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DIGITAL COMPUTER SIMULATION OF ELECTROMAGNETIC FIELD PROBLEMS
AS APPLIED TO THE DESIGN OF ELECTRICAL MACHINERY

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Summary

The problem of electromagnetic field calculation in the end-zone of electrical machines is considered in this paper. A three-dimensional field requires using new methods of computing because the existing methods with the concept of vector potential demand unacceptable computer time. The new method consists of replacing the eddy magnetic field by the potential field of magnetic charges for the calculation of field, forces and associated inductances. Electrical currents of the rotor and stator windings are replaced by magnetic charges. It is possible to use just one scalar potential function for the computation instead of three components of the vector potential. The method of scalar potential can be utilized for the calculation of not only the steady magnetic fields but also electromagnetic fields including the effect of eddy currents in conducting screens and plates. This new technique can be very effectively used for modeling and computation of fields in various types of electrical machinery.

1. Introduction

At the present time to obtain the definition of the design parameters of electric machines one has to calculate the electromagnetic fields. Numerical calculation of two-dimensional steady magnetic fields and harmonic electromagnetic fields can be accomplished with the help of one of the following techniques: finite-difference, finite-element, or integral-equation methods. One can use each of the methods for three-dimensional field calculations, in which case the main problems are those of computer time, storage, and

complexity of the computer program. Difficulties arise because of the necessity to compute three components of vector quantities instead of either a scalar function or one component of the vector potential as in the case of two-dimensional field problems. The concept of the vector magnetic potential which is used for two-dimensional field calculations may become practically unacceptable from the view point of computer memory storage for the calculation of three-dimensional magnetostatic as well as time-varying fields. The scalar magnetic potential can be used, not only for the evaluation of the field outside the conductors carrying electric current, but also inside the conductor as well.

The scalar potential method utilizing the imaginary magnetic charges has been used since 1960 in the USSR for the calculation and mathematical simulation of electromagnetic fields.¹⁻³ Significant problems including three-dimensional fields of large transformers, calculation of electromagnetic forces and inductances of large conventional and superconducting electric machinery^{2,3} have been solved with the help of imaginary magnetic charges. Essentially the same method is being developed in Great Britain⁴. The concept of the scalar potential as well as the imaginary magnetic charges can be effectively utilized in connection with any numerical method. In the present paper the general formulation of the problem in terms of the scalar potential will be considered and the computing models for the calculation of the fields in large electrical machinery will be developed.

2. The General Formulation

Let the unknown magnetic field intensity \vec{H} that is described by Maxwell's system of equations be replaced by the sum of two fields \vec{H}_1 and \vec{H}_2 :

$$\vec{H} = \vec{H}_1 + \vec{H}_2 \quad (1)$$

The field \vec{H}_2 is chosen in such a way that

$$\text{curl } \vec{H}_2 = \vec{J} \quad (2)$$

Because of the Maxwell's equation

$$\text{curl } \vec{H} = \vec{J} \quad (3)$$

one can write for the field \vec{H}_1 the following equation

$$\text{curl } \vec{H}_1 = 0 \quad (4)$$

which means that the field \vec{H}_1 is the potential one. Because of this we can introduce the scalar magnetic potential "u" for the calculation of the field \vec{H}_1 :

$$\vec{H}_1 = - \text{grad } u \quad (5)$$

The sources of the potential field can be found taking into account the Maxwell's equation

$$\text{div } \vec{H} = 0 \quad (6)$$

for a homogeneous medium:

$$\rho = \mu \text{div } \vec{H}_1 = -\mu \text{div } \vec{H}_2 \quad (7)$$

The Poisson's equation for the scalar magnetic potential is

$$\text{div } (\text{grad } u) = - \frac{1}{\mu} \rho \quad (8)$$

The field \vec{H}_2 can be defined as

$$\vec{H}_2 = \int \vec{J} \times d\vec{l} \quad (9)$$

where $d\vec{l}$ is an arbitrary directed vector. One can check the validity of the equation $\text{curl } \vec{H}_2 = \vec{J}$ for the \vec{H}_2 field when it is defined with the help of the integral.

