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NEW DEVELOPMENTS IN THE COMPUTER-AIDED ANALYSIS
OF THREE-DIMENSIONAL ELECTROMAGNETIC FIELD PROBLEMS
AS APPLIED TO THE DESIGN OF ELECTRICAL MACHINERY

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ABSTRACT

An accurate knowledge of the magnetic as well as other field distributions is of great importance in design optimization. In two dimensional problems with the current flow in only one direction, the magnetic field can be solved by computing a scalar potential or one component of the vector potential. The general formulation for three-dimensional solutions, including nonlinearities, is more complex and requires all three components of the vector potential as well as a scalar potential for the description of the fields. It is the purpose of this paper to present a general systematic, novel formulation at low frequencies feasible for the computer-aided design or analysis in terms of potential functions for three-dimensional numerical solutions of the nonlinear electromagnetic field problems that include either nonlinear magnetic materials or nonlinear electric materials. Special cases of the magnetostatic as well as eddy-current problems are discussed.

INTRODUCTION

The continuing rapid increase in the rating of the generating plant and the consequent increased electric and magnetic loadings of electrical machines are forcing manufacturers to work with greatly reduced safety factors in designing alternators. Design optimization calls for an accurate knowledge of the magnetic as well as other field distributions. As a result, a closer examination of the magnetic field and improved methods of predicting the field distribution become more important than ever before. The "field theory" as opposed to "circuit theory" has become increasingly important in many engineering problems.

The fundamental laws governing all electromagnetic fields can be expressed by the well-known Maxwell's equations which may be found in any textbook [1] on the subject. Assuming that the dielectric properties of any of the materials are of no significance in comparison with the conduction properties, as is true in low frequency problems, the dielectric effects or the displacement currents will be neglected, and the material regions will be considered as void of volume charge density. All the materials will be taken to be isotropic and homogeneous. Then the relevant partial differential equations may be written as follows:

$$\nabla \cdot B = 0 \quad ; \quad \nabla \times H = J \quad ; \quad \nabla \times E = - \frac{\partial B}{\partial t} \quad ; \quad \nabla \cdot J = 0 \quad . \quad (1,2,3,4)$$

The constituent relations are given as

$$B = \mu H \quad ; \quad J = \sigma E \quad . \quad (5,6)$$

It may be observed here that the current continuity relation given by (4) is a consequence of (2).

The general boundary conditions to be satisfied at the interfaces of stationary dissimilar media may be derived from the limiting integral forms of (1)-(4), and they are given as

$$B_{n1} - B_{n2} = 0 ; \quad H_{t1} - H_{t2} = J_s ; \quad E_{t1} - E_{t2} = 0 ; \quad J_{n1} - J_{n2} = 0 .$$

(7,8,9,10)

A surface current Density J_s will exist at the interface separating region 1 from region 2 only if one of the regions has infinite conductivity. Otherwise, if both regions have finite (including zero) conductivity, then (8) takes the form

$$H_{t1} - H_{t2} = 0 .$$

(8a)

It must be borne in mind that the boundary conditions need to be modified [2] at the interfaces between dissimilar media in relative motion.

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To allow for the magnetic nonlinearity, the permeability μ of the ferromagnetic materials will be taken as a single-valued function of the magnetic induction B , while the hysteresis effects are neglected. Constant conductivity σ of the medium will be assumed.

One can express B as the curl of A :

$$B = \nabla \times A$$

(11)

thereby satisfying (1). From (3) and (11), one obtains

$$E = - \frac{\partial A}{\partial t} - \nabla \phi$$

(12)

where ϕ is the electric scalar potential. Defining the reluctivity v as the reciprocal of the magnetic permeability μ , one can obtain the following from (2), (5), (6), (11), and (12):

$$\nabla \times [v(\nabla \times A)] = J$$

(13)

where J represents either the impressed current density sources or the conduction current densities existing due to induction phenomena given by:

$$J = - \sigma \left[\frac{\partial A}{\partial t} + \nabla \phi \right]$$

(14)

It follows then, as a consequence of (13) and (14), that ϕ has to satisfy

$$\nabla^2 \phi = - \frac{\partial}{\partial t} (\nabla \cdot A)$$

(15)

unless σ is zero.

