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# Matlab-based Finite Difference Frequency Domain Method for Microwave Breast Imaging



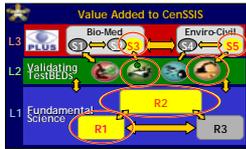
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## Abstract

The FDFD electromagnetic model computes wave scattering by directly discretizing Maxwell's equations along with specifying the material characteristics in the scattering volume. No boundary conditions are need except for the outer grid termination absorbing boundary. We use a sparse matrix Matlab code with loose generalized minimum residue (LGMRES) Krylov subspace iterative method to solve the large sparse matrix equation, along with the Perfectly Matched Layer (PML) absorbing boundary condition. The PML conductivity profile employs the empirical optimal value from [1-2]. This method is easily manipulated and general-geometry oriented, it is fast comparing to other models for solving the whole 3D computational grids.

The inverse scheme based on the forward FDFD model is also investigated. A novel matrix-based Born approximation is used instead of the traditional integral Born approximation. Tikhnov Regularization is employed. The good results have been obtained based on the simulated data from 2D FDFD TM model.

Microwave breast cancer detection is becoming a promising technique because of the high electrical contrasts between malignant tumors and normal tissue. This method investigates the electrical field properties of the 3D breast model with and without tumors at different frequencies, low frequency has big penetrating depth. The detection of tumor in 2D is presented.



## State of Arts

- Scalar Helmholtz wave equation in frequency domain are well computed with different boundary condition and inhomogeneous media in 2D ; 3D Fortran-based FDFD modeling is time and memory consuming with simple geometries;
- 2D Matlab-based FDFD methods deal with complicated geometries and isotropic, dispersive media;
- Our approach about 3D Matlab-based FDFD method is a valuable forward modeling for layered 3D inhomogeneous, dispersive media and high frequencies in reasonable memory and computational time.

## Opportunities for Technology Transfer

- The general purpose of this research is detecting the subsurface targets according to their EM properties. This model can be applied to the well-logging in the oil field by the induction (or resistivity) coupling voltage. The geometry for well logging is commonly anisotropic multi-layered & multi-faulted structure, which is suitable for the proposed model.
- This model can be also applied to other fields such as mine detection and tumor detection with the corresponding high and low frequencies.

## 3D FDFD Modeling

### > 3D matlab-based FDFD (finite difference frequency domain) method :

- Based on the general Maxwell's equations, the wave equation is 
$$\nabla^2 \vec{E} - \nabla(\nabla \cdot \vec{E}) + (\omega^2 \mu \epsilon + i\sigma\omega\mu) \vec{E} = 0$$
 where  $\mu = \mu_0$
- Equipped with the popular PML (perfectly matched layer) ABC (absorbing boundary conditions).
- Employing the Yee cell geometry as the grid structure of finite difference method.

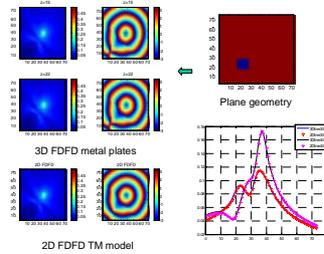
### > The applying mathematical method

The method finally leads to solving the problem of matrix equation:  $Ax=B$ ; where A is the coefficient matrix, B is the source column matrix and x is the unknown. A is a very large sparse matrix. Therefore the problem is suitable for the Krylov subspace iterative methods. One of them, LGMRES (Generalized minimum residue method), is employed after optimizing the structure of matrix A by multiplying the assisted matrix and doing some permutations.

## Application

### Comparing to 2D FDFD TMz model

- Uniform wet sand background with the relative permittivity  $\epsilon=20+1.06i$  at 1GHz
- The step size is 0.0045m
- The rectangular target with the relative permittivity  $\epsilon=2.63+0.016i$
- The grid size is 89x89 with 8 PML at each side for 2D and 89x89x29 for 3D with plates located at 3-5 and 24-26 along z direction
- Line source is located in the center(37,37) of the computational region in 3D model



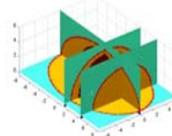
The comparison agrees very well to each other, the error is less than 3%.

## Breast Cancer Imaging

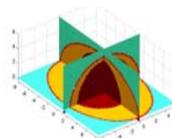
The relative permittivity for different dispersive breast tissues at 4 frequencies[7-8]:

freq	1.5G	2G	2.5G	3G
fat	5.2504 + 1.0792i	5.2180 + 0.9935i	5.1777 + 0.9780i	5.1307 + 0.9944i
fibroglandular	6.5219 + 2.6424i	6.4743 + 2.2212i	6.4157 + 2.0133i	6.3480 + 1.9067i
Tumor (HWC)	49.1976 + 17.7194i	48.7264 + 15.8215i	48.1420 + 15.1728i	47.4587 + 15.1000i
Muscle (chest wall)	57.6727 + 21.1531i	55.1529 + 20.3072i	52.6478 + 19.8237i	50.3367 + 19.3200i
skin	37.8866 + 13.5757i	37.5306 + 11.8164i	37.0961 + 11.0554i	36.5975 + 10.7527i

