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# A SINGLE STAGE KANBAN CONTROL SYSTEM WITH STATIC ROUTING

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## ABSTRACT

We consider a system with two discrete production lines where the output of each one can fulfill the demand for the same type of product. An example to this case is the hybrid-manufacturing environment, where a company manufactures new products and remanufactures returned products to meet the demand. The interarrival times for demand occurrences and service completions are exponentially distributed i.i.d. variables. Here, a single stage pull production control with two types of kanbans is utilized to model the system where a kanban type is dedicated for either manufacturing or remanufacturing process.

## INTRODUCTION

Many industrialized countries have adopted recent laws and regulations for preserving the environment which have limited the total amount of End-Of-Life (EOL) disposal of products by banning or making the disposal costs unattractive. These costs force manufacturers to find alternative ways of dealing with EOL issues. Therefore, remanufacturing has recently become a very real alternative for production planners. Many companies are involved in retrieving used products since it is more profitable to repair, refurbish, upgrade and sell them than to dispose off. Such production systems use both the parts retrieved from returned items and new parts as the input for the products they produce. Since traditional production and inventory control mechanisms do not take returns into account, they are not adequate to address such systems. Hence, new control mechanisms are needed that take returns into account.

Most of the models developed to include the control of returns are modifications of classical inventory techniques with control policies such as $(s, S)$ or $(s, Q)$. It is generally assumed that the return and demand processes are mutually independent. A single echelon inventory system with this assumption was first modeled using an $M/M/1/N$ queue by Heyman [3]. The objective was to determine the optimal inventory level $N$ that minimizes the total inventory cost. The model did not consider the lead times. A subsequent paper by Muckstadt and Isaac [8] considered lead times but ignored the disposal activity for a continuous $(Q, r)$ policy inventory model. The results obtained from the single-echelon model were applied to a multi-echelon model. Laan et. al. [5] added the disposal option to the single-echelon model in [8]. Their study included a comparison between inventory policies with and without disposal that showed disposal is a necessary action for cost minimization. Laan et. al. [4] compared several inventory control policies with disposal option and showed that a four-parameter control policy was optimal. Subsequently, Laan et. al. [7] showed that the pull control strategy was more cost effective than the push control strategy for inventory systems with return flows. A further study by Laan and Salomon [6] verified these results while adding the disposal option to the earlier model. Salomon et. al. [9] and Gungor and Gupta [2] give a more extensive overview of such systems in their papers.

In this paper, we model the problem as a production control system. We first assume that a make-to-stock production system produces both manufactured and remanufactured products on two distinct production lines to satisfy the demand. Here, the first production line manufactures strictly new products while the second production line remanufactures previously used items by combining new and refurbished components. We also assume that used items are restored into 'as good as new' shape. Thus, there exist two different output sources that are equally capable of fulfilling the demand.

## STATEMENT OF THE PROBLEM

A system as described above that combines both manufacturing and remanufacturing activities is called a 'hybrid production system'. In order to control such a system, we propose a modified single stage pull control mechanism using two types of kanbans that trigger either the manufacturing or the remanufacturing process. We assume that there is an infinite supply of raw materials for the new product manufacturing line. We also assume that there exists an infinite used items supply for the remanufacturing line. We model this system as a queueing network with a synchronization station at the end [1]. Thus, when a demand arrives at the synchronization station we match it with a product. Then, we detach the kanban and send it to the beginning of the process to trigger production. According to its type, the card starts either the manufacturing or the remanufacturing activity. When, at its arrival, the demand is not immediately met; it is backordered at a given backordering cost per unit time until a product is released from the hybrid production system.

## THE MODEL

The hybrid system is modeled using a two-dimensional Markov process to obtain the expected work in process, expected backorders and expected on-hand inventory. We define a hybrid production system with two mutually independent processes where one of them manufactures new products from raw materials while the other remanufactures returned items. Supplies for both raw materials and returned items are assumed abundant.