So far, the divergence of A and the gradient of ϕ have not been defined. In fact, they may be defined in any convenient manner as long as the basic equation (13) is not violated. Equation (13), in conjunction with (14),

may now be written as three component-coupled equations in Cartesian coordinate system as follows:

$$\left[\frac{\partial}{\partial y} \left\{ v \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \right\} - \frac{\partial}{\partial z} \left\{ v \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \right\} \right] = -\sigma \frac{\partial A_x}{\partial t} - \sigma \frac{\partial \phi}{\partial x} \quad (16a)$$

$$\left[\frac{\partial}{\partial z} \left\{ v \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \right\} - \frac{\partial}{\partial x} \left\{ v \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \right\} \right] = -\sigma \frac{\partial A_y}{\partial t} - \sigma \frac{\partial \phi}{\partial y} \quad (16b)$$

$$\left[\frac{\partial}{\partial x} \left\{ v \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) \right\} - \frac{\partial}{\partial y} \left\{ v \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \right\} \right] = -\sigma \frac{\partial A_z}{\partial t} - \sigma \frac{\partial \phi}{\partial z} \quad (16c)$$

As it is simpler and more convenient to have the uncoupled component equations for the numerical procedures, one may conveniently define the gradient of ϕ as follows in order to uncouple (16).

$$-\sigma \frac{\partial \phi}{\partial x} = \left[\frac{\partial}{\partial x} \left(v \frac{\partial A_x}{\partial x} \right) + \frac{\partial}{\partial y} \left(v \frac{\partial A_y}{\partial x} \right) + \frac{\partial}{\partial z} \left(v \frac{\partial A_z}{\partial x} \right) \right] \quad (17a)$$

$$-\sigma \frac{\partial \phi}{\partial y} = \left[\frac{\partial}{\partial y} \left(v \frac{\partial A_y}{\partial y} \right) + \frac{\partial}{\partial z} \left(v \frac{\partial A_z}{\partial y} \right) + \frac{\partial}{\partial x} \left(v \frac{\partial A_x}{\partial y} \right) \right] \quad (17b)$$

$$-\sigma \frac{\partial \phi}{\partial z} = \left[\frac{\partial}{\partial z} \left(v \frac{\partial A_z}{\partial z} \right) + \frac{\partial}{\partial x} \left(v \frac{\partial A_x}{\partial z} \right) + \frac{\partial}{\partial y} \left(v \frac{\partial A_y}{\partial z} \right) \right] \quad (17c)$$

It may be observed here that the vector quantity $[\nabla\{v(\nabla \cdot A)\}]$ has been implicitly defined in (17). As a consequence of this, (16) may be simplified as:

$$\left[\frac{\partial}{\partial x} \left(v \frac{\partial A_x}{\partial x} \right) + \frac{\partial}{\partial y} \left(v \frac{\partial A_x}{\partial y} \right) + \frac{\partial}{\partial z} \left(v \frac{\partial A_x}{\partial z} \right) \right] = \sigma \frac{\partial A_x}{\partial t} \quad (18a)$$

$$\left[\frac{\partial}{\partial x} \left(v \frac{\partial A_y}{\partial x} \right) + \frac{\partial}{\partial y} \left(v \frac{\partial A_y}{\partial y} \right) + \frac{\partial}{\partial z} \left(v \frac{\partial A_y}{\partial z} \right) \right] = \sigma \frac{\partial A_y}{\partial t} \quad (18b)$$

$$\left[\frac{\partial}{\partial x} \left(v \frac{\partial A_z}{\partial x} \right) + \frac{\partial}{\partial y} \left(v \frac{\partial A_z}{\partial y} \right) + \frac{\partial}{\partial z} \left(v \frac{\partial A_z}{\partial z} \right) \right] = \sigma \frac{\partial A_z}{\partial t} \quad (18c)$$

Equation (18) is not coupled and is easier to work with than the coupled set of (16). The three components A_x , A_y , and A_z have to be found from (18) and from the appropriate boundary conditions. The electric scalar potential ϕ does not come directly into the picture and need not be computed. It does, however, appear at the interfaces where the boundary condition given by (10) has to be imposed, in which case the definition of $(\nabla\phi)$ in terms of A , given by (17), will be taken advantage of.

It may be observed here that the formulation is rather simplified within the uniform constant-conductivity medium containing magnetic nonlinearity. However, the complexity presents itself at the boundary surfaces where the

conductivities change. The boundary conditions are really known in terms of conditions on the components of B , H , E , and J . These, when expressed in terms of conditions on the components of A through the relations such as (11), can be seen to be not uniquely specified. A specific case is discussed later so that this aspect may be appreciated further.

