

January 01, 2006

## A Six Sigma tolerancing approach for the design of an efficient closed-loop supply chain network

Surendra M. Gupta  
*Northeastern University*

Satish Nukala  
*Northeastern University*

---

### Recommended Citation

Gupta, Surendra M. and Nukala, Satish, "A Six Sigma tolerancing approach for the design of an efficient closed-loop supply chain network" (2006). . Paper 12. <http://hdl.handle.net/2047/d10010002>

This work is available open access, hosted by Northeastern University.



Laboratory for Responsible Manufacturing

## Bibliographic Information

Nukala, S. and Gupta, S. M., "A Six Sigma Tolerancing Approach for the Design of an Efficient Closed-Loop Supply Chain Network", ***Proceedings of the SPIE International Conference on Environmentally Conscious Manufacturing VI***, Boston, Massachusetts, pp. 123-129, October 1-3, 2006.

## Copyright Information

*Copyright 2006, Society of Photo-Optical Instrumentation Engineers.*

*This paper was published in Proceedings of SPIE (Volume 6385) and is made available as an electronic reprint with permission of SPIE. One print or electronic copy may be made for personal use only. Systematic or multiple reproduction, distribution to multiple locations via electronic or other means, duplication of any material in this paper for a fee or for commercial purposes, or modification of the content of the paper are prohibited.*

## Contact Information

Dr. Surendra M. Gupta, P.E.  
Professor of Mechanical and Industrial Engineering and  
Director of Laboratory for Responsible Manufacturing  
334 SN, Department of MIE  
Northeastern University  
360 Huntington Avenue  
Boston, MA 02115, U.S.A.

(617)-373-4846 **Phone**  
(617)-373-2921 **Fax**  
gupta@neu.edu **e-mail address**

<http://www.coe.neu.edu/~smgupta/> **Home Page**

# A Six Sigma Tolerancing Approach for the Design of an Efficient Closed-Loop Supply Chain Network

Satish Nukala and Surendra M. Gupta\*

(\*Corresponding Author)

Laboratory for Responsible Manufacturing, 334 SN, Department of MIE  
Northeastern University, 360 Huntington Avenue, Boston, MA 02115 USA  
Phone: (617)-373-4846, Fax: (617)-373-2921, E-mail: [gupta@neu.edu](mailto:gupta@neu.edu)

## ABSTRACT

Rapid technological developments and the growing desire of customers to acquire latest technology has led to a new environmental problem “*waste*”, comprising of both end-of-life products and used products that are disposed prematurely. As a result, both consumer and government concerns for the environment are driving many original equipment manufacturers (OEM) to engage in additional series of activities stemming from the reverse supply chain. The combination of forward/traditional supply chain and reverse supply chain forms the closed-loop supply chain. Contrary to a traditional/forward supply chain, a closed-loop supply chain involves more variability. In this paper, we explore the use of Motorola’s Six Sigma methodology to achieve better synchronization in a closed-loop supply chain network by tailoring the individual processes in a way that maximizes the overall delivery performance. A numerical example is considered to illustrate the approach.

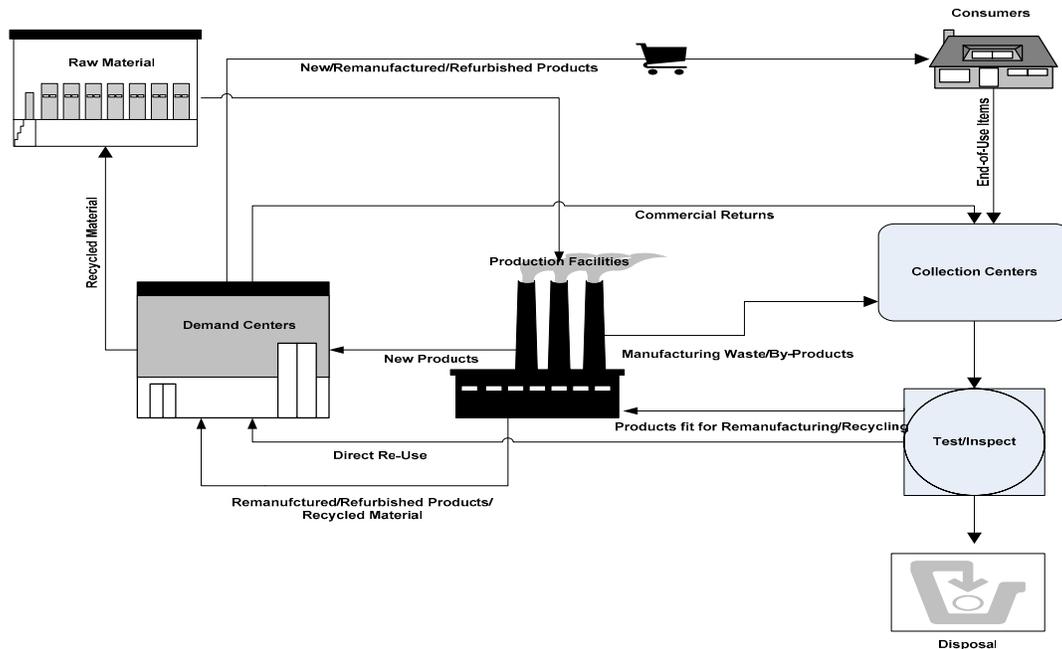
**Key words:** Closed-Loop Supply Chain Network, Six Sigma Tolerancing