### A Kanban Control with Static Routing

We consider a single stage kanban control for the remanufacturing\manufacturing process (*RMP*) as given in Figure 1. We use a total number of *K* kanbans where $K_R$ of them are dedicated for the remanufacturing process (*RP*) while the rest, $K_M = K - K_R$, are used to trigger the manufacturing process (*MP*). At the end of the process, the products are stored in the finished goods inventory *P*. The demand for finished goods occurs following a Poisson distribution with mean $l_D$. At the occurrence, the demand is satisfied by a product from *P*. If the demand is not satisfied, it is backordered and placed in queue *D*. Here, when a demand is satisfied, the detached kanban is sent to either *RP* or *MP* according to its type. Thus, the total number of work in process, $WIP(K_R, K_M)$, cannot exceed $K$, while $WIP_R(K_R, K_M)$ (the total number of work in process in *RP*) and $WIP_M(K_R, K_M)$ (the total number of work in process in *MP*) are limited to $K_R$ and $K_M$, respectively. In addition, the expected total number of parts in $WIP(K_R, K_M)$ and in the finished goods, $I(K_R, K_M)$ is always $K$. The *RP* and the *MP* are assumed to consist of either a balanced network of *M* machines in tandem or a single server with *b* parallel machines of the same speed. Hence, the throughput rates $l_R(n_R)$ and $l_M(n_M)$ are calculated using either [10]

$$l_{R(M)}\left(n_{R(M)}\right) = n_{R(M)}\, m \Big/ \left(n_{R(M)} + M - 1\right)$$

or

$$l_{R(M)}\left(n_{R(M)}\right) = \begin{cases} n_{R(M)}\, m, & n_{R(M)} < b, \\ b\, m, & n_{R(M)} \geq b. \end{cases}$$

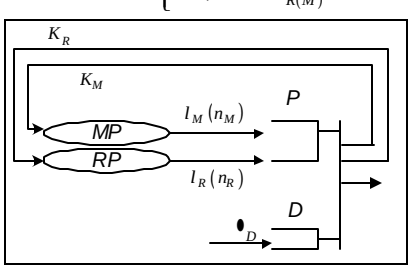


**Figure 1:** The *RMP* with dedicated kanbans

Let us define a two dimensional state space with $(x_I, x_R)$ where $x_I$ gives the total number of parts in queue *P* minus the total number of backorders in queue *D* and $x_R$ gives the total number of work in process in the remanufacturing subsystem. We develop the corresponding Markov chain as shown in Figure 2. Note that, the marginal probability distribution of $P_K^+(x_I)$ should give

$$\sum_{\forall x_I \geq 0} P_K^+(x_I) = 1, \tag{1}$$

where

$$P_K^+(x_I) = \sum_{x_R = \max(0, K_R - x_I)}^{\min(K_R, K - x_I)} P(x_R, x_I), 0 < x_I \leq K,$$

and

$$P_K^+(0) = \sum_{x_I \leq 0} P_K(x_I, K_R),$$

since

$$WIP(K_R, K_M) + I(K_R, K_M) = K.$$

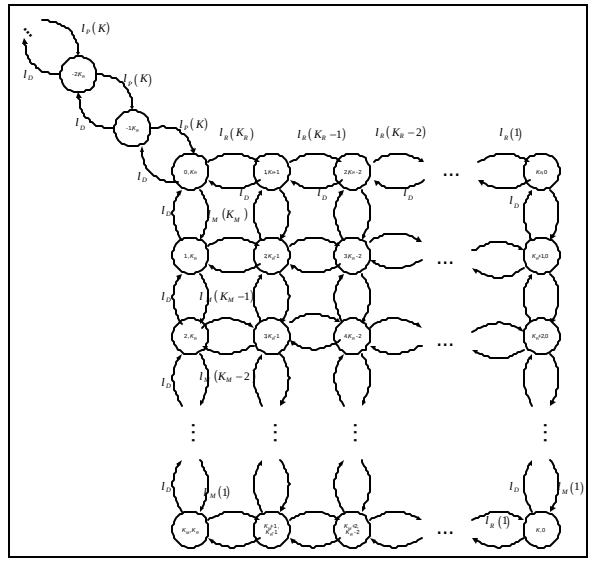

**Figure 2:** The Two-Dimensional Markov Chain of *RMP*.

The balance equations for this stochastic process are as follows:

$$\begin{aligned} P(x_I, x_R)\, l_D &= P(x_I - 1, x_R + 1)\, l_R(x_R + 1) \\ &\quad + P(x_I - 1, x_R)\, l_M(K - x_I - x_R + 1), \\ &\quad x_I = 1, \ldots, K, \\ &\quad x_R = 0, \ldots, K_R, x_M = 0, \ldots, K_M \\ P(x_I, K_R)\, l_D &= P(x_I - 1, K_R)\left(l_P(K)\right) \\ &\quad x_I \leq 0 \end{aligned} \tag{2}$$