A solution for the different fields, and particularly the boundary conditions in terms of the vector potential, become much simpler when the additional assumption that the vector potential has only one component can be made, e.g., A_z , as a consequence of which B_z goes to zero and E has all three components. Such a treatment has been applied [5] for the approximate three dimensional analysis of the solid rotor induction machine.

SPECIAL CASES

A. Magnetostatic Field Problems with Impressed Current Densities

For the particular case of magnetostatic field problems with impressed current densities, only (1) and (2) are needed. Equation (4) and hence (10) will be satisfied through the specification of the current densities.

In two dimensional problems [3], including discrete current carrying regions, one uses with advantage the concept of the magnetic vector potential A which has only one component and whose divergence is trivially zero. However, this is not the case in a three-dimensional analysis, since all the vectors have three components. Working with the vectors B and H , (1) and (2), which resolve into four component equations with three unknowns must be satisfied. This poses a serious problem for the numerical solution, which does not possess the advantage of determining the functional forms of the unknowns as in the analytical solutions. There does not seem to exist any numerical method to satisfy simultaneously all four equations with three unknowns at every lattice point. Turning to the vector A , (13) is the fundamental partial differential equation to be solved for.

The partial differential equations have to be transformed into the difference form for numerical work. The difference expressions can be developed [4] either by directly using the partial-differential equations and the first term of Taylor series, or by applying Ampere's law. Figure 1 shows a typical three-dimensional lattice. The distance between the point 0 and the points 1-6 are denoted by h_1 - h_6 , respectively. Figure 2 shows a typical lattice point 0 and its neighborhood; points I-VI are located at the middle points of the mesh lines $0\bar{1}$ - $0\bar{6}$, respectively; v_1 - v_6 are the reluctivities specified at the center points I-VI, respectively; J_0 is the current-density vector specified at the latter point 0.

A typical component equation of (13) given by

$$\left[\frac{\partial}{\partial x} \left(v \frac{\partial A}{\partial x} \right) + \frac{\partial}{\partial y} \left(v \frac{\partial A}{\partial y} \right) - \frac{\partial}{\partial x} \left(v \frac{\partial A}{\partial z} \right) - \frac{\partial}{\partial y} \left(v \frac{\partial A}{\partial z} \right) \right] = - J_z \quad (19)$$

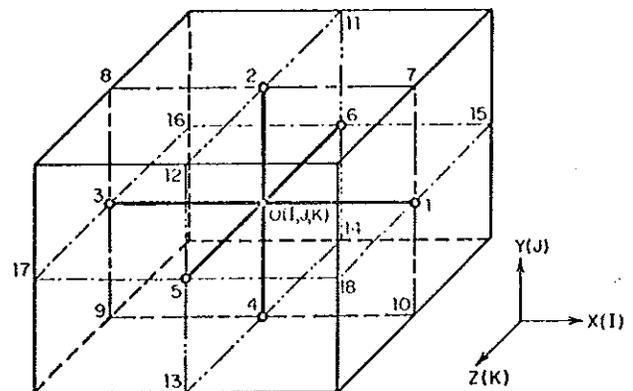


FIG. 1. TYPICAL THREE-DIMENSIONAL LATTICE.

will now be transformed into the difference form by replacing the partial derivatives by the central partial difference quotients.

A term like $\{(\partial/\partial x)[v(\partial A_z/\partial x)]\}$ is replaced by

$$\left[\left\{ v_1 \frac{(A_{z1} - A_{z0})}{h_1} - v_3 \frac{(A_{z0} - A_{z3})}{h_3} \right\} / \left(\frac{h_1 + h_3}{2} \right) \right].$$

A mixed derivative term like $\{(\partial/\partial x)(v(\partial A_x/\partial z))\}$ is replaced by

$$\left[\frac{v_1 (A_{x18} - A_{x15} + A_{x5} - A_{x6}) - v_3 (A_{x5} - A_{x6} + A_{x17} - A_{x16})}{(h_1 + h_3)(h_5 + h_6)} \right].$$