## 1. INTRODUCTION

Until recently, many original equipment manufacturers (OEM) spent more time and money in fine tuning their forward/traditional supply chains while ignoring their reverse supply chains. However, in today’s competitive world, with both the consumer and governments growing concern for the environment, it is no longer the case. Many OEM’s these days are engaged in additional series of activities stemming from the reverse supply chain. As a result, economically feasible production and distribution systems are established that enable remanufacturing of used-products in conjunction with the manufacturing of new products [1], [2].

The council of logistics management defines reverse supply chain as “the process of planning, implementing and controlling the efficient, cost-effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal.” The combination of forward and reverse supply chain forms a closed-loop supply chain (CLSC). Figure 1 shows a generic CLSC.

It is estimated that 14 to 20 million PCs are retired annually in the US, of which, 20 to 30% are resold, while the rest are discarded [3], [4]. The current growth in consumption results in increased use of raw material and energy which results in the depletion of world’s finite natural resources and increases the amount of waste generated. This environmental degradation is considerable and not sustainable by the earth’s eco-system [5]. This environmental issue, in addition to government regulations is the major driving force for companies to engage in the reverse supply chain activities. Apart from the environmental regulations, reverse supply chains reduce the operating costs by reusing products or components. For example, GE Aircraft Engines makes more in servicing its aircraft engines than it does initially selling them. Also, better management of reverse supply chain translates into better customer service and consequently, higher customer satisfaction. Some firms have also set up the reverse supply chains for altruistic reasons. Nike encourages consumers to bring their used shoes to the store from where they were purchased. These shoes are then shipped back to Nike, where, they are shredded and donated to make basketball courts and running tracks. By doing so, the companies enhance the value of their brand and also encourage people to purchase their products [6].



**Figure 1. A Generic Closed-Loop Supply Chain**

The different activities in a generic reverse supply chain include gatekeeping, collection, testing, remanufacturing/refurbishing/recycling, disposition and redistribution. In gatekeeping, it is decided which products to be allowed in the reverse supply chain, else, companies may be flooded with products that cannot be remanufactured/refurbished/recycled. Efficient gatekeeping is the first critical point for a successful reverse supply chain. Collection is associated with high uncertainty regarding the quantity, quality and timing of returns. Once collected, the products are subjected to testing/inspection for deciding whether or not to remanufacture/refurbish/recycle them. Disposition depends on the product configuration and condition. In redistribution, the company plans to sell the remanufactured/refurbished products, while the recycled materials can be used as raw materials for manufacturing new products.

Reverse supply chains differ from forward supply chains in many aspects and are complex to handle because of the inherent uncertainty involved in every stage of their planning. As a result, most of the existing forward supply chain systems are unable to handle reverse channel. Table 1 shows the comparison between forward and reverse supply chains provided by Rogers [7].

