

January 01, 2001

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### Recommended Citation

Korugan, Aybek and Gupta, Surendra M., "Adaptive kanban control mechanism for a single-stage hybrid system" (2001).. Paper 16.  
<http://hdl.handle.net/2047/d10003174>



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## Bibliographic Information

Korugan, A. and Gupta, S. M., "An Adaptive Kanban Control Mechanism for a Single Stage Hybrid System", *Proceedings of the SPIE International Conference on Environmentally Conscious Manufacturing II*, Newton, Massachusetts, October 28-29, pp. 175-181, 2001.

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# An Adaptive Kanban Control Mechanism for a Single Stage Hybrid System

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## ABSTRACT

In this paper, we consider a hybrid manufacturing system with two discrete production lines. Here the output of either production line can satisfy the demand for the same type of product without any penalties. The interarrival times for demand occurrences and service completions are exponentially distributed i.i.d. variables. In order to control this type of manufacturing system we suggest a single stage pull type control mechanism with adaptive kanbans and state independent routing of the production information.

## 1. INTRODUCTION

In many industrialized countries, new regulations geared towards preserving the environment have elevated the urgency to reuse the products at the end of their useful lives. Many companies are involved in retrieving used products, where they repair, refurbish and upgrade the products in order to sell them for profit. Such companies engage in either remanufacturing used products or producing both new and remanufactured products. Companies that engage in both activities are referred to as *hybrid* companies. The main characteristic of such production system is that the demand can be satisfied with substitutable products manufactured on different lines. Hence, both remanufactured and newly manufactured versions of a certain product coexist in the system. Here the demand is satisfied either with a newly manufactured product or with a remanufactured product that is assumed to be restored to 'as good as new' condition.

Most models developed to control such hybrid production systems adapt classical inventory techniques with control policies such as  $(s, S)$  or  $(s, Q)$ . A single echelon inventory system with this assumption was first modeled using an  $M/M/1/N$  queue by Heyman<sup>5</sup>. The objective was to determine the optimal inventory level  $N$  that minimizes the total inventory cost using a push production control. The model did not consider the lead times. A subsequent paper by Muckstadt and Isaac<sup>12</sup> considered lead times but ignored the disposal activity for a continuous  $(Q, r)$  policy inventory model. The results obtained from the single-echelon model were applied to a multi-echelon model. In a series of papers, starting with reference [9], where the disposal option to the single-echelon model in reference [12] is added, Laan *et al.* applied classical inventory control methods to hybrid systems. They conducted a comparative study between inventory policies with and without disposal that showed disposal is a necessary action for cost minimization. In reference [8], they analyzed several inventory control policies with disposal option and showed that a four-parameter control policy was optimal. Subsequently, in reference [11] they showed that the pull control strategy was more cost effective than the push control strategy for inventory systems with return flows. A further study by Laan and Salomon<sup>10</sup> verified these results while adding the disposal option to the earlier model. A more detailed overview of such systems is given by Salomon *et al.*<sup>13</sup> and Gungor and Gupta<sup>4</sup>.

In this paper, we consider a modified single stage pull control mechanism using an adaptive kanban control procedure to control material flow through a hybrid production system with two discrete processes that deal with manufacturing and remanufacturing activities, respectively. We make the assumption that used items are restored to 'as good as new' shape, which allows us to satisfy the demand using either of the new or the remanufactured items without compromise. We then model this system as a queueing network with a general manufacturing process that supplies a synchronization station as defined by Di Mascolo *et al.*<sup>3</sup>. In order to direct the production triggering information after a release of a finished product to the specific sub-process, we use a routing mechanism with a predefined routing probability  $r$ . Then, we perform the Markov Chain analysis for the control mechanism in a three-dimensional state space in order to find the performance measures of the system.

The outline of the paper is as follows: In Section 2, we describe the problem in detail. In Section 3, we analyze the adaptive kanban control policy using a Markov model. In Section 4, we measure the effectiveness of this policy using a numerical example and finalize the study by giving our concluding remarks in Section 5.