Equation (19) can now be written in difference form as given below after considerable rearrangement of terms, for the calculation of a component of the vector potential at the lattice point 0:

$$A_{z0} = M_3/N_3 \quad (20)$$

where

$$\begin{aligned} M_3 = & [4J_{z0} + v_1 \alpha_1 \{4A_{z1} - \alpha_5 h_1 h_5 (A_{x18} - A_{x15} + A_{x5} - A_{x6})\} \\ & + v_3 \alpha_3 \{4A_{z3} + \alpha_5 h_3 h_5 (A_{x5} - A_{x6} + A_{x17} - A_{x16})\} + v_2 \alpha_2 \{4A_{z2} - \alpha_5 h_2 h_5 \\ & \cdot (A_{y12} - A_{y11} + A_{y5} - A_{y6})\} + v_4 \alpha_4 \{4A_{z4} + \alpha_5 h_4 h_5 \cdot (A_{y5} - A_{y6} \\ & + A_{y13} - A_{y14})\}] \end{aligned}$$

and

$$N_3 = [4\{v_1 \alpha_1 + v_3 \alpha_3 + v_2 \alpha_2 + v_4 \alpha_4\}]$$

and where

$$\alpha_1 = \frac{2}{h_1(h_1 + h_3)}; \quad \alpha_3 = \frac{2}{h_3(h_1 + h_3)}; \quad \alpha_2 = \frac{2}{h_2(h_2 + h_4)}; \quad \alpha_4 = \frac{2}{h_4(h_2 + h_4)}.$$

This corresponds to the Z component of (13). The other expressions corresponding to the x and y components of (13) can be similarly developed. The numerical computation of the vector potential at a given lattice point makes use of the vector potentials at the surrounding 18 points. The difference algorithms are seen to be rather long and involved; due to which the computations would require much computer time before a reasonable convergence is obtained. The reluctivities as well as the current densities may eventually be defined for convenience either at the centers of the meshes in individual

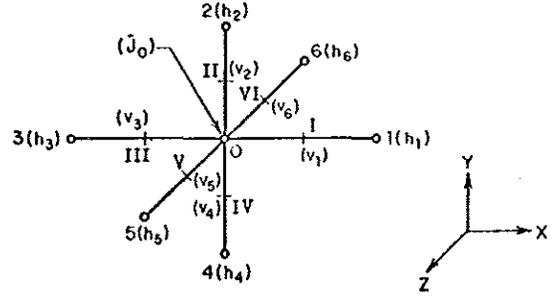


FIG. 2. TYPICAL LATTICE POINT AND ITS NEIGHBORHOOD.

planes [4] or at the centers of the parallelepipeds [5], and the difference algorithms may be modified accordingly.

The long difference algorithms are consequences of the coupled component equations of (13). If the component equations can be uncoupled, then it can be seen that the algorithms would require the information at the neighboring six lattice points only. Uncoupling the component equations of (13) can easily be achieved through the arbitrary definition of $[\nabla\{v(\nabla \cdot A)\}]$ given by (17) with the terms on the left hand side set equal to zero. This is equivalent to choosing the gradient of ϕ equal to zero, which is justified as E , given by (12), should in fact be zero in magnetostatic field problems. Then the following equations result.

$$\left[\frac{\partial}{\partial x} \left(v \frac{\partial A_x}{\partial x} \right) + \frac{\partial}{\partial y} \left(v \frac{\partial A_x}{\partial y} \right) + \frac{\partial}{\partial z} \left(v \frac{\partial A_x}{\partial z} \right) \right] = - J_x \quad (21a)$$

$$\left[\frac{\partial}{\partial x} \left(v \frac{\partial A_y}{\partial x} \right) + \frac{\partial}{\partial y} \left(v \frac{\partial A_y}{\partial y} \right) + \frac{\partial}{\partial z} \left(v \frac{\partial A_y}{\partial z} \right) \right] = - J_y \quad (21b)$$

$$\left[\frac{\partial}{\partial x} \left(v \frac{\partial A_z}{\partial x} \right) + \frac{\partial}{\partial y} \left(v \frac{\partial A_z}{\partial y} \right) + \frac{\partial}{\partial z} \left(v \frac{\partial A_z}{\partial z} \right) \right] = - J_z \quad (21c)$$

Using the central difference scheme, the following difference expression, corresponding to (21c) for the calculation of A_z at the lattice point 0 can be obtained by referring to Figs. 1 and 2:

$$A_{z0} = (M_1 + J_{z0})/N_1 \quad (22)$$

where

$$M_1 = \sum_{i=1}^6 v_i \alpha_i A_i$$

$$N_1 = \sum_{i=1}^6 v_i \alpha_i$$

and

$$\alpha_1 = \frac{2}{h_1(h_1 + h_3)} \quad ; \quad \alpha_3 = \frac{2}{h_3(h_1 + h_3)} \quad ; \quad \alpha_2 = \frac{2}{h_2(h_2 + h_4)}$$

$$\alpha_4 = \frac{2}{h_4(h_2 + h_4)} \quad ; \quad \alpha_5 = \frac{2}{h_5(h_5 + h_6)} \quad ; \quad \alpha_6 = \frac{2}{h_6(h_5 + h_6)}$$

The other expressions corresponding to (21a) and (21b) can be similarly obtained.

