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SCALAR POTENTIAL CONCEPT FOR CALCULATING  
THE STEADY MAGNETIC FIELDS AND EDDY CURRENTS

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ABSTRACT

The concept of scalar potential has been used for magnetic field calculation in the USSR during the last decade. It is based on treating the eddy component of the magnetic field separately and calculating the potential field with the help of the scalar potential, by introducing imaginary magnetic charges. This method has been developed for digital calculations of magnetic fields in various electromagnetic devices, by solving linear and nonlinear partial-differential equations through mathematical simulation. This paper presents the mathematical formulation of the method. It is advantageous to use the scalar potential method for two-dimensional eddy-current calculations because of the possibility to solve the equations just over the conductor surfaces. The general three-dimensional problem can be formulated in different ways. The scalar potential method is also used for the computer-aided analysis of eddy-current problems of conventional and superconducting electric machines.

INTRODUCTION

Two concepts of the potential are used for the electromagnetic field calculations at the present time: vector potential  $A$  and the scalar potential  $u$ . The magnetic and electric components can be calculated with the help of the vector potential. However one has some difficulties as equations for three components of vector potential have to be solved. This leads to increased computing time and computer memory. Another difficulty is that the equations for components of the vector potential may not be independent, because of the boundary conditions. Using the scalar potential one can obtain the Laplace equation for the scalar function  $u$ . On the one hand we have just one equation instead of three equations for the vector potential. But on the other hand it is impossible to use the scalar potential concept inside the region where the current density  $J$  is not equal to zero. The usual way in this case is that the conductor carrying electric current is changed in such a way that it becomes the current

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shell. But in the vicinity of the conducting region the accuracy of the calculation is not very high.

The scalar potential method utilizing the imaginary magnetic charges has been used since 1960 in the USSR for the calculation and mathematical simulation of electromagnetic fields<sup>1,2,3</sup>. The method allows the use of the scalar function inside the region where the current density is not equal to zero.

#### CALCULATION OF THE STEADY MAGNETIC FIELD

The unknown magnetic field intensity  $\vec{H}$  is replaced by the sum of two fields  $H_1$  and  $H_2$ :

$$\vec{H} = \vec{H}_1 + \vec{H}_2 \quad (1)$$

The field  $H_2$  is chosen in such a way that

$$\text{curl } \vec{H}_2 = \vec{J} \quad (2)$$

so one can write for the field  $H_1$  the equation

$$\text{curl } \vec{H}_1 = 0 \quad (3)$$

because of  $\text{curl } \vec{H} = \vec{J}$ .

The scalar magnetic potential  $u$  can be introduced for the calculation of the field  $H_1$ :

$$\vec{H}_1 = -\text{grad } u$$

The sources of the potential field can be found as

$$\rho = \text{div } \mu \vec{H}_1 = -\text{div } \mu \vec{H}_2 \quad (5)$$

because of  $\text{div } \mu \vec{H} = 0$ . The Poisson's equation for the scalar magnetic potential is

$$\text{div } (\mu \text{ grad } u) = -\rho \quad (6)$$

The field  $H_2$  can be defined as

$$\vec{H}_2 = \int_1 \vec{J} \times d\vec{l} \quad (7)$$

where  $d\vec{l}$  is an arbitrary directed vector. Under such a definition of the field  $H_2$ , the equation (2) is automatically satisfied.

The first step of the calculation is choosing the vector  $d\vec{l}$  and definition of the field  $H_2$  at any point. Then the volume density of magnetic charges has to be calculated. After that, equation (6) can be solved and the field  $H_1$  be found. The unknown field  $H$  equals  $H_1$  at any point where the current density equals zero and equals the sum of  $H_1$  and  $H_2$  if the current density  $\vec{J}$  is not equal to zero.

The vector  $d\vec{l}$  is an arbitrary one; so the field

$\vec{H}_2$  depends on the choice of the vector  $d\vec{l}$ . The field  $\vec{H}_2$  at the same time determines the volumes where the magnetic charges are present. Because of the possibility of multiple choices of the vector  $d\vec{l}$  (and therefore vector  $\vec{H}_2$ ) the problem of minimization of the sources of the potential field can be put. The minimum volume of the magnetic charges can minimize the computing time. This problem has been solved in reference (3).

In the general case of a three-dimensional field, one can write  $d\vec{l} = i dx$ . Then we have for the field  $\vec{H}_2$ :

$$\vec{H}_2 = \vec{j} \int_{x_0}^x J_z dx - \vec{k} \int_{x_0}^x J_y dx = \vec{j} H_{2y} + \vec{k} H_{2z} \quad (8)$$

in which  $J_z, J_y$  are the components of the current density.