**Table 1. Comparison between Forward and Reverse Supply Chains**

<b>Forward</b>	<b>Reverse</b>
Product quality uniform	Product quality not uniform
Disposition options clear	Disposition options unclear
Routing of products unambiguous	Routing of products ambiguous
Costs involved easily understood	Costs involved not easily understood
Product pricing uniform	Product pricing not uniform
Inventory management consistent	Inventory management inconsistent
Product life cycle manageable	Product life cycle less manageable
Financial management issues clear	Financial management issues unclear
Negotiations between parties straightforward	Negotiations less straightforward
Customer easily identifiable to market	Customer less easily identifiable to market
Process visibility more transparent	Process visibility less transparent

Much work has been done in the area of designing forward and reverse supply chains (for example see [8], [9]). However, not many models deal with both the forward and reverse supply chains together. While the issue of environmental consciousness is not addressed in the forward supply chain models, the models dealing with reverse supply chain assume that each incoming used product is economical to re-process and each available production facility is efficient enough to re-process the incoming used products. As a result, there is a risk of re-processing uneconomical used products in inefficient facilities. Pochampally and Gupta [10] addressed these drawbacks in a reverse supply chain and proposed a three phase mathematical programming approach for its strategic planning. Nukala and Gupta [11] extended Pochampally and Gupta's work to a closed-loop supply chain and proposed a unified single phase approach for its strategic and tactical planning. In this paper, we explore the use of Motorola's Six Sigma methodology to achieve better synchronization in a closed-loop supply chain network by tailoring the individual processes in the closed-loop supply chain in a way that maximizes the overall delivery performance. A numerical example is considered to illustrate the approach.

## 2. OVERVIEW OF SIX-SIGMA APPROACH

The quest for improved quality of products, processes, and indeed, all aspects of business performance, is the driving force behind Six-Sigma. Quality can be a confusing concept. From a design perspective it can be defined as a function of a specific measurable variable where the differences in quality reflect differences in the quantity of some product attribute. From a customers perspective the definition can be based on the presumption that what a customer wants determines quality while from operations perspective, quality can be defined as conformance to specifications. Six-Sigma can be best described as a business process improvement approach that seeks to find and eliminate causes of defects and errors, reduce cycle times and cost of operations, improve productivity, better meet customer expectations, and achieve higher asset utilization and returns on investment in manufacturing and service processes. It is based on a simple problem solving methodology, **DMAIC**, which stands for Define, Measure, Analyze, Improve and Control, that incorporates a wide variety of statistical and other process improvement tools.

Motorola pioneered the concept of Six-Sigma as an approach to measure product and service quality [12]. In the narrow statistical sense, Six-Sigma is a quality objective that specifies the variability required of a process in terms of the specifications of the product so that product quality and reliability meet and exceed the demanding customers' requirements. Six-Sigma measure equates to 3.4 or fewer errors or defects per million opportunities. One puzzling aspect of the Six-Sigma literature states that a process operating at Six-Sigma will produce 3.4 parts-per-million (ppm) non-conformances. However, if a standard normal table is consulted, one finds that the expected non-conformances are 0.002 ppm. The difference occurs because Motorola presumes that the process mean can drift 1.5 sigma in either direction. The area of a normal distribution beyond 4.5 sigma from the mean is indeed 3.4 ppm. The allowance of a shift in the process mean is important because Motorola believed that no process can be maintained in perfect control. In contrast to the Six-Sigma quality, the old Three-Sigma quality standard of 99.73% translates into 2700 ppm non-conformances, assuming a zero drift in the process mean. Figure 2 details the sigma levels and equivalent conformance rates.

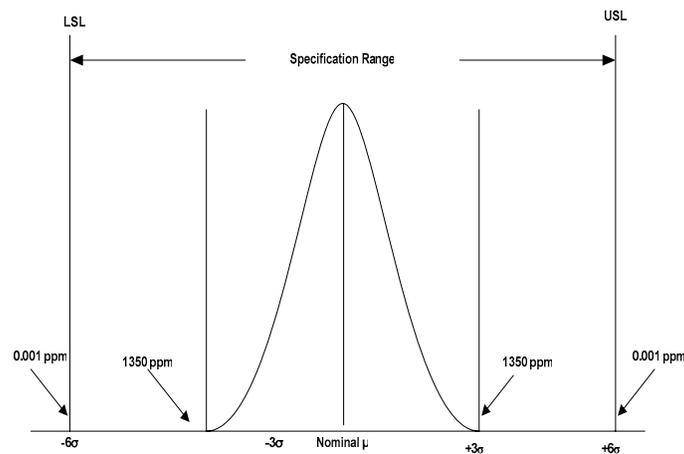
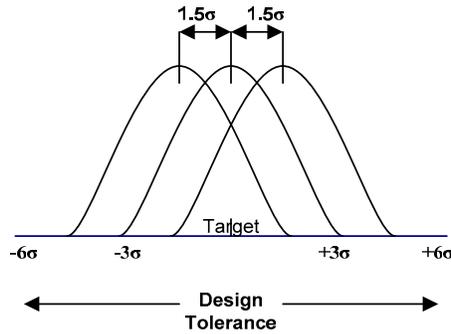