1) Linear Cases:

For the case of linear magnetostatic problems it can easily be seen that the Poisson's equation of the vector A applies, with the arbitrary definition of the divergence of A being equal to zero, which is in fact the Lorentz condition

$$\nabla \cdot A = 0 \quad (23)$$

$$\nabla^2 A + \mu J = 0 \quad (24)$$

Now it appears that one has again a system of four component equations with three unknowns. However, this difficulty can be overcome by viewing (13) as

$$\nabla(\nabla \cdot A) = \nabla^2 A + \mu J \quad (25)$$

and solving numerically the right-hand side to tend to zero. Thus one has to solve only the three component equations of (24) for the three unknown components of A , after obtaining the appropriate difference algorithms corresponding to (24).

It may be observed here that the uncoupling of the component equations would lead to simpler difference algorithms, which in turn means less computer time with reasonable convergence.

2) Boundary Conditions:

Since the partial differential equations and the difference expressions are set up with the variable A , the boundary conditions have to be expressed in terms of A using the relations such as (11). As a typical specific simple case, the boundary conditions at an xy plane of zero permeability will be discussed.

The normal z component of the flux density at an xy plane of zero permeability has to vanish. Accordingly,

$$B_z = \frac{\partial A}{\partial x} y - \frac{\partial A}{\partial y} x = 0 \quad (26)$$

Referring to Fig. 2, let the components of A at lattice point 5 be the unknowns. The distance h_5 is arbitrary and may conveniently be taken as equal to h_6 . Based on this, the difference expressions can be found by replacing the partial derivatives, evaluated at lattice point 0, by their central difference quotients:

$$\frac{A_{y1} - A_{y3}}{h_1 + h_3} - \frac{A_{x2} - A_{x4}}{h_2 + h_4} = 0 \quad (27)$$

In the same fashion, the condition on the divergence of A given by (23) can also be transformed.

$$\frac{A_{x1} - A_{x3}}{h_1 + h_3} + \frac{A_{y2} - A_{y4}}{h_2 + h_4} + \frac{A_{z5} - A_{z6}}{h_5 + h_6} = 0 \quad (28)$$

Since there are no additional conditions to be satisfied, one may conveniently choose

$$A_x = 0 \quad (29)$$

$$A_y = 0 \quad (30)$$

at all the lattice points on the xy plane. This can also be seen to be justified from the analytical solutions [4]. Because of the above choice,

(27) is satisfied and (28) yields

$$(A_{z5} - A_{z6}) / (h_5 + h_6) = 0$$

which implies that

$$A_{z5} = A_{z6} \quad . \quad (31)$$

Equations (29)-(31) are the boundary conditions in difference form in terms of A on an xy plane of zero permeability. The parallel components of A vanish and the normal component of A is continuous in the sense of the normal direction to the plane. The component A_z at the lattice point 0 can then be evaluated by using the appropriate difference expression corresponding to the Laplace's equation.

It can be seen that the boundary conditions, when expressed in terms of A , are not uniquely specified as the conditions on the components of A , as desired for the numerical analysis of the boundary-value problem. Thus one is forced to make a wise choice based on the approximate analytical solutions and available freedom. It is obvious that the difficulties will be greater if the boundary material regions of finite permeability with finite thickness, and the intersection of different boundary media have to be taken adequately into consideration.

B. Eddy-Current Problems

For charge-free nonlinear ferromagnetic material regions of finite conductivity, in which induction phenomena causes conduction current densities, one goes through the formulation set up earlier, and (13)-(18) apply. For linear material regions of finite permeability and conductivity, (17) will be replaced by the equivalent relationship given by

$$\nabla \cdot A = -\mu\sigma\phi \quad . \quad (32)$$

Equation (18) holds good with the reluctivity v which is treated as a constant. Two dimensional problems, with either linear or nonlinear magnetic media, can be solved [2] rather easily with the assumptions that the vector potential A has only one component and the scalar potential ϕ does not exist.