The region in which  $\vec{H}_2$  is not equal to zero, can be infinite when, for example, the magnetic field of single conductors is calculated. It means that the volume which is occupied with the magnetic charges is infinite as well. It is important therefore to restrict the field  $\vec{H}_2$  by some surfaces. If on one side of the surface the field  $\vec{H}_2$  is not equal to zero, but along the other side it should equal zero, it means that the normal and tangential components of the  $\vec{H}_2$  field have to have a jump and the single layers of magnetic charge and electric current have to be placed along the surface. The surface density of magnetic charges equals:

$$\sigma = \mu \text{Div } \vec{H}_2 \quad (9)$$

in which Div represents the surface divergence of the vector. The surface current equals the tangential component of the  $\vec{H}_2$ :

$$I = H_{2t} \quad (10)$$

#### CALCULATION OF THE FIELD VARYING WITH TIME

The equation  $[\text{curl curl } \vec{H}_2 = \text{curl } \vec{J}]$  can be written in cartesian coordinates as

$$\text{grad}_k \text{div } \vec{H}_2 - \text{div grad } H_{2k} = \text{curl}_k \vec{J}, \quad (11)$$

(k = x, y, z)

One can write

$$\text{curl}_k \vec{J} = \text{curl}_k \gamma \vec{E} = -\mu \gamma \frac{\partial H}{\partial t} \vec{k} + (\text{grad } \gamma \times \vec{E})_k \quad (12)$$

For the harmonic functions and the complex values:

$$\text{grad}_k \text{div } \vec{H}_2 - \text{div grad } \dot{H}_{2k} + j\omega\mu\gamma\dot{H}_{2k} = j\omega\mu \text{grad}_k \dot{u} + (\text{grad } \gamma \times \vec{E})_k \quad (13)$$

The problem of eddy current calculation is formulated with the equation (13) and the equation for the scalar potential:

$$\text{div} (\mu \text{grad } \dot{u}) = -\dot{\rho} = \text{div } \mu \vec{H}_2 \quad (14)$$

Four equations can be written: three equations are for the projections of the equation (13) and one equation (14) is for the scalar potential. At the same time the number of unknown values is three: two components of the field  $\vec{H}_2$  and the scalar potential. This means that the problem can be formulated in different ways when two different components of equation (13) are used.

For two-dimensional problems, when the field  $\vec{H}_2$  consists of just one component, the number of equations is two; one of them is for the scalar potential and the other is one of the component equations of Eq. (13). The boundary conditions for the scalar potential  $u$  can be written in a conventional manner. When the vector  $dl$  is  $dl = \dot{i}dx$  and  $k$  is  $x$ , the system of equations consists of equation for the scalar  $u$  and for the surface density of magnetic charges along the conductor surface. One can write the equations for the eddy current calculation of thin plate ( $\mu = \mu_0$ ), two flat surfaces of which are parallel to the plane  $xoz$  and the thickness is in  $y$ -direction. Assuming only two components ( $J_x$  and  $J_z$ ) of eddy current inside the plate, with  $J_y$  equal to zero, and choosing the direction of the vector  $dl = \dot{i}dx$ , one has

$$\vec{H}_2' = \dot{j} \int J_z dx$$

Using the  $y$ -projection of the equation (13), one gets

$$\frac{\partial^2 \dot{H}_{2y}}{\partial x^2} + \frac{\partial^2 \dot{H}_{2y}}{\partial z^2} = +j\omega\mu_0\gamma(\dot{H}_{2y} - \frac{\partial \dot{u}}{\partial y}) \quad (15)$$

The second equation is for the scalar potential given by Eq. (14). The field  $\dot{H}_{2y}$  is equal to zero over the side surface of the conducting plate.

If the  $x$ -projection of the equation (13) is used, one should write the equations for the scalar magnetic potential  $u$  and for the surface density of magnetic charges which are situated over two surfaces parallel to the plane  $xoz$ :

$$\nabla^2 \dot{u} - j\omega\mu_0\gamma\dot{u} = -j\omega\mu_0\gamma \dot{u}(0) - \frac{\dot{\sigma}}{\mu} \quad (17)$$

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$$\nabla^2 \dot{\sigma} = + j\omega\mu_0\gamma \frac{\partial \dot{u}}{\partial y}$$

The value  $\dot{u}(0)$  represents the scalar potential over the yz-plane. The boundary condition for the  $\dot{\sigma}$  is that it should be zero.

#### REFERENCE

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