## 2. PROBLEM DESCRIPTION AND MODEL

We consider a hybrid make-to-stock production system with two mutually independent processes that service a single type of demand, where one of them manufactures new products from raw materials while the other remanufactures returned items. The manufacturing (remanufacturing) process is assumed to consist of either a balanced tandem network of  $M_M(M_R)$  stations or a single station with  $C_M(C_R)$  parallel servers with equal service rates. After a product leaves the production process, it is placed in the finished goods inventory. Without loss of generality we assume that there are raw materials and returned items whenever needed. The demand process follows a Poisson distribution with rate  $\lambda_D$  and all service times follow an exponential distribution with the same mean  $\mu_M(\mu_R)$ .

When a demand is satisfied, the freed kanban triggers the release of a part into the production system. Yet, at this instant, the decision of which sub-process to trigger arises. We use a routing mechanism with a predefined routing probability for this decision. Whenever the production release information is sent, it is directed to the remanufacturing process with probability  $r$  and to the manufacturing process with probability  $(1 - r)$ . Throughout this paper we call this triggering procedure ‘dynamic routing’ mechanism. We define a single stage production control system with the adaptive kanban control procedure and represent the control policy using a queuing network with a synchronization station at the end <sup>2, 6</sup>. The control policy uses  $K + E$  kanbans to control a production system as given in Figure 1, where  $K$  is the minimum number of kanbans the system carries, while  $E$  represents the extra kanbans which are stored in queue  $A$  when not in use. The decision for activation and deactivation of these kanbans are based on the effective finished goods inventory level  $I(t)$  of the production system at time  $t$ , where  $I(t)$  is given by the total number of parts in the finished goods queue  $P$  less the backorder level in queue  $D$ . Let  $L$  and  $C$  be the release and capture levels of an extra kanban, respectively, while  $E(t)$  be the number of extra kanbans not in use at time  $t$ . At the arrival of a demand at time  $t$ , before it is satisfied by a finished product, an extra kanban is released if  $I(t) \leq L$  and  $E(t) > 0$ . Like a freed kanban, it is sent to the remanufacturing process ( $RP$ ) with probability  $r$  or to the manufacturing process ( $MP$ ) with probability  $(1 - r)$  (Figure 2). Then, the demand is satisfied with a finished product upon its availability and the attached kanban card is sent to trigger the production of a part. On the other hand, if at time  $t$   $I(t) > C$  and  $E(t) < E$  then the kanban released after the satisfaction of demand is captured and stored in queue  $A$ . Note that  $L < C$  for this system to function properly.

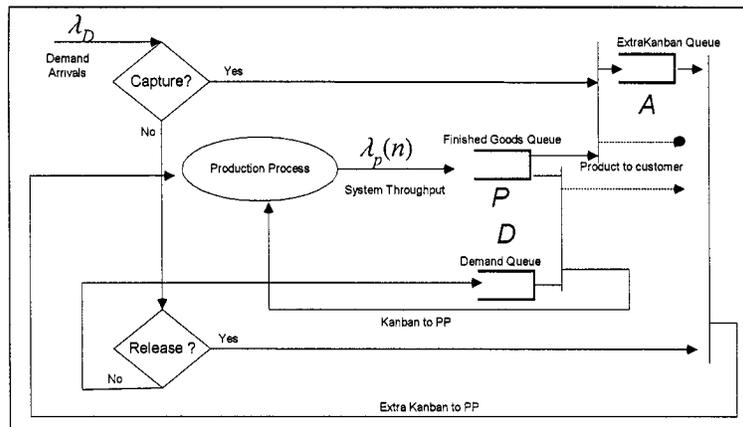


Figure 1: Single-stage adaptive kanban control of a production process(PP)