In the general case of a three-dimensional field, one can write $d\vec{l} = \vec{i} dx$. Let the vector of current density \vec{J} consist of three components in cartesian coordinates J_x, J_y, J_z . Then the field \vec{H}_2 is

$$\vec{H}_2 = \vec{j} \int_{x_0}^x J_z dx - \vec{k} \int_{x_0}^x J_y dx = \vec{j} H_{2y} + \vec{k} H_{2z} \quad (10)$$

consisting of two components. Similar formulae can be written for cylindrical coordinates:

$$d\vec{l} = \vec{j} r d\phi; \quad \vec{H}_2 = \int_{\phi_0}^{\phi} \vec{J} \times \vec{j} r d\phi = -\vec{i} \int_{\phi_0}^{\phi} J_z r d\phi + \vec{k} \int_{\phi_0}^{\phi} J_r r d\phi \quad (11)$$

(the electric current density has three components J_r, J_ϕ, J_z)

3. Algorithm for the calculation of the steady three-dimensional field

The magnetic field in the end-zone of an electric machine is essentially a three-dimensional one. The property of periodicity of electric currents and magnetic field in this zone allows the field to be described by a two-dimensional equation. The current density of the field winding has at least two components, while the armature current has three components of electric current density.

The field due to each of the components of electric current has to be calculated if the concept of vector potential is used. Using the technique of the scalar potential method the electric currents can be replaced by magnetic charges. The sources of the field become scalar quantities and the calculation is simplified.

The geometry of the end-zone region is shown in Fig. 1. There are two components of electric current of the field winding J_x and J_z . Choosing the direction of integration as $d\vec{l} = \vec{k}dz$, the field \vec{H}_2 consists of one component

H_{2y} :

$$\vec{H}_2 = -\vec{j} \int J_x dz = -\vec{j} H_{2y} \quad (12)$$

The indefinite integral can be calculated here because of the infinite length of the machine in the z-direction. The volume density of magnetic charges is

$$\rho = -\mu \operatorname{div} \vec{H}_2 = -\mu \frac{\partial H_{2y}}{\partial y} \quad (13)$$

This value is zero at any point except for the points over the surface 1 and 2 which are restricting the field winding. The volume density of magnetic charges over these surfaces is $\pm \infty$; but the surface density can be defined

as

$$\sigma = -\mu \operatorname{Div} \vec{H}_2 = \pm \mu H_{2y} \quad (14)$$

which is a finite value and in which $\operatorname{Div} \vec{H}_2$ represents the surface divergence of the field \vec{H}_2 . The computing model of the field winding consists of two single layers of magnetic charges which are placed over the two surfaces restricting

the field winding.

The current density of the armature winding consists of three components J_x, J_y, J_z . For region I of the armature winding we have just one component of the field \bar{H}_2 [See Eq. (12)] because there is just one component of the current density J_x . For this part of the armature winding $\rho = \pm \infty$, and $\sigma = \pm \mu H_{2y}$. As for the tilt region of the winding there are three components of electric current and the field \bar{H}_2 here consists of two components:

$$\bar{H}_2 = \bar{i} \int J_y dz - \bar{j} \int J_x dz \quad (15)$$

The volume density of magnetic charges is zero at any point except for the points along the surfaces 3,4,5. The surface density of magnetic charges is $\sigma = -\mu \text{Div } \bar{H}_2 = \pm \mu H_{2n}$ over the surfaces, in which H_{2n} is the normal component of the field \bar{H}_2 toward the surfaces 3,4,5. Like the computing model of the field winding the computing model of the armature winding consists of just single layers of magnetic charges. One can use the solution of Poisson's equations of the scalar potential for the calculation of the magnetic field in a homogeneous medium and integrate over the surfaces restricting the field winding and armature winding. After finding the scalar potential "u" and the field $\bar{H}_1 (= -\text{gradu})$, the unknown field \bar{H} can be calculated as being equal to \bar{H}_1 at any point where \bar{J} is equal to 0, and as the sum of \bar{H}_1 and \bar{H}_2 at any point where \bar{J} is not equal to zero.

The calculation procedure can be simplified however, if one takes into account the property of periodicity of currents and magnetic fields in the end-zone. Using this property one can integrate Poisson's equation along the lines instead of the surfaces.

