

January 01, 2006

## Electromagnetic information theory

Fred K. Gruber  
*Northeastern University*

Edwin A. Marengo  
*Northeastern University*

---

### Recommended Citation

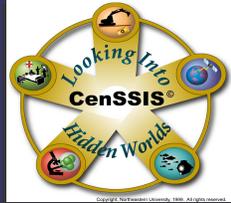
Gruber, Fred K. and Marengo, Edwin A., "Electromagnetic information theory" (2006). *Research Thrust R2 Presentations*. Paper 17.  
<http://hdl.handle.net/2047/d10008226>

This work is available open access, hosted by Northeastern University.



# Electromagnetic Information Theory

Fred K. Gruber and Edwin A. Marengo  
Department of Electrical and Computer Engineering, Northeastern University



## Abstract

Our research program is concerned with the development of an information theory of classical electromagnetic fields with applications to wireless communications, remote sensing, and radar. In the present work, emphasis is given to the derivation of upper bounds for the Shannon information capacity of a wireless communication channel formed by a rather general receiving antenna and a transmitting antenna whose support is assumed to be contained within a mathematical spherical volume of a given radius. Due to reciprocity the results also apply as fundamental bounds for the information capacity of a wave sensor of a given size, be it an antenna, the eye, or any wavefield-measuring device. This work includes numerical results illustrating the derived theory. A discussion of applications of the derived theory and numerical results to wavefield imaging, e.g., number of degrees of freedom of the data, and classes of recoverable object profiles, using near or far electromagnetic fields, is also given.

Specifically, this work has applications to MIMO wireless communications systems, MIMO radars, in determining resolution limits of sensing systems such as synthetic aperture radar and other imaging systems and it also clarifies the possibility of super-resolution [1] in certain situations. More generally, the developments are derived from the first principles' point of view provided by the classical electromagnetic theoretic framework, and are therefore fundamental for both analysis and design of a variety of wireless communication, remote sensing and radar systems.

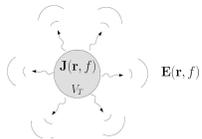
## State of the Art

- Miller [2], [3] studied the orthogonal communication channels between two arbitrary volumes in free space radiating a scalar waves and, in a later generalization, [4] for volumes radiating electromagnetic waves.
- The scalar version of Miller's work was later extended by Hanlen and Fu [5] to include the effect of scatterers in the propagation path.
- Gustafsson and Nordebo [6] analyzed the fundamental limitations in the capacity of an arbitrary electromagnetic antenna under Rayleigh fading channel and white Gaussian noise taking into account antenna theory and broadband matching.
- Poon et al. [7] derived expressions for the NDF of a communication system with different array geometries and with a channel model based on measurements in urban and indoor environments.

## Challenges and Significance

- In this work we present the initial steps in using Shannon's information theory to characterize the fundamental limits in the information transfer of a source of a given size under physically meaningful constraints.
- In addition to the obvious applications in communication systems, the information theoretical concept used in this work are important in other applications like remote sensing where, for instances, we can characterize the performance of imaging systems in terms of the amount of information about the object contained in the image rather on how much the image resembles the object.
- Important challenges include the consideration of specific systems as well as appropriate constraints.

## Technical Approach



## Multipole Expansion

$$\mathbf{E}(\mathbf{r}, f) = \sum_{j,l,m} a_{j,l,m}(f) \mathbf{A}_{j,l,m}(\mathbf{r}, f)$$

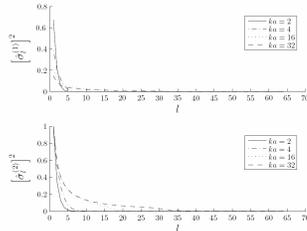
$$\sum_{j,l,m} \equiv \sum_{j=1}^{\infty} \sum_{l=1}^{\infty} \sum_{m=-l}^l$$

$$a_{j,l,m}(f) = \int_{V_f} \mathbf{B}_{j,l,m}^\dagger(\mathbf{r}, f) \mathbf{J}(\mathbf{r}, f) d^3r$$