The throughput of the *PP* is estimated by a load dependent stochastic process with rate  $\lambda_p(n)$ . Since *PP* consists of the Manufacturing Process (*MP*) and the Remanufacturing Process (*RP*) (Figure 2), the total throughput  $\lambda_p(n)$  is a combination of the throughput rates  $\lambda_M(n)$  and  $\lambda_R(n)$  of these two respective sub-processes. Since each sub-process is modeled either by a balanced tandem network or a single station with parallel servers of the same speed, their throughputs can be calculated using either  $\lambda_{M(R)}(n) = n\mu / (n + M_{M(R)} - 1)$ <sup>14</sup> or  $\lambda_{M(R)}(n) = \begin{cases} n\mu, & n < C_{M(R)} \\ C_{M(R)}\mu, & n \geq C_{M(R)} \end{cases}$ , respectively. The emphasis of each sub-network throughput in the total system throughput is determined by the routing probability  $r$  of the production triggering mechanism.

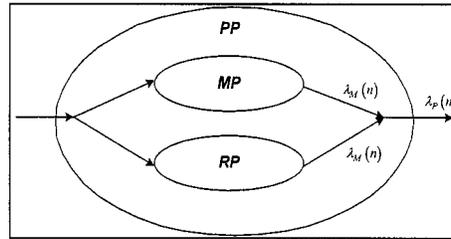


Figure 2: Production Process in Detail

### 3. ANALYSIS OF THE ADAPTIVE KANBAN CONTROL POLICY ON A HYBRID SYSTEM

In the case where  $E = 0$ , the general single stage pull type production system in Figure 1 reduces to a simple kanban control policy. Here, the control variables would be the kanban size  $K$  and the dynamic routing probability  $r$ . Since there are no extra kanbans, the release and capture levels of the system are no longer in effect. When a demand arrives, it is satisfied by a product subject to availability. The kanban is detached and sent to *RP* with probability  $r$  or to *MP* with probability  $(1 - r)$ . We define a stochastic process,  $x(t) = [x_I, x_R]$ , that records the changes in the on hand inventory of the system ( $x_I(t) = x_I$ ) and the work in process level, ( $x_R(t) = x_R$ ) of the remanufacturing sub-process for  $t \geq 0$ . The two dimensional Markov chain of this system is given in Figure 3. Here, if we can calculate the throughput,  $\lambda_p(n)$ , of the production process, *PP*, we can reduce the dimension of the Markov chain by one, which essentially becomes a birth-death process for a single stage kanban system (Korugan and Gupta<sup>7</sup>). Such systems are modeled as closed queueing systems with  $K$  customers (Di Mascolo et al.<sup>3</sup>). Let  $x(t) = [x_I]$ , with  $x_I$  equal to the total number of parts in queue  $P$  minus the total number of backorders in queue  $D$ . We can define a birth-death process as in Figure 4, which represents a simplified version of a synchronization process defined by Dallery<sup>1</sup>. Here,  $P_K(x_I)$  denotes the stationary probability of state  $x_I$  with  $K$  kanbans in the system. When  $\lambda_D / \lambda_P(K) < 1$ , these probabilities exist and are given as,

$$\begin{aligned} \lambda_D P_K(x_I) &= \lambda_P(\min(K - x_I + 1, K)) P_K(x_I - 1) \\ \sum_K P_K(x_I) &= 1 \end{aligned} \tag{1}$$

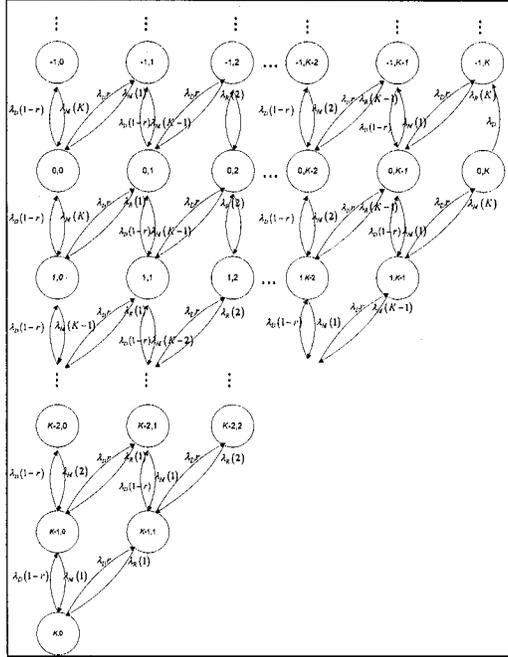


Figure 3: The Markov Chain model of the RMP system.