If the current density of the field winding is presented as

$$J_x = \sum_{\nu} J_{\nu} \cos \frac{\nu \pi}{T} z \quad (16)$$

the surface density of the magnetic charges for the model of the rotor winding

$$\sigma = \pm \sum_{\nu} \frac{\mu \tau}{\nu \pi} J_{m\nu x} \sin \frac{\nu \pi}{\tau} Z \quad (17)$$

The different signs \pm correspond to different surfaces restricting the winding. The surface density of magnetic charges which replace the currents of the armature winding can be written in a similar way. For the armature current density one has

$$J = \sum_{\nu} J'_m \cos \frac{\nu \pi}{\tau} Z + J''_m \sin \frac{\nu \pi}{\tau} Z \quad (18)$$

There are two components of the armature current density J'_m and J''_m because of the current's phase change along the x-direction. The surface density of magnetic charges consists of two components σ' and σ'' as well and is equal to

$$\sigma = \pm \mu H_{2n} = \pm \mu (H_{2x} \sin \beta + H_{2y} \cos \beta) \quad (19)$$

Thus the electric currents of the field and armature winding are replaced with the surface magnetic charges. The potential field which is described by the equation (for a homogeneous medium) in cartesian coordinates

$$\frac{\partial^2 U_{m\nu}}{\partial x^2} + \frac{\partial^2 U_{m\nu}}{\partial y^2} - \left(\frac{\nu \pi}{\tau}\right)^2 U_{m\nu} = -\frac{\rho_{m\nu}}{\mu} \quad (20)$$

can be solved with the help of any numerical method. The equation for the scalar potential is valid for any of the space harmonic ν and for ('') and ('') components of the potential as well. For the scalar potential u , one has

$$u = \sum_{\nu} U'_{m\nu} \cos \frac{\nu \pi}{\tau} Z + U''_{m\nu} \sin \frac{\nu \pi}{\tau} Z \quad (21)$$

The solution of the Poisson's equation for the scalar magnetic potential can be written as

$$u = \frac{1}{4\pi\mu} \int \frac{\rho dv}{r} \quad (22)$$

If the scalar potential method is used the integral over the volume is replaced by the surface integral.

$$u = \frac{1}{4\pi\mu} \int \frac{\sigma ds}{r} \quad (23)$$

Further simplification of the solution can be made if the property of periodicity of magnetic charges is taken into account. The integration can be performed along

$$U_{mV} = \frac{1}{4\pi\mu} \int \frac{\tau_{mV} \cos \frac{\sqrt{\pi}}{\tau} Z dz}{r} \quad (24)$$

if the field of scalar potential is created by the line with the magnetic charge changing according to the law $\tau = \tau_{mV} \cos \frac{\sqrt{\pi}}{\tau} Z$. The integral can be calculated for any value of ν :

$$U_{mV} = \frac{\tau_{mV}}{2\pi\mu} K_0 \left(\frac{\sqrt{\pi}}{\tau} r \right) \quad (25)$$

in which $K_0 \left(\frac{\sqrt{\pi}}{\tau} r \right)$ is the modified Bessel function.

This solution can be considered as the fundamental one of the equation when the field is created by linear charge distributed as a harmonic function along the line. The potential field \bar{H}_1 is $H_{1k} = -\text{grad}_k u$, ($k = x, y, z$):

$$\begin{aligned} H_{1mVx} &= -\frac{\tau_{mV}}{2\pi\mu} \cdot \frac{\sqrt{\pi}}{\tau} \frac{\partial r}{\partial x} K_1 \left(\frac{\sqrt{\pi}}{\tau} r \right) \\ H_{1mVy} &= -\frac{\tau_{mV}}{2\pi\mu} \cdot \frac{\sqrt{\pi}}{\tau} \cdot \frac{\partial r}{\partial y} K_1 \left(\frac{\sqrt{\pi}}{\tau} r \right) \\ H_{1mVz} &= \frac{\tau_{mV}}{2\pi\mu} \cdot \frac{\sqrt{\pi}}{\tau} \cdot K_0 \left(\frac{\sqrt{\pi}}{\tau} r \right) \end{aligned} \quad (26)$$

The solution for the " U_{mV} " and field " H_{1mVu} " can be used for the calculation of the magnetic field which is created by magnetic charges attributed to electric currents of the field and armature winding. These solutions can be used effectively while taking the properties of a ferromagnetic shield into account, with the help of an integral equation. The appropriate integral equation can be easily written and solved. The replacement of the electric currents by the surface magnetic charges allows the effective use of the integral equation method.

