model, implies quite directly that there is hysteresis in the plot of flux,  $\phi$ , versus current. This point will be stressed again below, but for the moment think of the situation that would transpire if you had a current,  $I_0$ , flowing in a superconducting ribbon and then reduced that current to zero. According to rule #2, you would have half the filaments carrying positive current and the other half carrying negative current. These currents will be in layers, as we show below, so at zero current there is a net B field within the superconductor, with net energy storage. The direction of that field will depend on whether previously  $I_0$  or  $-I_0$  was flowing. Accordingly, there is memory and hysteresis in a type-II superconductor.

## 2. Derivation of the loss-rate equation

The loss formula used in reference (1) is essentially that derived by Dunn et al (3) and Fournet et al (4). We present our own derivation here to provide the reader (and ourselves) with the results in practical units. The first part of the derivation follows that of Bean (2), from which the proper final form can easily be extracted by carefully adding the lossless surface sheet of current.

The basic premises of the derivation are:

1. A hard superconductor is an array of superconducting filaments that are pinned in place in a matrix of low conductivity normal metal.
2. Each filament has only three states corresponding to +1, 0, -1. Once a particular volume has been exposed to magnetic flux, only the +1 and -1 states will be found. Assuming a homogeneous superconductor, this means that the local current density is either  $+J_c$  or  $-J_c$ .

These assumptions of quantized filaments of current leads directly to a rather conventional looking hysteresis loop for the magnetization loop of a hard superconductor (one operating in the type II mode). From the hysteresis loop, one may calculate the losses for cyclic excitation. Consider, for example, a plane sheet of superconducting material excited from one side by a time-dependent magnetic field.

To derive the hysteresis loop, we begin from the point in the cycle when the sheet current density reaches its maximum positive value,  $\sigma_m$  amperes/m. At this moment, in view of assumption 2, the current is carried uniformly in a sheet of depth  $\delta = \sigma_m / J_c$ .

Also, directly from Ampere's law, the magnetic field,  $H(x)$ , is given by:

$$H(x) = H_0(1-x/\delta) \text{ where } H_0 = \sigma_m \quad (1)$$

It is worthwhile to note how different this situation of a uniform current sheet is from either a thick, normal metal (where the current amplitude would follow a form,  $\exp[-(\alpha+j\beta)x]$ ) or a type-I superconductor where the entire current is carried in an exponentially tapered sheet of very small depth (of the order of a few hundred angstroms) at the surface.

To have the sheet current density decrease from its initial maximum value, the change must move in from the surface (as with any conductor). In view of assumption 2, the change is accomplished by pushing a layer of current density,  $-J_c$ , in from the surface. Figure 1 below shows the current distribution when the sheet current density has fallen to  $\sigma_m/2$ .

$H(x)$  at the same moment is also shown.

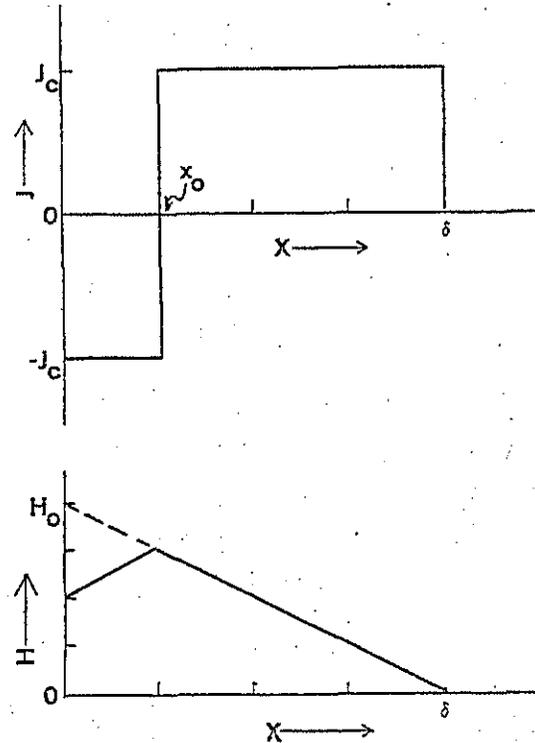


Figure 1:  $J(x)$  and  $H(x)$  when  $\sigma$  has decreased from  $\sigma_m$  to  $\sigma_m/2$ .

Defining the point where the current density switches from  $-J_c$  to  $+J_c$  as  $x_0$  (see figure 1), one may write the general expression for  $\sigma$  and the average magnetic field,  $\bar{H}$ , within the active region:

$$\sigma = \sigma_m [1 - 2x_0/\delta] \quad [\text{n.b.: } \sigma_m = H_0] \quad (2)$$

$$\bar{H} = (1/2)[(H_0 - J_c x_0)(1 - x_0/\delta) + (2H_0 - 3J_c x_0)(x_0/\delta)] \quad (3)$$

Eliminating  $x_0$  between these two equations and taking advantage of the relationship  $\sigma_m = H_0 = J_c \delta$ , one obtains:

$$\bar{H} = (H_0/4)[1 + 2\sigma_R - \sigma_R^2] \text{ where } \sigma_R = \sigma/\sigma_m \quad (4)$$

The total magnetic flux within the active region is given by:

$$\phi = \mu_0 \bar{H} \delta = \mu_0 \bar{H} H_0 / J_c \quad (5)$$

At this point one can draw the magnetization curve from  $H_0$  to  $-H_0$ .