where, $l_P(K) = l_R(K_R) + l_M(K_M)$. Here, when $l_D / l_P(K) < 1$, the steady state probabilities exist and are given as:

$$0 \leq x_I \leq K,$$

$$P(x_I, x_R) = \frac{\prod_{n_R=1}^{K_R} l_R(n_R) \prod_{n_M=1}^{K_M} l_M(n_M)}{\prod_{m_R=1}^{x_R} l_R(m_R) \prod_{m_M=1}^{K - x_I - x_R} l_M(m_M)} \frac{P(0, K_R)}{l_D^{\,x_I}} \tag{3}$$

$$x_I < 0,$$

$$P(x_I, K_R) = P(0, K_R)\left(\frac{l_D}{l_P(K)}\right)^{-x_I}$$

Using the steady state probabilities we calculate the expected inventory level as,

$$I(K_R, K_M) = \sum_{x_I=0}^{K} x_I \; P_K^+(x_I) \tag{4}$$

and the expected work in process as,

$$WIP(K_R, K_M) = \sum_{x_I=0}^{K} (K - x_I) P_K^+(x_I). \tag{5}$$

In addition, the expected work in process for the remanufacturing sub-process can be calculated as,

$$WIP_R(K_R, K_M) = \sum_{x_I=0}^{K} \sum_{x_R=\max(0, K_R - x_I)}^{\min(K_R, K - x_I)} x_R \, P(x_I, x_R), \tag{6}$$

and the expected work in process for the manufacturing subprocess can be easily calculated using

$$WIP(K_R, K_M) = WIP_R(K_R, K_M) + WIP_M(K_R, K_M) \tag{7}$$

In order to calculate the expected backorders, we first calculate the marginal probability distribution for $P_K^-(x_I)$, where $P_K^-(x_I) = P(x_I, K_R)$ for $x_I < 0$ and $P_K^-(0) = \sum_{x_I \geq 0, \forall x_R} P(x_I, x_R)$. Then we give the expected backorders of the system as,

$$\begin{aligned} B(K_R, K_M) &= \sum_{x_I=0}^{\infty} x_I P_K^-(-x_I) \\ &= P(0, K_R) \frac{\frac{l_D}{l_P(K)}}{\left(1 - \frac{l_D}{l_P(K)}\right)^2}. \end{aligned} \tag{8}$$

## A NUMERICAL EXAMPLE

Consider a *RMP* with an arbitrary network where the remanufacturing subsystem has a tandem network with $M = 3$ machines and the manufacturing subsystem has a single service station with $b = 2$ parallel machines. Let each machine have exponentially distributed processing times with $m = 0.5$. The demand occurs following a Poisson process with $l_D = 0.4$. Also $K_R = 4$ and $K_M = 3$. We first calculate the throughputs of each subsystem as follows:

$$l_R(n_R) = 0.5 n_R / (n_R + 2), \; n_R = 1, \ldots, 4$$

$$l_M(n_M) = \begin{cases} 0.5 n_M, & n_M < 2, \\ 1, & n_M \geq 2, \end{cases} \quad n_M = 1, 2, 3.$$

Since, $l_D / (l_R(4) + l_M(3)) \cong 0.3 < 1$, we can calculate the steady state probabilities using (2). From (1) and (3) we find $P(0,4) = 0.0188$. Next, we find $B(K_R, K_M) = 0.0115$ by evaluating (8). Then we obtain the marginal probability distribution of $P_K^+(x_I), 0 \leq x \leq K$, using (1) and (3) as $\{ P_K^+[0] = 0.0268, \; P_K^+[1] = 0.0625, \; P_K^+[2] = 0.168, \; P_K^+[3] = 0.2808, \; P_K^+[4] = 0.2167, \; P_K^+[5] = 0.145, \; P_K^+[6] = 0.0763, \; P_K^+[7] = 0.0239 \}$. Substituting these values in (4) and (5) we find $I(K_R, K_M) = 3.4576$ and $WIP(K_R, K_M) = 3.5424$, respectively. Finally, using (3) and (6) we obtain $WIP_R(K_R, K_M) = 2.6813$ for the expected work in process in the remanufacturing system.

## CONCLUSIONS

In this paper, we introduced a two-parameter kanban control mechanism for a hybrid manufacturing process with dedicated production resources for each subprocess. We defined a two dimensional Markov model for this system where steady state probabilities exist when the stability condition is met. Then we derived the expected work in process, the expected inventory level and the expected backorders using the steady state probabilities.

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