4. Calculation of the field varying with time

In cartesian coordinates the equation $\text{curl curl } \vec{H}_2 = \text{curl } \vec{J}$ can be written as

$$\text{grad}_k \text{div } \vec{H}_2 - \text{div grad } H_{2k} = \text{curl}_k \vec{J}, \quad (k = x, y, z) \quad (27)$$

Because of $\vec{J} = \gamma \vec{E}$ and $\vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$, one has

$$\text{curl}_k \gamma \vec{E} = -\mu \gamma \frac{\partial \vec{H}}{\partial t} + (\text{grad } \gamma \times \vec{E})_k \quad (28)$$

We will consider harmonic functions and replace the vectors by their complex values, while making use of $\vec{H} = -\text{grad } u$:

$$\text{grad}_k \text{div } \vec{H}_2 - \text{div grad } H_{2k} + j\omega\mu\gamma H_{2k} = j\omega\mu \text{grad } \dot{u} + (\text{grad } \gamma \times \vec{E})_k \quad (29)$$

This equation and the equation for the scalar magnetic potential

$$\text{div grad } \dot{u} = \text{div } \vec{H}_2 = -\frac{\rho}{\mu} \quad (30)$$

formulate the problem.

If the direction of the vector $d\vec{l}$ is chosen as $d\vec{l} = \vec{k} dz$, the vector \vec{H}_2 consists of two components, namely H_{2x} and H_{2y} . The number of unknown values is three: H_{2x} , H_{2y} and the scalar potential. One can write at the same time four equations: one for the scalar potential and three for x, y, z components of the first equation.

It means that one can choose different formulations of the problem when different projections of the first equation are used in different coordinate systems.

One of the important problems is the calculation of eddy currents inside the end plate of the end zone of electric machines. The magnetic field in the end zone is excited by the electric currents flowing through the field and armature windings. These currents can be replaced by magnetic charges using the procedure given in Section 3. So the distribution of field and armature currents, as well as the equivalent magnetic charges will be considered to be known.

The plate is situated near the ferromagnetic core of the machine and decreases the axial flux on the core surface. Supposing that there are two components of eddy currents inside the plate, J_y and J_z , one needs to use

Just one projection of the equation for the field \vec{H}_2 .

Let the "z" projection of this equation be written as ($d\vec{l} = \vec{k}dz$, $\vec{H} = \vec{i} \int j_y dz$, $H_{2z} = 0$, $H_{2y} = 0$):

$$\text{grad}_z \text{div} \vec{H}_2 = j\omega\mu\gamma \frac{\partial \dot{u}}{\partial z} + (\text{grad } \gamma \times \vec{E})_z \text{ or}$$

$$\text{div} \vec{H} = j\omega\mu\gamma \dot{u} + \int \frac{\partial \gamma}{\partial x} \dot{E}_y dz \quad (31)$$

Because $\frac{\partial \gamma}{\partial x} = 0$ at any point inside the plate, the volume density of magnetic charges is

$$\dot{\rho} = -\mu \text{div} \vec{H}_2 = -j\omega\mu^2 \gamma \dot{u} \quad (32)$$

One has $\frac{\partial \gamma}{\partial x} = \pm \infty$ over two surfaces of the plate, which means that there are single layers of magnetic charges and the density is:

$$\dot{\sigma} = -\mu \text{Div} \vec{H}_2 = \pm \mu \dot{H}_{2y} = \pm \mu \gamma \int \dot{E}_y dz \quad (33)$$

Calculating the second derivative of σ with respect of y and z , and adding these values one can write

$$\frac{\partial^2 \dot{\sigma}}{\partial y^2} + \frac{\partial^2 \dot{\sigma}}{\partial z^2} = \pm \mu \gamma \text{curl}_x \vec{E} = \pm j\omega\mu^2 \gamma \frac{\partial \dot{u}}{\partial x} \quad (34)$$

if the derivative $\frac{\partial \dot{u}}{\partial x}$ is calculated outside the plate.