The expression equivalent to (4) for the increasing-flux path is:

$$\bar{H} = (-H_0/4)[1 - 2\sigma_R - \sigma_R^2] \quad (6)$$

With (4, 5, and 6) one may plot a conventional  $\phi$  vs.  $I$  style of magnetization curve. In figure 2 below, the solid dash line represents a plot of  $\phi$  vs.  $\sigma$  according to equations (4, 5, and 6). The area within the loop is the sheet energy density (joules/m<sup>2</sup>) dissipated in a single cycle. The superconductor is carrying a peak current of  $\sigma_m$  amperes/m, and within the current-carrying portion of the superconductor, a peak magnetic flux of  $\phi_m$  webers/m is found. [Note that the "per meter" for  $\phi_m$  flux is "per meter of conduction"; that length is parallel to the current. For the current which is spread laterally across the ribbon, there are so many amperes per meter with that "meter" being perpendicular to the current in the plane of the current.]

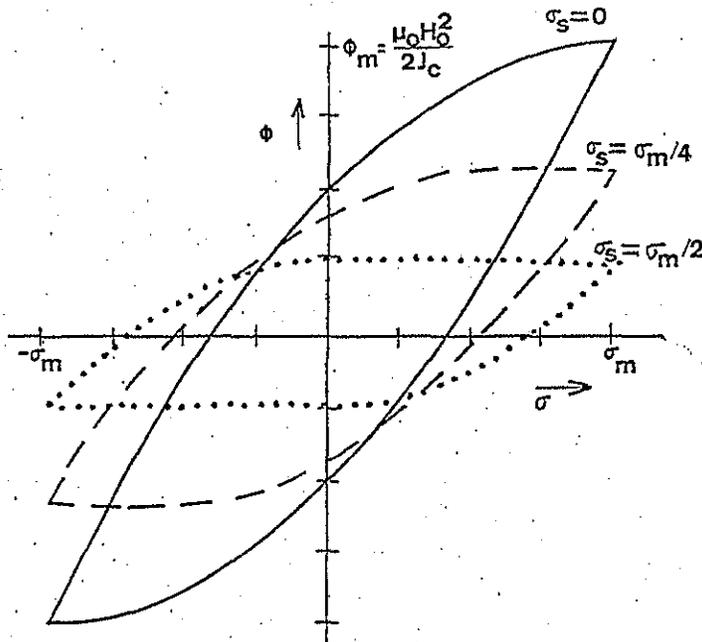


Figure 2: Magnetization curves for a type-II superconductor for various values of  $\sigma_s$ .

Integration to determine the area within the hysteresis loop yields:

$$E = (2/3)\mu_0 H_0^3 / J_c \quad \text{joules/m}^2 \quad (7)$$

(7) is, apart from the conversion to MKS units, the result given by Bean<sup>(2)</sup>. To get the hysteretic losses under sinusoidal excitation, one simply multiplies (7) by the frequency. At 60 Hz and with the  $J_c$ 's employed by Wilkinson ( $J_c = 10^5$  A/cm<sup>2</sup>), a film carrying a peak current of  $5 \times 10^4$  A/m would be dissipating 1 Kw/m<sup>2</sup>! It was losses such as these that led Wilkinson (see (1) for a discussion) to discount superconducting windings. However, the measured value<sup>(6)</sup> of the intrinsic losses under these conditions of frequency and current

tion of the losses over those envisaged by Wilkinson.

The first is simply that  $J_c$  is really about one and a half orders of magnitude higher than Wilkinson's figure. The second factor, which is the one that reduces the losses to zero at this relatively low current loading, is the presence of a lossless sheet of current traveling on the surface of the superconductor. This current is predicted by Abrikosov<sup>(5)</sup> and mentioned by Bean<sup>(2)</sup>, but there was little evidence for it in the materials available in the early sixties. Recent measurements<sup>(6)</sup> on properly prepared samples of Nb<sub>3</sub>Sn tape show that this surface current has the impressively high value of about  $5.5 \times 10^4$  A/m (peak). Thus, the current proposed above is less than the maximum surface current, and the flux would not penetrate into the tape. If one exceeds this maximum surface current,  $\sigma_s$ , flux and current will penetrate into the bulk in just the fashion described above.

Noting once again that all changes in current must propagate from the outside inward, so that the first current to reverse is the surface current, one

may write the proper versions of (4) and (6) to include the effects of  $\sigma_s$ :

For the path from  $\phi_m$  to  $-\phi_m$ : ( $\sigma \leq \sigma_m - 2\sigma_s$ )

$$\phi = (\phi_m/2) \left[ 1 + 2 \left( \frac{\sigma_0 + \sigma_R}{1 - \sigma_0} \right) - \left( \frac{\sigma_0 + \sigma_R}{1 - \sigma_0} \right)^2 \right] \quad \text{webers/m} \quad (8)$$

where  $\sigma_0 = \sigma_s / \sigma_m$  and  $\sigma_R = \sigma / \sigma_m$ .

Similarly, for the path from  $-\phi_m$  to  $\phi_m$ : ( $\sigma > -\sigma_m + 2\sigma_s$ )

$$\phi = (-\phi_m/2) \left[ 1 - 2 \left( \frac{\sigma_0 + \sigma_R}{1 - \sigma_0} \right) + \left( \frac{\sigma_0 + \sigma_R}{1 - \sigma_0} \right)^2 \right] \quad (9)$$

Straight-line segments connect the end points at the top and bottom of the two curvilinear line segments.