$$\int_{V_r} \mathbf{B}_{j,l,m}^\dagger(\mathbf{r}, f) \mathbf{B}_{j',l',m'}(\mathbf{r}, f) d^3r = [\sigma_l^{(j)}(f)]^2 \delta_{j,j'} \delta_{l,l'} \delta_{m,m'}$$

$$[\sigma_l^{(1)}(f)]^2 = \frac{1}{2l+1} \left\{ (l-1) [\sigma_{l-1}^{(2)}(f)]^2 + (l+2) [\sigma_{l+1}^{(2)}(f)]^2 \right\}$$

$$[\sigma_l^{(2)}(f)]^2 = \frac{(\eta k)^2 a^3}{2l(l+1)} [j_l^2(ka) - j_{l-1}(ka) j_{l+1}(ka)]$$



## Singular Value Decomposition

Let  $\mathcal{Y}$  be the space of square-summable vectors and  $\mathcal{X} = L_2(V_f)$

Define:

$$(\mathbf{J}\mathbf{J}^\dagger)_{\mathcal{X}} \equiv \int_{V_f} \mathbf{J}(\mathbf{r}, f) \mathbf{J}^\dagger(\mathbf{r}, f) d^3r \quad \langle \mathbf{a} | \mathbf{a}' \rangle_{\mathcal{Y}} \equiv \sum_{j,l,m} a_{j,l,m}^\dagger(f) a'_{j,l,m}(f)$$

$$P: \mathcal{X} \rightarrow \mathcal{Y} \quad a_{j,l,m}(f) = (P\mathbf{J})(j, l, m, f) = \langle \mathbf{B}_{j,l,m} | \mathbf{J} \rangle_{\mathcal{X}}$$

$$P^\dagger: \mathcal{Y} \rightarrow \mathcal{X} \quad (P^\dagger \mathbf{a})(\mathbf{r}, f) = \sum_{j,l,m} a_{j,l,m}(f) \mathbf{B}_{j,l,m}(\mathbf{r}, f)$$

SVD:

$$(P\mathbf{u}_{j,l,m}(f), j', l', m', f) = \sigma_l^{(j)}(f) v_{j,l,m}(j', l', m', f)$$

$$(P^\dagger v_{j,l,m}(f), \mathbf{r}) = \sigma_l^{(j)}(f) \mathbf{u}_{j,l,m}(\mathbf{r}, f)$$

$$\mathbf{u}_{j,l,m}(\mathbf{r}, f) \equiv \frac{\mathbf{B}_{j,l,m}(\mathbf{r}, f)}{\sigma_l^{(j)}(f)} \quad v_{j,l,m}(j', l', m', f) = \delta_{j,j'} \delta_{l,l'} \delta_{m,m'} \delta_{m,m'}$$

$$\mathbf{a} = (P\mathbf{J}) = \sum_{j,l,m} v_{j,l,m} \sigma_l^{(j)}(f) \langle \mathbf{u}_{j,l,m} | \mathbf{J} \rangle_{\mathcal{X}}$$

$$b_{j,l,m}(f) = \langle \mathbf{u}_{j,l,m} | \mathbf{J} \rangle_{\mathcal{X}}$$

$$a_{j,l,m}(f) = \sigma_l^{(j)}(f) b_{j,l,m}(f)$$

## Information Capacity

Space information

$$a_{j,l,m}(f) = \sigma_l^{(j)}(f) b_{j,l,m}(f) + n_{j,l,m}(f)$$

$n_{j,l,m}(f)$  represents additive Gaussian noise with power spectral density  $N_{j,l,m}(f)$ .