The system of equations to be calculated consists of two equations; one of them is for scalar potential and the other is for the surface density of magnetic charges.

$$\frac{\partial^2 \dot{u}}{\partial x^2} + \frac{\partial^2 \dot{u}}{\partial y^2} + \frac{\partial^2 \dot{u}}{\partial z^2} - j\omega\mu\gamma \dot{u} = -\frac{\dot{\rho}_k}{\mu} - \frac{\dot{\sigma}}{\mu}$$

$$\frac{\partial^2 \dot{\sigma}}{\partial y^2} + \frac{\partial^2 \dot{\sigma}}{\partial z^2} = \pm j\omega\mu^2 \gamma \frac{\partial \dot{u}}{\partial x} \quad (35)$$

The function $\dot{\rho}_k$ which characterizes the known electric currents of the windings is the periodical one along z -coordinate and can be written as

$$\dot{\rho}_k = \dot{\rho}'_{kmv} \cos \frac{v\pi}{\tau} z + \dot{\rho}''_{kmv} \sin \frac{v\pi}{\tau} z \quad (36)$$

The functions \dot{u} and $\dot{\sigma}$ can be presented in a similar way. Using such a representation, the system for the amplitudes \dot{u}'_{mv} , $\dot{\rho}'_{kmv}$, $\dot{\sigma}'_{mv}$ to be solved is

$$\frac{\partial^2 \dot{u}'_{mv}(')}{\partial x^2} + \frac{\partial^2 \dot{u}'_{mv}(')}{\partial y^2} - [j\omega\mu\gamma + (\frac{v\pi}{\tau})^2] \dot{u}'_{mv}(') = - \frac{\dot{\rho}'_{kmv}(')}{\mu} - \frac{\dot{\sigma}'_{mv}(')}{\mu}$$

$$\frac{\partial^2 \dot{\sigma}'_{mv}(')}{\partial y^2} - (\frac{v\pi}{\tau})^2 \dot{\sigma}'_{mv}(') = + j\omega\mu^2\gamma \frac{\partial \dot{u}'_{mv}(')}{\partial x} \quad (37)$$

The first equation has to be solved over the surfaces (' and ('), and the second one along two lines. The calculating problem can be formulated either in terms of the finite-difference, finite-element or integral equation method. Several problems have been solved with the help of imaginary magnetic charges by applying the scalar potential concept and overcoming the computational difficulties of the vector potential formulation. The equations in the paper are written in cartesian coordinates but one can derive the equations in cylindrical coordinates. The choice of appropriate coordinate system depends on the problem to be solved and the type of electric machine.

5. Conclusion

The electromagnetic field in electric machines can be successfully calculated with the help of imaginary magnetic charges allowing the use of just the scalar function, namely scalar magnetic potential. The computing models consist of single layers of magnetic charges when cartesian coordinates are used. The integral equation method can be effectively applied in connection with the scalar potential method for the calculation of the steady magnetic field and eddy current.

Some solved problems show the possibility and advantages of the use of this

technique for the electromagnetic field calculations in electric machinery.

6. References

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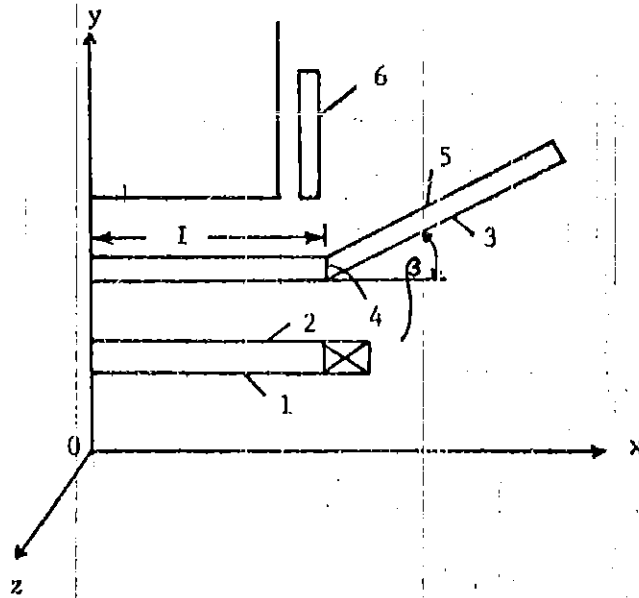


Figure 1: End-Zone Region