Two examples of the curve predicted by equations (8 and 9) are shown in figure 2. They are drawn for the same total current but with varying fractions of that current being carried by the surface current sheet. Note that as the surface current fraction increases, the loop collapses into the  $\sigma$ -axis, with the area of the loop going to zero. Alternatively, as the surface current becomes a less important fraction of the total current, the loop takes on the form predicted by the simple Bean theory(19).

Computing the area of the loop for an arbitrary  $\sigma_s$  between 0 and  $\sigma_m$ , one obtains the relationship:

$$E = (2\mu_0/3J_c)[(\sigma_m - \sigma_s)^3 + 3\sigma_s(\sigma_m - \sigma_s)^2] \quad (10)$$

Note that  $\sigma_m = H_0$ , so (10) may be readily set in terms of the magnetic field. Furthermore, note that if  $\sigma_0 = 0$ , (10) reduces to Bean's simpler form of (7).

Reasonable values for  $J_c$  and  $\sigma_s$  for  $Nb_3Sn$  are  $5 \times 10^{10}$  amperes/m<sup>2</sup> and  $5.6 \times 10^4$  amperes/m respectively (6). That makes the coefficient in (10)  $1.7 \times 10^{-17}$  V-sec-m/A<sup>2</sup>. At the maximum current capacity of a typical thin tape (tapes of various thickness are available), say  $\sigma = 3.5 \times 10^5$  A/m, the loss rate per cycle becomes:  $E = 0.57$  joules/m<sup>2</sup>. Thus, at 60 Hz the loss rate will be about  $42$  W/m<sup>2</sup>; important but far less than the kilowatts/m<sup>2</sup> that Wilkinson thought would obtain.

### 3. The inutility of transposition in AC applications

The thickest conductor that is practical in a transformer winding is noticeably thinner than the skin depth. To get greater current capacity than one such conductor can give, one uses multiple strands and arranges the strands so that all carry equal current. That arrangement is obtained by appropriate transposition of the strands so that each strand "sees" the same flux linkage. Unfortunately, such a scheme presumes a finite skin depth. As we have discussed above, for all practical purposes, the skin depth for a type-I superconductor is zero. Since a superconducting tape of normal thickness has, perforce, two sides, flux from an outer layer cannot penetrate through an inner one to contribute to the total flux inside. Of course, Ampere's circuital law is just as valid with superconducting materials as with normal ones. Thus, if you place two coils, each carrying  $I$  amperes through with  $N$  turns per meter, in a concentric, coaxial configuration, the field inside the inner coil will indeed have its conventional value,  $H = 2NI$ . Unfortunately, Ampere's law is satisfied not by having the fields from the inner and outer coils add, but rather by having a current of  $2I$  flowing on the inner surface of the inner conductor. Since the purpose of the two layers is to distribute the current loading equally between them, multiple layers of type-I superconducting material serve no purpose.

Figure 3 illustrates the issues involved. It shows two concentric superconducting tapes one meter wide each carrying a net current,  $I$ . The outer one carries the current  $I$  on its inner surface. Ampere's law requires that the field inside this outer tape be  $I$  amp-turns/m. However, the Meissner effect requires that  $H$  be zero inside the inner tape. Thus, a current of  $-I$  must be flowing on the outer surface of the inner tape. The net current flowing on the inner tape

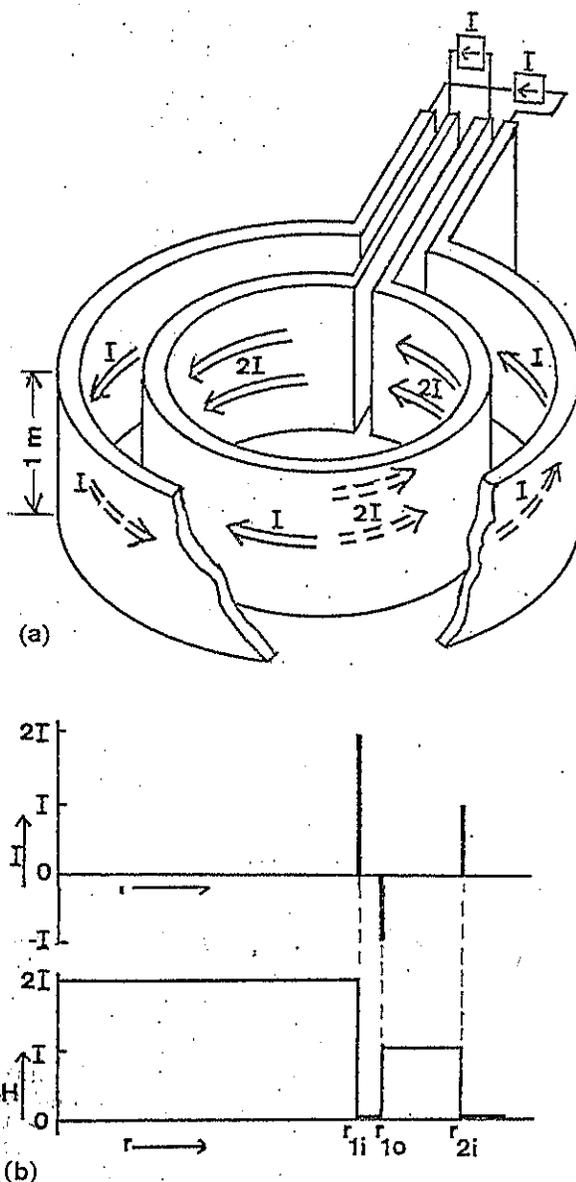


Figure 3: Sheets of current in layers of type-I superconductors. (a) 3 dimensional illustration. (b) Plots of the current,  $I$ , and the magnetic field,  $H$ , as functions of the radial distance,  $r$ .

is  $I$ , so there must be a current of  $+2I$  flowing on the inner surface of the inner tape. Were the purpose of the two tapes to have a current of only  $I$  on any surface with a total of  $2I$  flowing, the Meissner effect would clearly have frustrated the designer.