L2 norm and radiated power constraint:

$$C = \sum_{j,l,m} \left[ \log \frac{[\sigma_l^{(j)}(f)]^2}{N_{j,l,m}(f)} \left( \frac{1}{\lambda_1 + \lambda_2 l(l+1) [\sigma_l^{(j)}(f)]^2} \right)^+ \right]$$

where  $\lambda_1$  and  $\lambda_2$  are two non-negative constants chosen to satisfy the inequalities

$$\sum_{j,l,m} P_{j,l,m} - \mathcal{E} \leq 0 \quad \sum_{j,l,m} l(l+1) [\sigma_l^{(j)}(f)]^2 P_{j,l,m} - P \leq 0$$

$$\lambda_1 \left( \sum_{j,l,m} P_{j,l,m} - \mathcal{E} \right) = 0$$

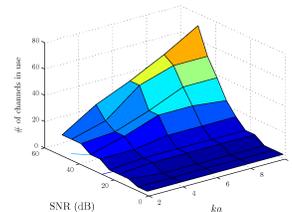
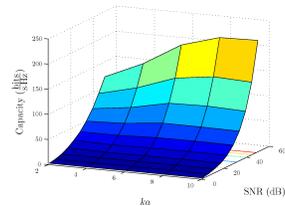
$$\lambda_2 \left( \sum_{j,l,m} l(l+1) [\sigma_l^{(j)}(f)]^2 P_{j,l,m} - P \right) = 0$$

$$P_{j,l,m} = \left( \frac{1}{\lambda_1 + \lambda_2 l(l+1) [\sigma_l^{(j)}(f)]^2} - \frac{N_{j,l,m}(f)}{[\sigma_l^{(j)}(f)]^2} \right)^+$$

where  $P_{j,l,m} = E |b_{j,l,m}(f)|^2$   
 $N_0 = 0.1$  and  $ka = 5$

C (bits/Hz)	# of chan.	P'	P' slack	E'	E' slack
24.6625	11	100	0	100	73.9907
24.6625	11	100	0	50	23.99
18.3563	8	100	38.59	10	0
32.544	14	200	0	50	0
34.4257	18	300	16.1583	50	0
34.4257	18	400	116.2225	50	0
18.4832	9	50	0	100	87.1604
18.4832	9	50	0	200	187.1604

P=200  
E=50



## Space-time Information

$$a_{j,l,m}(t) = \sigma_l^{(j)}(t) \otimes b_{j,l,m}(t) + n_{j,l,m}(t)$$

L2 norm and radiated power constraint:

$$C = \sum_{j,l,m} \int_{\omega} d\omega \left[ \log \frac{[\sigma_l^{(j)}(f)]^2}{N_{j,l,m}(f)} \left( \frac{1}{\lambda_1 + \lambda_2 l(l+1) [\sigma_l^{(j)}(f)]^2} \right)^+ \right]$$

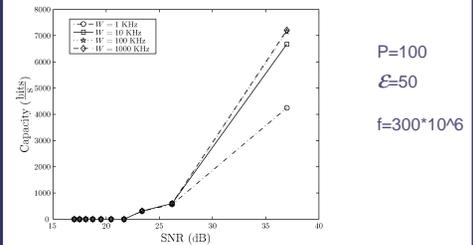
where  $\lambda_1$  and  $\lambda_2$  are two non-negative constants chosen to satisfy the inequalities

$$\sum_{j,l,m} \int_{\omega} d\omega \left( \frac{1}{\lambda_1 + \lambda_2 l(l+1) [\sigma_l^{(j)}(f)]^2} - \frac{N_{j,l,m}(f)}{[\sigma_l^{(j)}(f)]^2} \right)^+ d\omega - \mathcal{E} \leq 0$$

$$\sum_{j,l,m} \int_{\omega} d\omega l(l+1) \left( \frac{1}{\lambda_1 + \lambda_2 l(l+1) [\sigma_l^{(j)}(f)]^2} - \frac{N_{j,l,m}(f)}{[\sigma_l^{(j)}(f)]^2} \right)^+ d\omega - P \leq 0$$

$$\lambda_1 \left[ \sum_{j,l,m} \int_{\omega} d\omega \left( \frac{1}{\lambda_1 + \lambda_2 l(l+1) [\sigma_l^{(j)}(f)]^2} - \frac{N_{j,l,m}(f)}{[\sigma_l^{(j)}(f)]^2} \right)^+ d\omega - \mathcal{E} \right] = 0$$

$$\lambda_2 \left[ \sum_{j,l,m} \int_{\omega} d\omega l(l+1) \left( \frac{1}{\lambda_1 + \lambda_2 l(l+1) [\sigma_l^{(j)}(f)]^2} - \frac{N_{j,l,m}(f)}{[\sigma_l^{(j)}(f)]^2} \right)^+ d\omega - P \right] = 0$$