Figure 2. Sigma Levels and Conformance Rates

Figure 3 explains the theoretical basis for Six-Sigma in the context of manufacturing operations. In the figure, the area under the tail of one of the shifted curves beyond the six sigma range is only 0.0000034, or 3.4 ppm. Thus, if the process mean can be controlled to allow a one way shift of at most 1.5 standard deviations of the target, then a maximum of 3.4 ppm non-conformances can be expected. If it is held exactly on target, only one defect per billion would be expected beyond the six sigma range in either tail. If shifts can occur in both directions, then the defect rate at a six sigma level would be at most 6.8 ppm, and if held exactly on target, only two per billion [13].

Further analysis of the Six-Sigma program makes use of process capability indices  $C_p$  and  $C_{pk}$ . These capability indices give us the measure of how close the quality of product is in comparison with the performance objective. Process capability compares the output of an in-control process to the specification limits using the capability indices. The ratio of the spread between the process specifications to the spread of the process values, as measured by the six standard deviation units is used for this comparison Process capability index,  $C_p$ , defined as:



**Figure 3. Theoretical Basis for Six-Sigma**

$$C_p = \frac{U - L}{6\sigma} \quad (1)$$

where,  $U$  and  $L$  represent the upper and lower specification limits and the denominator represents the total width of the process distribution ( $\sigma$  is the process standard deviation). When  $C_p = 2$  and mean is centered at  $(U+L)/2$ , then the probability of conformance can be shown to be 0.002 failures per million. Also,  $(U-L)/2$  is the tolerance,  $T$ . From equation (1) we have:

$$\sigma = \frac{U - L}{6C_p} = \frac{T}{3C_p} \quad (2)$$

However,  $C_p$ , does not account for process centering. It does not give any indication of how far away from the target value,  $\tau$ , has the process mean,  $\mu$ , shifted. The index,  $C_{pk}$ , captures the effect of the shift in the process mean.  $C_{pk}$  is defined as:

$$C_{pk} = C_p (1 - k), \text{ where } k = \frac{|\tau - \mu|}{\left(\frac{U - L}{2}\right)} \quad (3)$$

$$\text{Tolerance interval} = (\tau_Y - T_Y, \tau_Y + T_Y) \quad (4)$$

The factor  $k$  shows the amount of the tolerance consumed by the mean shift [14].  $C_{pk}$  measures how close the process is to its target. A process may be performing with minimum variation, but it can be away from its target towards one of the specification limit, in which case its  $C_{pk}$  value will be less while its  $C_p$  value will be high. On the other hand a process may be on target, but the variation in the process might be high (however, still lower than the tolerance band). In such

case also its  $C_{pk}$  value will be low while its  $C_p$  value will be high.  $C_{pk}$  value will be high only when the process is consistently meeting its target with minimum variation.

Motorola's Six-Sigma program assumes a one-sided mean shift of  $1.5 \sigma$ . If  $C_p = 2$  and  $C_{pk} = 1.5$ , the probability of conformance can be shown to be 0.9999966, which translates into a defect rate of 3.4 parts per million. Thus  $C_p \geq 2$  and  $C_{pk} \geq 1.5$  imply six-sigma quality under the assumption of  $1.5 \sigma$  mean shift. In the presence of a mean shift, the Six-Sigma program suggests the use of the following dynamic root mean square formula [15]:

$$\sigma_Y^2 = \sum_{i=1}^n \sigma_i^2; \sigma_i = \frac{T_i}{3C_{pki}} \quad (5)$$

where,  $\sigma_i$ 's and  $T_i$ 's are the standard deviations and tolerances of the individual processes and  $\sigma_Y$  is the overall process standard deviation.

### 3. SUPPLY CHAIN SYNCHRONIZATION FOR SIX-SIGMA PERFORMANCE

In this section, we describe two different design experiments. While the former determines the range of nominal values for the lead times of different internal business processes for a six-sigma delivery performance, the latter determines a variance pool (hence their process capabilities) for the lead times of the different internal business processes for a six-sigma delivery performance.