The figure shows independent current generators establishing the currents. The closure of the several current loops is not specified. One possible closure for the "extra" current  $I$  on the inner tape is across the end of the tape near the generator. Note that that makes a closed current loop entirely on the superconducting tape, the current in that loop being just the value that reduces the net interior field to zero. For the case of transposed windings, each strand of which spends a certain portion of the coil length in the inner position, the closure of the shielding current occurs at the point where the tape is bent for transposition.

The conclusion to be drawn from this discussion

is that if you want to operate in a type-I mode and use superconducting layers that are thicker than a few hundred Angstroms, every layer of so many ampere-turns must be opposed by an equal and opposite number of ampere-turns with major insulation between them. Transposition will not work. Reference 1 discusses the consequences of this rule. [The alternative of using many super-thin layers should not be dismissed as forever impractical -- relatively minute areas of such thin films are routine in semiconductor device technology -- but the manufacturing of hundreds of meters of 300 Å thick ribbon with the required perfection strikes as well beyond the present state-of-the-art.]

### C. REFRIGERATION REQUIREMENTS

In comparing conventional and superconducting power systems, the refrigeration subsystem looms as possibly the critical component. What are its costs and how reliable will it be? The uncertainty arises because the refrigerator is the subsystem that utilizes the least mature technology. In the past, there has been little demand for large, efficient and highly reliable cryogenic refrigerators, so relatively little effort has been expended in developing them. Such a situation suggests that there may well be an opportunity for substantial improvement in the state of the art. At the same time, it leaves one with a feeling of uncertainty about how far to extrapolate the present technology and costs to get an accurate estimate of their future value. Since we have no special expertise in this discipline, our approach has been to calculate what we would need for a system using only state-of-the-art components and recent prices for delivered systems. These are the numbers that we present below.

The 570 MVA step-up transformer of reference 1 would dissipate truly negligible power in the windings themselves. We give estimates in the next two sections of the total loss within the three phase system of about 1 Kw. That is less than 0.1% of the losses present in the windings of a conventional transformer of similar rating. Before greeting that fact with dancing in the streets, it is well to note that the fine print says "in the windings". Those windings are at a temperature of 5 to 10 K. Getting that 1 Kw out of the dewar involves a fight with both Carnot and reality. To get some idea of what the final cost would be, one must make some reasonable assumptions on the cooling system's operation.

The heat that must be removed from the dewar in steady state operation comes from three quite different sources. These comprise:

- 1) heat leaks through the dewar walls
- 2) heat leaks through the pot heads
- 3) heat dissipated in the insulating dielectric and the flux steering rings

The leak rate through the walls of a high quality, modern helium storage dewar -- one that uses "super insulation" and a vapor-cooled shield -- is conservatively rated at 0.1 watt/m<sup>2</sup> of dewar surface. The dewar for the 570 MVA transformer of reference 1 has a surface area of 58 m<sup>2</sup>, so the leak rate through the dewar wall itself is a most modest 6 w. (7)

The pot-head loss is larger. American Magnetics quotes Efferson leads (vapor-cooled feed throughs) up to 5000 amperes as having a loss rate of 1.4 watts/kiloampere. In a recent paper on potheads for a superconducting transmission line (8), Mauser et al get a value about 10% less on a pothead designed to carry 14 Ka. Using the American Magnetics value for the total of all of the leads of 28.27 Ka, gives a dissipation rate of 40 watts.

The dielectric losses are larger still. They are estimated for particular arrangements in the next two sections, with the sum being about 1 Kw.

To extract that heat, Q, from the bowels of the dewar using an all gas system (no change of phase), Carnot requires that we expend work, W<sub>c</sub>, given by:

$$(W_c/Q) = \left[ \frac{T_0}{(T_H - T_L) / \ln(T_H/T_L)} - 1 \right] \quad (11)$$

where T<sub>L</sub> is the temperature of the helium as it enters the dewar, T<sub>H</sub> is the exit temperature, and T<sub>0</sub> is the ambient. Since the heat is predominantly in the dielectric, and it is only the superconducting tapes that must be kept near T<sub>L</sub>, T<sub>H</sub> can be relatively high, as long as the coldest gas passes over the tapes. Using T<sub>H</sub> = 20 K and T<sub>L</sub> = 6 K and assuming a 300 K ambient gives a Carnot multiplication factor of 24.8.

Reality also extracts its toll. Figure 4 from a recent study by EPRI (9) indicates that at the 1 Kw level, efficiencies of operating units range between 18 and 30% of the Carnot values. That's quite a range, but taking the 20% value as a conservative figure, one obtains a refrigerator input of 124 Kw/Kw of heat extracted.

