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Charlos Fabián Benítez-Quiroz
University of Puerto Rico - Mayaguez

Shawn D. Hunt
University of Puerto Rico - Mayaguez

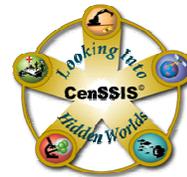
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Determining the Need for Dither when Re-Quantizing a 1-D Signal

Carlos Fabián Benítez-Quiroz and Shawn D. Hunt
Electrical and Computer Engineering Department
University of Puerto Rico, Mayagüez Campus



ABSTRACT

This poster presents novel methods for determining if dither is needed when reducing the bit depth of a one dimensional digital signal. These are statistical based methods in both the time and frequency domains, and are based on determining whether the quantization noise with no dither added is white. If this is the case, then no undesired harmonics are added in the quantization or re-quantization process. Experiments showing the effectiveness of the methods with both synthetic and real one dimensional signals are presented.

Introduction

Digital signals have become widespread, and are favored in many applications because of their noise immunity. This is a result of their being discrete in both time and amplitude. Once the signal has been discretized, the signal can be stored or transmitted without additional noise being added. There are many applications however, where the discretization in both time and amplitude needs to be changed in the discrete domain. The process of lowering the amplitude resolution of a signal is called re-quantization.

If the signal amplitude change is large from sample to sample, then it is generally assumed that the re-quantization error will be a uniformly distributed i.i.d. sequence. In this case, the quantization error is independent of the signal being quantized. This assumption does not hold for all cases, however, and re-quantizing a signal can introduce various undesirable artifacts, namely, additional harmonics related to the signal being re-quantized. This harmonics can be viewed in the power spectral density (PSD) of the error signal. Figure 1 shows the PSD of a .03rad/sam sine wave after re-quantizing from 24 to 7 bits.

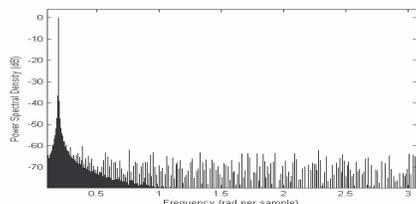


Figure 1 PSD of a sine wave after re-quantizing from 24 to 7 bits

To avoid these unwanted harmonics, dither is generally added to the signal before quantizing. The purpose of the dither signal is to ensure that the quantization error is uncorrelated with the signal being quantized. Ideally, the quantization signal is independent of the signal being quantized, but it is generally accepted that having uncorrelated first and second moments is sufficient.

State of the Art

The state of the art when re-quantizing a signal is to always add dither before the quantizing stage. The disadvantage of this is that dither is a noise signal, and so adds additional noise when used. Specifically, 6dB SNR is lost when dither is added. In many real cases, dither is not required, and adding dither decreases the SNR unnecessarily.

Tests In The Time Domain

A sufficient condition for not introducing unwanted harmonics during re-quantization is that the quantization noise be an i.i.d. sequence. Using an autocorrelation estimator, the whiteness condition can be measured. The autocorrelation estimator for an error signal q with k lags of difference is denoted by r_k , where r_k is normalized between -1 and 1. Table 1 show the statistic used for each estimator.

Test In The Frequency Domain

The test in frequency is based on the Fourier transform of the sample autocorrelation. The statistic used is the square of the norm of the DFT multiplied by scale factor. This is also summarized in Table 1.

Table 1. Summary of the statistics used in the tests.

Test	Domain	Estimator used	Distribution of Estimator	Summary of Method
Box-Pierce Test	Time	$Q_{bp} = N \sum_{k=1}^m r_k^2$	$Q_{bp} \sim \chi_m^2$	Say the quantization error is white when Q_{bp} is greater than a set threshold.
Ljung-Box Test	Time	$Q_{lb} = N(N+2) \sum_{k=1}^m \frac{r_k}{N-k}$	$Q_{bp} \sim \chi_m^2$	Say the quantization error is white when Q_{bp} is greater than a set threshold.
Frequency Test	Frequency	$A(l) = \sum_{n=-N}^{N-1} q(n) \cos\left(\frac{2\pi ln}{N}\right)$ $B(l) = \sum_{n=-N}^{N-1} q(n) \sin\left(\frac{2\pi ln}{N}\right)$ $Y(l) = \frac{2}{N\sigma^2} (A^2(l) + B^2(l))$ $-\frac{N}{2} < k < \frac{N}{2}$	$Y(l) \sim \chi_2^2$	A goodness of fit test is used to determine if the estimator has a Chi-Squared distribution.

Results

One synthetic test signal was a full scale 0.03rad/sam cosine with 24 bit precision. The tests were used to determine at what bit level the quantization noise ceases to be white. Here, the signal is re-quantized to different bit levels varying from 22 down to 1 bit. The test signal has a length of 10000 points and 2500 lags are used in the autocorrelation tests. The acceptance threshold for the p-value used was 0.1 as show in figure 2. The probability value (p-value) can be interpreted as the probability of the test statistic being larger than that obtained by chance alone, assuming that the null hypothesis is true.

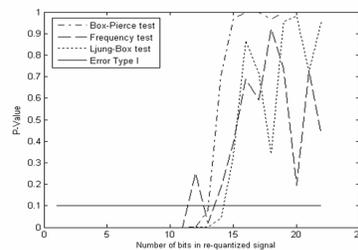


Figure 2: P-values of different levels of quantization for a sine signal.

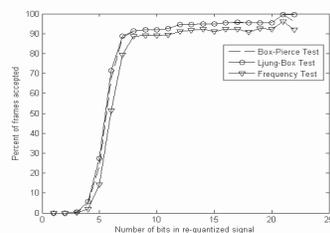


Figure 3: Percentage of frames accepted as having white quantization noise

Another test signal was used real audio with 24 bit precision and a 44.1 kHz sampling rate. The tests were applied and the percentage of accepted frames is shown in figure 3

Experiments were also run after adding dither. Because of the added dither, all tests should reveal that no additional dither is needed. For the tests, segments from the previous experiment that are known to need dither are dithered and these used as the test signal.

	Box-Pierce test	Ljung-Box Test	Frequency Test
Uniform PDF	1.0000	0.4176	0.7886
Gaussian PDF	1.0000	0.7044	0.9626
Triangular PDF	1.0000	0.7867	0.6143

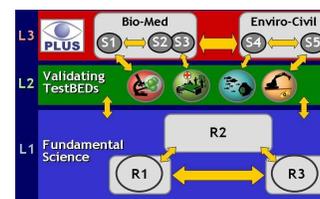
Table 2: p-values for different dithers.

Opportunities for Technology Transfer

The work developed here is applicable to all digital signals. Having correctly dithered signals is necessary at both the analog to digital conversion stage, or at any later stage when reducing the bit depth. Some examples are:

- when reducing bit depth to save on communication or storage costs
- A/D converters,
- sample rate converters.

REVELANCE TO CENSSIS STRATEGIC RESEARCH PLAN



- The hypothesis test for white noise can be used to measure if the re-quantization error has unwanted harmonics (R1).
- Re-quantization with no dither can be used for store and transmit the error with the best signal to noise ratio(R3).

Future Work

- Determine an optimal dither, in the sense of maximizing the SNR, when dither is needed.
- Determine the optimal dither for real time signal processing.

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