All the field equations associated with the eddy currents can be seen to contain the partial derivative term with respect to the time variable. Through a suitable transformation of coordinates, it is possible in some cases [2],[4],[5] to replace the time derivative terms by spatial derivatives so that the time variable need not be considered explicitly in the digital computer programs. Depending on the type of iterative procedure used, the difference scheme for the first order terms representing the induced current sources has to be chosen properly from the viewpoint of stability and convergence. The forward difference scheme for such terms appears to work well [4] along with the successive point iteration method. Including the time variable in the digital computer program complicates matters considerably and seems to demand prohibitively large computer time. Further research work needs to be done for finding practical methods that would enable one to consider the time as a variable. The author has been looking into the possibility of using the hybrid computers for this purpose, but with no encouraging results as yet.

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For this case, constant permeability μ of the medium will be assumed. To allow for the electric nonlinearity, the conductivity σ will be taken as a single valued function, being dependent on J and E .

One can express J as the curl of F thereby satisfying (4). It may be pointed out here that the potential F can be used only if the divergence of J is equal to zero, whereas A has no such restrictions. Then, one obtains

$$H = F - \nabla\Omega \quad (33)$$

where Ω is the magnetic scalar potential. One can then obtain the following equation from (3), (5), (6), and (33):

$$\nabla \times \left[\frac{\nabla \times F}{\sigma} \right] = -\mu \cdot \frac{\partial}{\partial t} [F - \nabla\Omega] \quad (34)$$

If μ is not zero, it follows that

$$\nabla^2\Omega = \nabla \cdot F \quad (35)$$

The reciprocal of the conductivity may be replaced by the resistivity. For an electric nonlinear medium, (21) may be resolved into three uncoupled component equations through the suitable definition of $\nabla \cdot F$ and $\nabla\Omega$. However, for a linear medium in which the conductivity is independent of E and J , (21) may be simplified as:

$$\nabla^2 F - \nabla(\nabla \cdot F) = \mu\sigma \frac{\partial}{\partial t} [F - \nabla\Omega] \quad (36)$$

The calculation of the potential Ω may be alleviated and the formulation be done in terms of F alone by defining

$$\nabla \cdot F = \mu\sigma\Omega \quad (37)$$

in which case (36) becomes simply

$$\nabla^2 F = \mu\sigma \frac{\partial F}{\partial t} \quad (38)$$

which can be resolved into three uncoupled component equations.

CONCLUDING REMARKS

The relevant partial differential equations and the boundary conditions at the interfaces have been discussed for the nonlinear magnetostatic and eddy-current problems at low frequencies with particular reference to the three-dimensional numerical solutions. A systematic and simple formulation feasible for the computer-aided design or analysis is presented. The formulation may be done either in terms of magnetic vector potential A or in terms of an electric vector potential F . The former approach is desirable in dealing with nonlinear magnetic materials, while the latter is appropriate for the case of nonlinear electric materials.

Though the development in this paper has been done in a Cartesian coordinate system for the purpose of illustrating the principle involved, it may be similarly developed either in circular cylindrical coordinate system or any other.

The techniques presented here are of a fundamental nature and can be applied for three-dimensional analyses of various kinds of fields satisfying analogous mathematical relations. The particular system of equations that is

presented here can be used as a general mathematical and computer aid for developing, modifying, and optimizing the designs of magnets and various parts of electric machinery from an electromagnetic field point of view. The three-dimensional field distribution can be rather accurately determined with the use of high-speed digital computers, while taking into account the intricate shapes of the windings and boundaries having different values of varying permeability, and the reaction of the induced eddy currents. The formulations of the purely magnetostatic or eddy-current problems in either two or three space variables come out as special cases of the general formulation.

The author has had experience in applying the presented techniques for analyzing the endzone fields of high speed aerospace alternators [4] as well as the solid rotor induction machines [5] under certain simplifying assumptions. The difficulties experienced lie mainly in specifying the boundary conditions uniquely in terms of the potential functions, considering the time as a variable, and the slow convergence of the iterative procedures. Including the time variable in the digital computer program complicates matters considerably and seems to demand prohibitively large computer time. Further research work needs to be done for finding practical methods that would enable one to consider the time as a variable. The author has been looking into the possibility of using either the method of fractional steps [6] for digital-computer numerical solutions or the hybrid computers for this purpose.

The method of fractional steps [6], which has been well developed in Russia but not used so far in other countries, including U.S.A., appears to be very encouraging and can be claimed to be probably the ultimate savior for the multi-dimensional numerical solutions. It would also be desirable to develop methods that can take account of the hysteresis effects and the isotropic properties of materials such as grain-oriented sheet steels.

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