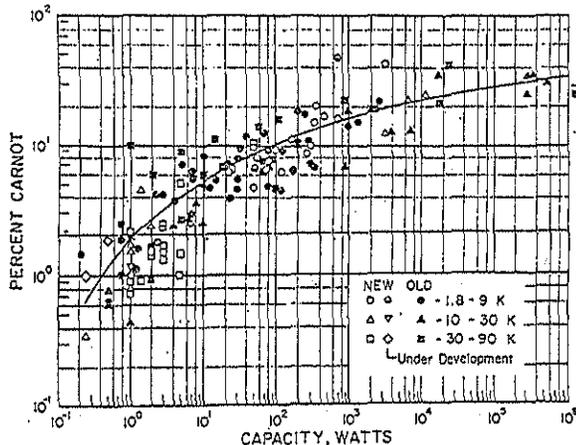


Figure 4: Efficiency of low temperature refrigerators as a function of refrigeration capacity. (From ref. 9)

Figure 5 from the same EPRI-sponsored study indicates what such a refrigerator would cost in 1973 dollars. The figure is \$160,000. The significance of this cost appears when it is compared to the total cost for a 570 MVA conventional transformer with the same electrical specifications as the one in reference 1. At today's prices, such a unit would cost about \$1,300,000. The refrigeration equipment is clearly a very important fraction of the cost for units of this size. However, this is usually the case. Superconducting systems come into their own only at very high to super high power levels. The transformer is no exception. All that this section has attempted to demonstrate is that with today's technology, the refrigeration requirements for the transformer are modest in terms of power and within bounds in terms of cost. We have made no claim to predicting the future.

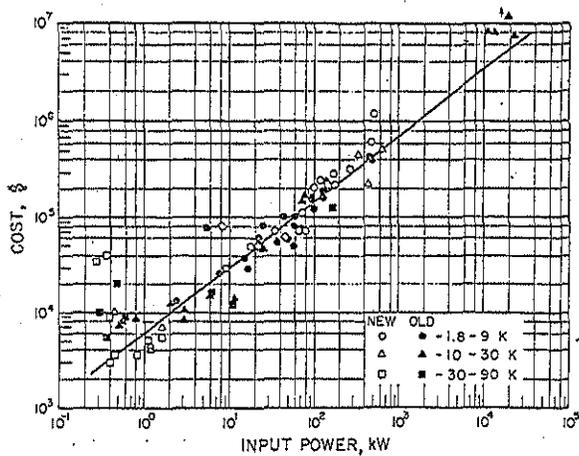


Figure 5: Cost of low temperature refrigerators as a function of installed input power. (From ref. 9)

#### D. COIL CONSTRUCTION IN A SUPERCONDUCTING POWER TRANSFORMER

Conventional transformer coils in large power transformers employ a very compatible two-phase system of liquid oil, solid pressboard and solid, wrappable paper to achieve simultaneously void-free electrical insulation, efficient heat extraction, and great resistance to the compressive forces generated during short circuits. Failure modes in properly designed transformers usually derive either from the contamination of the oil or from mechanical failure of the paper after a relatively long period of thermally-induced deterioration<sup>(10)</sup>. One may properly ask: Would the coils of a superconducting transformer be equally reliable?

Considering that this is a totally new transformer technology, the prospect is really rather encouraging. For once, cryogenic operation makes things inherently easier. It turns out that the obvious structural insulating material -- glass-reinforced epoxy -- has mechanical and electrical properties that are excellent at room temperature and that generally improve with decreasing temperature<sup>(11)</sup>. Large, thick castings of such material have been used for both mechanical support and dielectric isolation in several of the potheads being designed and tested for use on superconducting transmission lines<sup>(8)</sup>. These potheads operate with one end at 300°K and the other at about 5°K. As long as reasonable care is employed in cooling the composite to its operating temperature, the reinforced plastic retains its integrity and proves to be generally compatible with and bondable to the conductors and supports found in the cryogenic environment. It seems reasonable to surmise that excellent coils can be designed and executed using glass-reinforced epoxy as both the primary insulation and the mechanical support. It also seems reasonable to surmise that doing it right the first time will be a non-trivial exercise.

The two principal problems that must be dealt

with in the coil design are heat dissipation throughout the bulk of the coil and current crowding in the end turns. Though these issues are classical enough, the superconducting regime will require some rather novel solutions. In this section, we discuss the cooling issues. Current crowding is the subject of the next section.

The principal heating loads that the coil designer must deal with are the initial cool-down from ambient to cryogenic temperature and the removal of heat liberated by dielectric losses. [The losses associated with the type-II operation of the windings during transient short circuits are of such short duration that they must be dealt with by thermal mass. It is not really a refrigeration issue.] The cool-down problem is to provide heat extraction that is uniform enough to keep the thermal contraction stresses within reasonable bounds. Most of the contraction takes place between 300°K and 77°K, and for the typical glass-reinforced epoxy<sup>(11)</sup>,  $\Delta L/L$  will be a few parts in  $10^3$ . The designer must trade off more cooling channels for a quicker uniform cool-down against the reduction in dielectric strength of gaseous helium compared to the epoxy it displaces.