#### 3.1 Finding a Nominal Pool

We consider the following internal business processes in a reverse supply chain:

1. Procurement; involves obtaining used-products from the consumers at the collection centers.
2. Inspection/Testing; involves determining the condition of the products collected in order to determine whether or not to remanufacture/refurbish/recycle the product.
3. Disassembly.
4. Remanufacture/Refurbish.
5. Transport the remanufactured products to the markets.
6. Delivery.

We assume the firm only deals with remanufacturing or refurbishing the used-product. A third-party recycler takes care of the products meant for recycling.

In finding a nominal pool, the tolerances for the lead times of the above business processes of the reverse supply chain are given as also the nominals of the lead times of some of the business processes (in this case,  $\tau_2, \tau_3, \tau_4$ ). The nominal value of the overall process as well as its tolerance is also known. The problem now is to find a range of values for the other nominals (in this case,  $\tau_1, \tau_5, \tau_6$ ), so as to achieve a six-sigma delivery performance. This has its implications on the choice of suppliers, carriers and other logistics providers.

#### Numerical Example

Let  $\tau_Y$  (overall target for the reverse supply chain) = 100 days;  $T_Y$  (reverse supply chain's tolerance) = 12 days;  $T_1 = 3$ ;  $T_2 = 4$ ;  $T_3 = 1$ ;  $T_4 = 2$ ;  $T_5 = 2$ ;  $T_6 = 1$ . Also, let the nominal value of Inspection/Testing, Disassembly and Remanufacturing/Refurbishing be 20, 25 and 30 days respectively. It is now required to find a range of values for the pool of other nominals ( $\tau_1 + \tau_5 + \tau_6$ ) such that the probability of delivery is at least 0.9999966 within the delivery window, which is  $(\tau_Y - T_Y, \tau_Y + T_Y) = (88, 112)$ .

We assume that the individual business processes are six-sigma processes, which implies a  $C_{pk}$  value of 1.5 for each process. Using equation (5), the standard deviations of the individual processes as well as the overall supply chain are found to be  $\sigma_1 = 0.67$ ,  $\sigma_2 = 0.87$ ,  $\sigma_3 = 0.22$ ,  $\sigma_4 = 0.44$ ,  $\sigma_5 = 0.44$ ,  $\sigma_6 = 0.22$ ,  $\sigma_Y = 1.299$ . Knowing the supply chain's overall target and the individual processes standard deviations, we can compute a range of values for  $(\tau_1 + \tau_5 + \tau_6)$  over which six-sigma delivery performance is guaranteed. It is found that as long as  $(\tau_1 + \tau_5 + \tau_6)$  is in the range (23.5, 26.5); we obtain a six-sigma delivery performance. That is, for this range of values, the probability of  $Y$  to be in the range (88, 112) is at least 0.9999966. The maximum probability is attained at 25 days. We can choose any value within that range, and the chosen value can then be allocated to  $\tau_1, \tau_5$ , and  $\tau_6$  in the best possible way, based on expert's judgment.

### 3.2 Finding a Variance Pool

Here, the nominals of the lead times of the business processes considered in the reverse supply chain are given as also the tolerances for the lead times of some of the business processes. The nominal value of the overall process as well as its tolerance is also known. The problem now is to find a variance pool, hence the process capabilities, that can be distributed across the individual business processes whose tolerances are not known.

#### Numerical Example

Let  $\tau_Y$  (overall target for the reverse supply chain) = 100 days;  $T_Y$  (reverse supply chain's tolerance) = 12 days;  $\tau_1 = 20$ ;  $\tau_2 = 5$ ;  $\tau_3 = 25$ ;  $\tau_4 = 30$ ;  $\tau_5 = 12$ ;  $\tau_6 = 8$ . Also, let the tolerance of the lead time of Procurement, Disassembly and Remanufacturing be 3, 2 and 1 respectively.