When we compare the balance equations of both the two-dimensional and the one-dimensional Markov chains, we find the total throughput of the system as,

$$\lambda_P(K - x_I) = \sum_{n_R=0}^{K-x_I} \frac{P_{K,r}(x_I, n_R)}{P_{K,r}(x_I)} (\lambda_M(K - x_I - n_R) + \lambda_R(n_R)), \quad x_I = 1, \dots, K$$

$$\lambda_P(K) = \sum_{n_R=0}^K \frac{P_{K,r}(x_I, n_R)}{P_{K,r}(x_I)} (\lambda_M(K - n_R) + \lambda_R(n_R)), \quad x_I \leq 0, \quad (2)$$

$$\lambda_M(0) = 0, \lambda_R(0) = 0.$$

Thus, the problem reduces to the calculation of the conditional probability distribution

$$P_{K,r}(n_R | x_I = n_I) = \frac{P_{K,r}(n_I, n_R)}{P_{K,r}(n_I)}, \quad n_I = 0, \dots, K \quad (3)$$

Here, at time  $t$  given that there are  $x_I = n_I$  finished products available, the probability of  $RP$  having  $n_R$ ,  $0 \leq n_R \leq n_w$ ,  $n_w = K - n_I$  needs to be calculated. This probability is calculated using a binomial distribution with parameters  $(n_w, r)$ . Hence, we can find the throughput of the system as <sup>7</sup>,

$$\lambda_P(n_w) = \sum_{n_R=0}^{n_w} \binom{n_w}{n_R} r^{n_R} (1-r)^{(n_w-n_R)} (\lambda_R(n_R) + \lambda_M(n_w - n_R)). \quad (4)$$

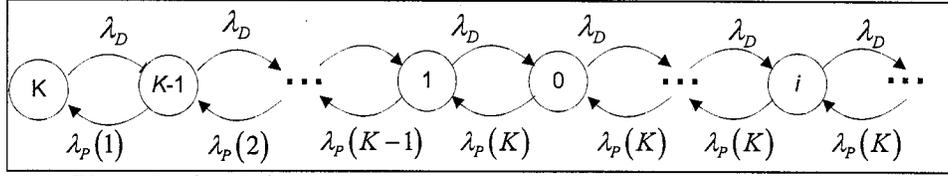


Figure 4: The Birth-Death Process of a General Single Stage Kanban Control

When  $E > 0$  however, the problem gets more complicated, since it is necessary to trace the number of extra kanbans  $x_E(t)$  in circulation at time  $t$ . Therefore, the system is modeled as a three-dimensional Markov process  $x(t) = [x_I, x_R, x_E]$ , where,  $x_I(t)$  gives the total number of parts in queue,  $P$ , minus the total number of backorders in queue,  $D$ ,  $x_R(t)$  gives the total number of work in process in the remanufacturing subsystem, and  $x_E(t)$  gives the number of extra kanbans in the system at time  $t$ . Our objective is to reduce the three dimensional Markov process to a two dimensional Markov process  $x(t) = [x_I, x_E]$ , by calculating the expected total throughput  $\lambda_p(n_w)$  of the production system, with  $x_w(t) = n_w$  being the system load of  $PP$  at time  $t$ . In order to achieve this goal, we need to show that the total throughput is calculated the same way as in the  $E = 0$  case. Thus, the problem is reduced to showing that  $\lambda_p(n_w)$  is independent of the control parameters  $L$  and  $C$  of the  $(K, E, r, L, C)$  control mechanism. If this is true, it enables us to apply the throughput calculation in (4) to the three dimensional system. Then we only need to look at a two dimensional Markov process that counts the number of finished products and the active spare kanbans. Since the analysis of such processes is given by Tardif and Maasiedvaag<sup>15</sup>, using their results, we will be able to calculate the steady state probabilities for the  $E > 0$  case.