The dielectric loss issue is an interesting one. Certainly no designer of conventional power transformers ever worried about the dielectric loss tangent. But when one goes to a superconducting winding operating in the type-I mode, the only loss remaining in the coil structure is the dielectric loss. It is a remarkably small fraction of the  $I^2R$  losses in a conventional coil, but small amounts of power look bigger when you have to pump them up from 10°K to 300°K. The expression for the local dissipation rate is given by:

$$P_D = E^2 \omega \epsilon \tan \delta \text{ watts/unit volume} \quad (12)$$

where  $E$  is the electric field,  $\omega$  the angular frequency,  $\epsilon$  the dielectric constant, and  $\tan \delta$  the loss tangent. For a typical, suitable epoxy<sup>(12)</sup>, the measured values for  $\epsilon$  and  $\tan \delta$  are  $4.7\epsilon_0$  and  $3.4 \times 10^{-4}$  respectively.

To give some idea of what these values mean in terms of a heat load for an operating transformer, we have calculated the load that would be found for a reasonable coil design for the transformer discussed in reference 1. In that reference, the transformer winding was designed using a radial build that kept the inductance of all the coil pairs equal. There were reasons for doing that, but it really isn't a very good design. To get a more realistic loss value, we redesigned the coil using the following criteria to obtain the radial build:

- 1) the 14 layers were paired so that the flux density in adjacent layers was the same (i.e., layer 1 matched layer 2, 3 matched 4, etc.);
- 2) the maximum electric stress was the same in every pair of layers (This approximates a good solution for surge stress distribution and also gives a reasonably low value for the dielectric loss integrated over the whole coil.); and
- 3) the total coil volume (and exterior dimensions) had to be the same as in reference 1 to meet the design reactance value.

Noting that the transformer was  $\Delta$ -Y, and using the data of table 3, reference 1, one may obtain the maximum voltage in any pair of layers. Using right-circular-cylinder coil forms, criterion #1 puts that voltage at the "top" of the even-numbered layers. Then, using the maximum design stress of table 3, reference 1 (17,560 V/cm or 44.6 V/mil) one may obtain the radial

build of the even numbered layers. The radial build of the odd-numbered layers is obtained from criterion #1. The values so obtained are found in Table 1.

One may then solve for the loss in a given coil-pair layer using equation (12) and, for simplicity, a plane approximation to the field distribution in a given layer. The resulting expression for the dielectric loss in the n'th layer is:

$$P_n = (\alpha A_n h / 3 \Delta r_n^2) [\Delta V'^2 (n^3 - (n-1)^3) + V_{Lg}^2 + \Delta V' V_{Lg} (3\sqrt{3}/2) (n^2 - (n-1)^2)] \quad (13)$$

where  $\alpha = \epsilon \omega \tan \delta$ ,  $A_n$  is the cross sectional area of the dielectric material separating the coils of the n'th layer, h is the coil height,  $\Delta r_n$  is the radial build of the n'th layer,  $\Delta V'$  is the voltage rise across a single layer of turns on the high voltage windings, and  $V_{Lg}$  is the line-to-neutral voltage of the low voltage ( $\Delta$ ) winding. Noting that the transformer is designed for 50 hz operation, the several coefficients and the values of  $P_n$  are given in Table 1.

The sum of the  $P_n$  is the very modest number of 262 watts per coil. Since an equivalently rated conventional winding would be dissipating something on the order of 1 Mw per coil, it is not surprising that in the past dielectric losses have been disregarded. However, when one multiplies those 262 watts by the refrigeration inefficiency factor, one obtains a load of about 32 Kw per coil, small still compared to 1 Mw, but quite substantial when pricing out the refrigeration equipment.

Practical designs would probably emphasize the reduction of the dielectric losses to a much greater degree, choosing the dielectric with greater care, using tapered radial builds to keep  $E^2$  to a minimum and possibly using somewhat warmer helium which reduces the inefficiency factor. The purpose of the presentation here is to give some quantitative idea of what the problem is and roughly how big it is. The final column in Table 1 shows how far we got toward minimizing the total loss by getting the losses to be uniformly distributed among the layers. It is clear that there is still some room for improvement if one could relax criterion #2. To get much improvement beyond that, a tapered radial build or a dielectric with lower losses would have to be employed.

coil number n	value of the square bracket in equ. 2 in (Volts) <sup>2</sup>	radial build $\Delta V_n$ meters	cross section $\pi(V_n^2 - V_{n-1}^2)$ $A_n$ meters <sup>2</sup>	$P_n$ watts	$T_n$ watts/m <sup>3</sup>
1	.9780 x 10 <sup>9</sup>	.0134	.0328	.896	8.04
2	3.700	.0130	.0329	3.62	32.4
3	9.145	.0342	.0913	3.60	11.6
4	13.50	.0316	.0913	6.20	20.0
5	21.67	.0555	.175	6.22	10.5
6	30.92	.0502	.175	10.84	18.2
7	41.81	.0771	.300	10.64	10.4
8	54.34	.0690	.300	17.26	16.9
9	68.49	.0983	.479	17.11	10.5
10	84.29	.0878	.479	26.41	16.2
11	101.7	.1190	.727	26.29	10.7
12	120.8	.1066	.727	38.92	15.8
13	141.5	.1390	1.054	38.91	10.9
14	163.8	.1253	1.054	55.33	15.5

values used:  
 $\omega = 100\pi$   
 $\epsilon = 4.7 \epsilon_0$   
 $\tan \delta = 3.41 \times 10^{-4}$   
 $h = 3.394$  m  
 $ah = 1.51 \times 10^{-11}$  amperes/volt  
 $\Delta V' = 16,500$  volts  
 $V_{Lg} = 12,700$  volts

$r_0 = 0.383$  m  
 $r_{max} = 1.40$  m  
 Total = 262 watts

Table 1. Values developed for the estimation of dielectric losses.