## Accomplishments up Through Current Year

Since January 2005 we have been working on several fundamental problems in remote sensing that have resulted in publications in peer review journals:

1. In [8] we reformulated and extended the treatment of time-reversal imaging of multiply scattering developed in [9] to the estimation of the target scattering strengths using the Foldy-Lax multiple scattering model.
2. In [10] we proposed a new non-iterative analytical alternative to the iterative numerical solution of the target scattering strength estimation proposed in [8].
3. In [11] we describe an alternative signal subspace method which is based on search in high-dimensional parameter space and which is found to outperform the time-reversal approach in number of localizable targets and in estimation variance.
4. This theory has been generalized to extended target and it is currently under review for publication at the IEEE Transactions on Image Processing.

## Opportunities for Technology Transfer

1. While we emphasize the transmit mode information capacity in this work, due to reciprocity, the results also apply as fundamental bounds for the (receive mode) information capacity of a remote sensing system.
2. A few of the concepts used in this work, like mutual information and degrees of freedom, have been applied before in very specific sensing systems like ground-based telescopes [12] and synthetic aperture radars [13].
3. We are interested in developing a systematic methodology for determining the fundamental limitations of general remote sensing systems operating at given constraints.

## References

- [1] M. Bertero and C. D. Mol, "Super-resolution by data inversion," in Progress in Optics (E. Wolf, ed.), vol. XXXVII, pp. 129-178, Elsevier, Amsterdam, 1996.
- [2] D. A. B. Miller, "Spatial channels for communicating with waves between volumes," Optics Letters 23(21), 1645-1647 (November 1998).
- [3] D. A. B. Miller, "Communicating with Waves Between Volumes: Evaluating Orthogonal Spatial Channels and Limits on Coupling Strengths," Applied Optics 39(11), 1661-1699 (Apr. 2000).
- [4] R. Piestun and D. A. B. Miller, "Electromagnetic degrees of freedom of an optical system," The Journal of the Optical Society of America A 17(5), 892-902 (May 2000).
- [5] L. Hanlen and M. Fu, "Wireless communication systems with spatial diversity: a volumetric model," Wireless Communications, IEEE Transactions on 5(1), 133-142 (2006).
- [6] M. Gustafsson and S. Nordebo, "On the spectral efficiency of a sphere," Report TEAT-7127, Lund Institute of Technology, 2004.
- [7] A. Poon, R. Brodersen, and D. Tse, "Degrees of freedom in multiple-antenna channels: a signal space approach," Information Theory, IEEE Transactions on 51(2), 523-536 (2005).
- [8] A. J. Devaney, E. A. Marengo, and F. K. Gruber, "Time-reversal-based imaging and inverse scattering of multiply scattering point targets," Journal of the Acoustical Society of America, Vol. 118, p. 3129-3138, 2005.
- [9] F. K. Gruber, E. A. Marengo, and A. J. Devaney, "Time-reversal imaging with multiple signal classification considering multiple scattering between the targets," J. Acoust. Soc. Am. 115, 3042-3047, 2004.
- [10] E. A. Marengo and F. K. Gruber, "Non-iterative analytical formula for inverse scattering of multiply scattering point targets," Journal of the Acoustical Society of America, to appear, 2006.
- [11] E. A. Marengo and F. K. Gruber, "Subspace-based localization and inverse scattering of multiply scattering point targets," EURASIP Journal on Advances in Signal Processing, Vol. 2007, Article ID 17342, 16 pages, 2007.
- [12] S. Prasad, "Information capacity of a seeing-limited imaging system," Optics Communications, 2000, 177, 119-134.
- [13] F. Dickey, L. Romero, J. DeLaurentis, & A. Doerry, "Super-resolution, degrees of freedom and synthetic aperture radar," Radar, Sonar and Navigation, IEE Proceedings., 2003, 150, 419-42.