We assume that the individual business processes are six-sigma processes, which implies a  $C_{pki}$  value of 1.5 for each process. Also, a six-sigma delivery performance implies  $C_{pY} = 2$  and  $C_{pkY} = 1.5$ . Using equation (5) the standard deviations of the individual processes whose tolerances are known (Procurement, Disassembly and Remanufacturing in this example) as well as the overall supply chain are found to be  $\sigma_1 = 2$ ,  $\sigma_3 = 0.44$ ,  $\sigma_4 = 0.22$ ,  $\sigma_Y = 2.67$  respectively. Again, using equation (5) we get the variance pool,  $(\sigma_2^2 + \sigma_5^2 + \sigma_6^2)$ , for the three processes whose tolerances are unknown (Inspection/Testing, Transportation and Delivery in this example) as 2.89. This variance pool can now be distributed among the individual processes based on engineering judgment which assures a six-sigma delivery performance for the supply chain. That is, for this variance pool, the probability of  $Y$  to be in the range (88, 112) is at least 0.9999966.

## 4. CONCLUSIONS

Reverse supply chains differ from forward supply chains in many aspects and are complex to handle because of the inherent uncertainty involved in every stage of their planning. As a result, synchronization among individual processes in a reverse supply chain is of paramount importance in achieving a high level of delivery performance. In this paper, we explored the use of Motorola's Six-Sigma methodology to achieve better synchronization in a reverse supply chain network by tailoring the individual processes of the supply chain in a way that maximizes the overall delivery performance. Two different design experiments were detailed. While the former determined the range of nominal values for the lead times of different internal business processes for a six-sigma delivery performance, the latter determined a variance pool (hence their process capabilities) for the lead times of the different internal business processes for a six-sigma delivery performance. A numerical example was considered to illustrate the design experiments. This methodology can be extended to a closed-loop supply chain to synchronize individual business processes in the forward as well as the reverse channel. For example, this methodology can be applied to manufacturers leasing their products, in which case, procurement lead times are known precisely.

## REFERENCES

- [1] Lambert, A. J. D. and Gupta, S. M., *Disassembly Modeling for Assembly, Maintenance, Reuse, and Recycling*, CRC Press, Boca Raton, FL, 2005.
- [2] Savaskan, R. C., Bhattacharya, S. and Van Wassenhove Luk, N., "Closed-Loop Supply Chain Models with Product Remanufacturing", *Management Science*, **50** (2), 239-252, 2004.
- [3] Goodrich, M., "Making electronic recycling connections", *Recycling Today* **37** (9), 64-89, September 1999.
- [4] Jung, L. B., "The conundrum of computer recycling", *Resource Recycling*, 38-45, May 1999.
- [5] Beamon, B. M., "Designing the green supply chain", *Logistics Information Management*, **12** (4), 332-342, 1999.
- [6] Roy, A., "How Efficient is Your Reverse Supply Chain?", *Effective executive*, ICFAI University Press, 52-55, January 2003.
- [7] Rogers, D., "RLEC Project Plans", Livonia, MI, p. 18, October 2001.
- [8] Fleischmann, M., *Quantitative Models for Reverse Logistics: Lecture Notes in Economics and Mathematical Systems*, Springer-Verlag, Germany, 2001.
- [9] Talluri, S. and Baker, R. C., "A multi-phase mathematical programming approach for effective supply chain design", *European Journal of Operations Research*, **141**, 544-558, 2002.
- [10] Pochampally, K. K. and Gupta, S. M., "Strategic Planning of a Reverse Supply Chain network", *International Journal of Integrated Supply Management*, **1** (4), 421-441, 2005.

- [11] Nukala, S. and Gupta, S. M., "A Single Phase Unified Approach for Designing a Closed-Loop Supply Chain Network", *Proceedings of the Seventeenth Annual Conference of Production and Operations Management Society*, CD-ROM, 2006.
- [12] Harry, M. J. and Stewart, R. *Six Sigma Mechanical Design Tolerancing*, Technical Report, Government Electronics Group, Motorola Inc., Scottsdale, AZ, 1988.
- [13] Evans, J. R. and Lindsay, W. M., *An Introduction to Six Sigma & Process Improvement*, Thomson South-Western, 2005.
- [14] Antony, J., Swarnkar, R. and Tiwari, M. K., "Design of Synchronized Supply Chains: A Genetic Algorithm Based Six Sigma Constrained Approach", *International Journal of Logistics Systems and Management*, **2** (2), 120-141, 2006.
- [15] Narahari, Y., Viswanadham, N. and Bhattacharya, R., "Design of Synchronized Supply Chains: A Six Sigma Tolerancing Approach", *Proceedings of IEEE International Conference on Robotics & Automation*, 1151-1156, 2000.