### E. CURRENT CROWDING IN THE END TURNS

The curvature of the electric and magnetic fields at the ends of the coils in a conventional high-voltage power transformer introduces a special problem in heat extraction. The curvature of the H field induces eddy currents and their associated  $I^2R$  losses just at the point where the maximum electrical stress is to be found. Thus, the added heat must be extracted through a greater length of thermally insulating dielectric. The conventional solution is to add more copper to the end turns to reduce the total ohmic loss in the end regions to acceptable levels.

Even though  $I^2R$  losses as such are absent in the superconducting tapes, there is a closely related problem in the design of a superconducting transformer winding. In the superconducting case, one is concerned that the superposition of the eddy currents onto the load current could drive the end turns into type II or even into normal (ohmic) operation. In this section, we discuss several techniques for handling the eddy currents in the end turns, using our 3- $\phi$  570 MVA step-up transformer as a model. Recall that in that design, each phase employs a winding made up of 14 layers. Each layer contains a fully opposed pair of windings (same NI in each) separated by major insulation. Adjacent layers are paired so that leakage flux going up in one layer flows down the other. Figure 6 is a sketch of the magnetic field map that one would find at the end of one such pair of layers. The lines labeled equipotentials are really the normals to the magnetic field lines, H. Their density at the conductor surfaces equals the sheet current density in the conductor. Observe that we have chosen to put the low voltage (LV) windings in the center. The reasons for that choice will be apparent shortly.

Inspection of the figure shows that there would be an increase in the current density in the LV windings, but a paucity of current at the edges of the HV windings. The increased current density in the LV windings must be properly accommodated, and two different situations must be considered. The first situation is the steady state where rated load current must be carried with the entire winding in the type-I mode. The second situation is the transient, maximum short circuit where no portion of the superconducting tape should switch into the normal (ohmic) state.

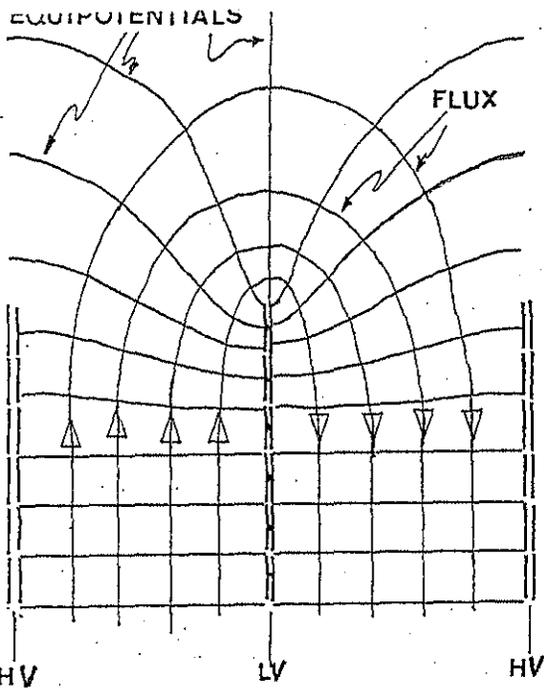


Figure 6: Lines of flux and normals to the magnetic field ("equipotentials") for a matched pair of winding layers. The right and left hand LV windings are at the same electric potential.

The designer has three relatively independent parameters that he may manipulate to deal with the curving magnetic field lines of figure 6. The first, as Wilkinson<sup>(13)</sup> suggested over a decade ago, is to curve the conductor itself to better fit the field. With our design, in contrast to his, that is relatively easy to accomplish provided that there is no voltage difference between the two adjacent middle layers. Since the HV layers are in series and the LV layers in parallel, we would clearly choose to put the LV windings in the middle. Having done so, we may readily shape the end turns to follow an elliptical cross section that should conform very closely to the flux lines themselves. This could well be enough to reduce the current crowding to unimportant proportions, but a positive answer will fit only one specific example of tape parameters, shape and current loading.

A second possibility arises if the problem of excessive current density occurs only under short circuit operation. In that case, with the tape operating in the type-II mode, where the current penetrates into the bulk of the superconductor, one may accommodate extra current by using a thicker tape. Such a solution carries with it a penalty of noticeably higher hysteretic losses in the end windings. As equation (10) shows, the loss rate will go up roughly as the cube of the tape thickness. Accordingly, that type of solution can be extended only a limited way.

The third parameter that the designer may manipulate is the flux path itself. People have tried to ameliorate the flux curvature problem in the past by using high permeability materials. In high voltage transformers the problem with that method is that the high- $\mu$  metals used to manipulate the flux must be insulated from the potential or potentials of the windings they protect, and that insulation precludes getting the high permeability materials close enough to do any good. [In low voltage applications, however, the method can be used to virtually eliminate

any unwanted flux curvature. This was precisely the method employed by Forsyth and Morgan<sup>(14)</sup> for their special purpose transformer designed for the measurements reported in (6). For that application, the uniformity of the field was critical. Excellent uniformity was obtained by terminating the cylindrical windings at discs of laminated transformer steel.]