**Lemma:** The total throughput of the system,  $\lambda_p(n_w)$ , is independent of the release,  $L$ , and capture,  $C$ , levels of the dynamic kanban control policy  $(K, E, r, L, C)$ .

**Proof:** Let us assume that the opposite is true. That is, for two sets of control parameters in an  $RMP$  system,  $\pi = \{(K_1, E_1, r_1, L_1, C_1), (K_2, E_2, r_2, L_2, C_2)\}$ , where only  $L_1 \neq L_2$  and  $C_1 \neq C_2$ , and for a state  $x_t = [n_I, n_R, n_E]$ , the throughputs  $\lambda_p^1(n_w^1) \neq \lambda_p^2(n_w^2)$ . For this inequality to hold, at least one of either sub-process throughputs or the conditional probabilities should be unequal. We first assume  $P_{\pi_1}^1(n_R | n_I, n_E) \neq P_{\pi_2}^2(n_R | n_I, n_E)$ . Let us first calculate  $P_{\pi_1}^1(n_R | n_I, n_E)$ . Here, the probability of having  $n_R$  parts in the remanufacturing process at time  $t$  is to be found, where  $x_I(t) = n_I$ ,  $x_E(t) = n_E$  and the control parameters  $\pi_1 = (K_1, E_1, r_1, L_1, C_1)$  are given. To calculate this probability we have to find out how many parts are in process in the whole system. In the  $E = 0$  case,  $K_T(t) = x_I(t) + x_w(t)$  and  $K_T(t) = K$  for all  $t \geq 0$ , with  $K_T(t)$  being the total number of active kanbans in  $PP$ . In  $E > 0$  however, for  $L, C < \infty$  and  $L < C$ ,  $K \leq K(t) \leq K + E$ . That is,  $K(t) = K + x_E(t)$ , where  $0 \leq x_E(t) \leq E$ . Then we can write,  $x_w(t) = K + x_E(t) - x_I(t)$ . Thus, the conditional probability we are trying to find may be redefined as  $P_{\pi_1}^1(x_R(t) = n_R | x_w(t) = n_w) = \binom{n_w}{n_R} r^{n_R} (1-r)^{(n_w - n_R)}$ . This is in fact the same conditional probability as in the  $E = 0$  case. Since this function is not dependent on  $C$  and  $L$ ,  $P_{\pi_1}^1(n_R | n_I, n_E) = P_{\pi_2}^2(n_R | n_I, n_E)$ . Then we assume the sub-process throughputs are unequal. The total  $PP$  workload is given as,  $n_w = K - n_I + n_E$ ,  $n_w^1 = n_w^2$  and therefore, for any  $0 \leq n_R^1 = n_R^2 \leq n_w^1 = n_w^2$ , the throughput of each sub-process  $\lambda_R^1(n_R^1) = \lambda_R^2(n_R^2)$  and  $\lambda_M^1(n_w^1 - n_R^1) = \lambda_M^2(n_w^2 - n_R^2)$ , and therefore our initial assumption is false.

Thus, the load dependent throughput calculation of  $PP$  in (4) can be used for  $E \geq 0$  where,

$$n_w = \begin{cases} K - n_I, E = 0 \\ K - n_I + n_E, E > 0. \end{cases} \quad (5)$$