### Source (white star)



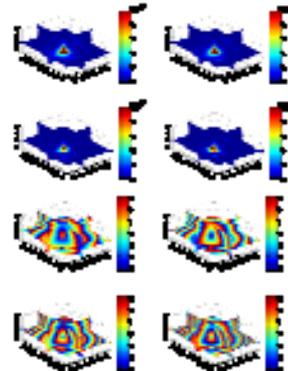
### Tumor (blue)



- Breast geometry at supine position, the breast immersed in the media with  $\epsilon=2.6$ ;
- Semi-ellipsoid model for breast terminated at the planar chest wall
- System of transmitter and receivers surrounding the breast
- Transmitter: magnetic dipole source with z polarization at (4.0cm,-5.1cm, 2.5cm)
- The 2mm-radius tumor located in (1cm, -2.0cm, 2.0cm)

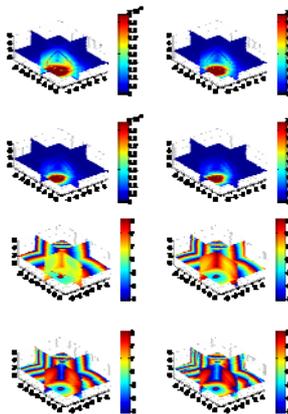
## Analysis:

- From the results, the skin make a big contribution for the total electrical field. Therefore, it is important to choose suitable surrounding medium to minimize reflection from the skin.
- The breast tissues are dispersive and lossy, the penetrating depth  $\delta(\omega) = (1/(\sigma\omega\mu))^{1/2}$  (the place at  $(1/\epsilon)$  of breast fat surface intensity, the relative intensity vs. depth  $d$  is  $(1/\epsilon)^2(d/\delta)$ , for 3GHz,  $\delta=1.2cm$  for breast fat.



Magnitude of scattering field z component of tumor : it decay very fast due to the high decay rate

Phase of scattering field z component of tumor : clearly shows that the higher frequency has shorter penetrating depth.



Magnitude of total field: z component

Phase of total field: z component

## Inverse Problems: FDFD matrix-based Inversion

- Based on the Matlab-based FDFD forward model:  $E_s = A X$

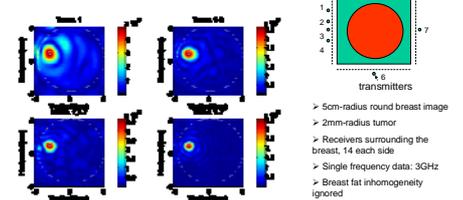
$E_s$  : measured E-scattering field data;

A : equals  $(A_0^{-1}E_0)$ , where  $A_0$  is related to the background coefficient matrix,  $E_0$  is background E-field;

k : approximation of perturbation to the background:  $(1+E_0^{-1}\Delta)E_0$ , where  $\Delta$  is the difference of square wavelamper between region with objects and region without objects (background field).

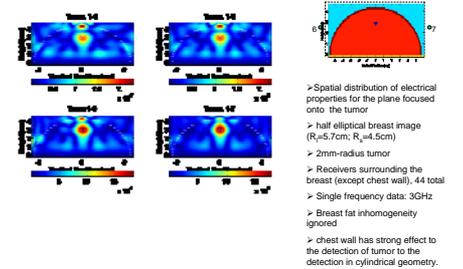
- Robustness with respect to the measurement noise.

### I. Tumor detection in Cylindrical Breast Geometry (2D)



- 5cm-radius round breast image
- 2mm-radius tumor
- Receivers surrounding the breast, 14 each side
- Single frequency data: 3GHz
- Breast fat inhomogeneity ignored

### II. Tumor detection in vertical plane perpendicular to chest wall (2D)



- Spatial distribution of electrical properties for the plane focused onto the tumor
- half elliptical breast image ( $R_1=5.7cm, R_2=4.5cm$ )
- 2mm-radius tumor
- Receivers surrounding the breast (except chest wall), 14 total
- Single frequency data: 3GHz
- Breast fat inhomogeneity ignored
- chest wall has strong effect to the detection of tumor to the detection in cylindrical geometry.

## Conclusion and Future works:

- 3D FDFD model is general-geometry objected and fast solver for the whole region computation;
- Microwave breast imaging is investigated with full 3D version:
  - distance of transmitter and receiver to the tumor is guiding the level of signal detection from tumor due to the penetrating length ;
  - Skin have important contribution to the total reflected electrical field. The further work to minimize effect of the skin will be done.
- Microwave breast tumor detection in 2D:
  - Tumor in Cylindrical breast geometry has a good recovery;
  - Chest wall has a strong effect on tumor recovery which causes a big noise.

**Future plan:** Extension investigation on microwave breast imaging ; 2D and 3D inverse algorithm to detect the breast tumor. More medical application in FDFD model due to its high inhomogeneity-handling properties, Multilayer inhomogeneous, dispersive media modeling and detection.

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