For the configuration shown in figure 6, which would be unique to superconducting windings, manipulating the flux in the critical region at the ends of the middle windings is rendered relatively easy by the fact that there is no variation of potential across the top of these windings. One could place a flux-steering ring of laminated high permeability transformer steel immediately adjacent to the end turn on top and bottom as shown in figure 7.

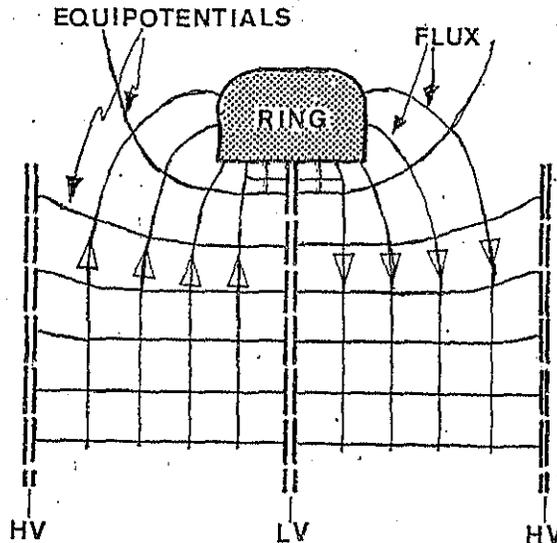


Figure 7: The configuration of figure 6 with the fields modified by the inclusion of a flux steering ring at the end of the LV coil.

Since the superconducting coils are inherently structures with a relatively low leakage flux density<sup>(1)</sup>, the peak flux density in the dielectric under short circuit conditions will be far less than the saturation flux density of modern transformer steels. Accordingly, a relatively thin ring will suffice to straighten the field in the vicinity of the conductor surface. In fact, in the 570 MVA step-up transformer we have used as an example, the peak short circuit flux density is less than 0.5 T. That might well permit one to employ ferrite ceramics for the flux steering rings. Such materials<sup>(15)</sup> have very high permeabilities and very low losses, but their saturation flux densities are relatively low. Accordingly, one would have to balance the advantage of having an insulating ceramic against the added volume and possibly weight and cost that such a ceramic would entail. Finally, critical to the choice of either material is the total loss rate under steady state operation at cryogenic temperature. Data is wanting in this regard for both high permeability steels as well as the ferrite ceramics.

#### F. SOME PRELIMINARY CONCLUSIONS ON ENERGY SAVINGS

In his 1966 paper<sup>(16)</sup> Wilkinson began his discussion of superconductors in power-system applications with the statement: "In broad terms, the normal conductor-loss in a large transformer, when capitalized, is a sum that is comparable with the transformer's own cost; losses of this magnitude are clearly

worth saving." With the extraordinary inflation in the cost of energy that has recently taken place, that statement is true a fortiori. Since we have already obtained first order estimates of the core mass<sup>(1)</sup> and the refrigeration requirements of a 570 MVA step-up transformer, we may readily determine the energy savings to be obtained to the same degree of precision as the input data.

For a 570 MVA unit of conventional design, the loss rate would be about 4 MW (0.7%) at rated load. If the loss ratio (copper losses to core losses) were 2/1, there would be 2.7 MW of copper loss and 1.3 MW of core loss when the transformer was operating at rated load. Copper losses would fall off as  $I^2$ , while the core losses would be independent of load. For the superconducting transformer, under the assumption of lossless type-I operation up to and including rated load, the loss rate would be almost independent of load. (As discussed in section C, there is an  $I^2$  dependent term -- the feedthrough loss -- but it is very much less than the current-independent losses.) Our estimates of the refrigeration load ranges between 0.08 and 0.12 MW, depending on refrigerator efficiency. We will use the larger one. In reference 1 we determined that we could use a core that weighed only 62% of Wilkinson's conventional design. At equal excitation, the core loss is proportional to mass or volume. Thus, for the superconducting design, the core losses would be about 0.81 MW. Accordingly, the approximate loss rate for the superconducting transformer is 0.93 MW, relatively independent of load. It should be stressed that these estimates are all based on recent state-of-the-art figures. No future technological wonders have been invoked to come up with an advantageous comparison.

The relative savings in energy for several levels of loading are compared in table 2. The figures show that a substantial saving in energy would indeed be realized.

Power Level	Copper losses	Core losses	Total losses	losses <sub>s.c.</sub>
				losses conventional as %
100%	2.7 MW	1.3 MW	4.0 MW	23%
75%	1.5	1.3	2.8	33%
50%	0.7	1.3	2.0	47%
0%	0	1.3	1.3	72%

Table 2: Losses in a conventional 570 MVA step-up transformer with 2/1 loss ratio compared with the losses in a superconducting transformer of the same rating.

Using the numbers from table 2, as well as Wilkinson's estimate that the energy losses over the life of a transformer capitalize to roughly the cost of the transformer itself, and assuming that his figure corresponds to 75% loading, it would appear that the superconducting transformer could return 2/3's of the price of a conventional transformer in energy savings. Granted that one would have to deduct from those savings the capital cost of the refrigeration plant and its maintenance cost, it still seems that the superconducting transformer may well be very competitive, particularly in the larger sizes. If one adds to this the general public good of saving 9360 barrels of oil per annum per transformer in equivalent energy savings, we think a reasonable ab initio case can be made that superconducting transformers will be cost effective.

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