Finally, the following balance equations derived from the Markov Chain given by Tardif and Maasiedvaag<sup>15</sup> give the steady state probabilities when  $\lambda_D/\lambda_P(K+E) < 1$ :

$$\begin{aligned} \text{For, } & 0 \leq x_E \leq E, x_I \leq K + x_E \\ \lambda_D P(x_I, x_E) = & \lambda_P \left( \min \{K + x_E, K + x_E + 1 - x_I\} \right) P(x_I - 1, x_E) \\ & + \lambda_D \left[ \left( \sum_{i=\max\{x_I, C\}+1}^{K+x_E+1} P(i, x_E + 1) - \Psi(x_E) \sum_{j=\max\{x_I, C\}+1}^{K+x_E} P(j, x_E) \right) \right. \\ & \left. + \Theta(x_I) \lambda_D \left( \sum_{v=x_I}^{L-1} P(v+1, x_E - 1) - \Psi(E - x_E) \sum_{v=x_I}^{L-1} P(v+1, x_E) \right) \right] \end{aligned} \quad (6)$$

where,

$$\Psi(x_E) = \begin{cases} 0, x_E = 0 \\ 1, x_E > 0 \end{cases} \quad \text{and} \quad \Theta(x_I) = \begin{cases} 0, x_I \geq L \\ 1, x_I < L \end{cases}$$

Without loss of generality we assume,  $P(x_I, x_E) = 0$  for  $\forall x_E \cap K + x_E < x_I \cup 0 \leq x_E < E \cap x_I < R - x_E$ .

#### 4. NUMERICAL EXAMPLE

Consider an *RMP* with an arbitrary network where the remanufacturing subsystem has a tandem network with  $M = 3$  machines and the manufacturing subsystem has a single service station with  $b = 2$  parallel machines. Let each machine have exponentially distributed processing times with  $\mu = 0.5$ . Let the demands occur following a Poisson process with  $\lambda_D = 0.4$ . Also let  $K = 4$ ,  $E = 2$ ,  $L = 1$  and  $C = 3$  while the routing probability  $r = 0.6$ . Here, we first calculate the throughputs of each subsystem as:

$$\begin{aligned} \lambda_R(n_R) &= 0.5 n_R / (n_R + 2), n_R = 1, \dots, 6 \\ \lambda_M(n_M) &= \begin{cases} 0.5 n_M, & n_M < 2, \\ 1, & n_M \geq 2, \end{cases} n_M = 1, \dots, 6 \end{aligned}$$

Then we find the expected total throughput by applying these sub-system throughputs and (5) to (4). We observe that  $\lambda_D/\lambda_P(K+E) = 0.27 < 1$  and hence the system is stable. Following, we utilize the total throughput to find the steady state probabilities by solving the balance equations simultaneously. Then we calculate the performance measures of the hybrid production system as  $I(K, E, r, L, C) = 2.91$ ,  $WIP(K, E, r, L, C) = 1.43$  and  $B(K, E, r, L, C) = 0.007$ . Here we also realize that  $K + \bar{E} = I(K, E, r, L, C) + WIP(K, E, r, L, C)$ , where  $\bar{E} = 0.33$  gives the average number of extra kanbans used. In order to demonstrate the effectiveness of this control policy we also calculate the  $E = 0$  case for the same routing probability and for  $K = \{4, 5\}$ . We find for  $K = 4$ ,  $I(K, r) = 1.19$ ,  $WIP(K, r) = 2.81$  and  $B(K, r) = 0.2$ , and for  $K = 5$ ,  $I(K, r) = 1.82$ ,  $WIP(K, r) = 3.18$  and  $B(K, r) = 0.08$ . This comparison clearly shows that the adaptive kanban control policy is more efficient.

## 5. CONCLUSIONS

In this paper, we introduced a single stage five parameter adaptive kanban control policy for hybrid manufacturing environments. We have shown that the throughput of such systems depend only on the three of the five parameters. Hence, the three-dimensional Markov process of such systems may be reduced to a Markov process with two dimensions, which essentially models the adaptive kanban control policy for generic production systems. Then we defined a balance equation generation rule for such systems. We concluded our work with a numerical example in which we compared the performance measures of the production process for both adaptive and ordinary kanban control policies. This comparison revealed the advantages of the adaptive kanban policy.

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