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Developments in finite-element method for calculating electromagnetic fields in electrical machines

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DEVELOPMENTS IN FINITE-ELEMENT METHOD FOR CALCULATING ELECTROMAGNETIC FIELDS IN ELECTRICAL MACHINES

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ABSTRACT

In order to meet the specifications of an electrical apparatus, the designer must be able to analyze and modify several fields of interest such as the electromagnetic fields, with the ultimate aims of safety, reliability, simplicity, efficiency and economy. The numerical finite-element method is proving to be an accurate, economical and useful design tool for the field analysis in electrical machinery. This paper presents briefly the formulation of the finite-element technique for solving two- and three-dimensional field problems in nonlinear devices, and reviews various applications made so far, while putting forth some new developments. These developments along with the progress made thus far in implementing the finite-element method show immense promise.

INTRODUCTION

An accurate knowledge of the electromagnetic as well as other field distributions is of great importance in design optimization. The continuing rapid increase in the rating of the generating plants, transformers, direct current machines, accelerator magnets and a host of other electromechanical devices, and the consequent increased electric and magnetic loadings of the energy-conversion apparatus are forcing manufacturers to work with greatly reduced safety factors in designing the modern power apparatus and systems. The present-day machine designer is concerned with the prediction of electric and magnetic fields under various operating conditions, with a view to evaluating saturated reactances, transient characteristics, short-circuit forces, stray-load and eddy-current losses, end-zone effects, excitation requirements under open-circuit, short-circuit and full-load conditions, surge-voltage phenomena and insulation strength, load regulation and commutation characteristics in direct-current machinery, and many others, in order to meet the challenges of his profession.

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During the past two decades, numerical methods have been greatly stimulated by the advent of high-speed digital computational robots which have made routine solutions possible to a high degree of accuracy for many types of problems for which the solutions would otherwise be extremely or even prohibitively laborious. There has been considerable research activity for the numerical analysis and determination of the electromagnetic fields in electrical machinery through the solution of Maxwell's equations, while taking full account of the magnetic saturation. Since rigorous solution of the field problem by analytical methods is limited to simple geometric shapes of the region of interest and boundaries, and idealized magnetization characteristics of the ferromagnetic material parts, the machine designers have taken recourse to numerical methods.

Concepts of both scalar and vector magnetic potentials have been used for computing the electromagnetic fields. Numerical approaches involving finite-difference method, finite-element method, and integral-equation techniques are evident in the literature. The finite-element method along with the magnetic vector potential concept is mainly considered in this paper for discussion. It is based on variational formulation in which a corresponding energy functional for the nonlinear case is minimized over the entire region. This functional can usually be identified with the stored energy in the system for most of the engineering applications, while the Euler equation of the functional yields the original partial differential equation to be solved. The potential function is defined in discretized sub-regions of the field region, and the minimization is performed by means of the finite-element method, while the resultant set of nonlinear algebraic equations is solved through iterative schemes such as Newton-Raphson method.

With z-directed current densities and consequently z-directed magnetic vector potential, for a magnetostatic field problem posed in two-dimensional xy Cartesian coordinate system, the nonlinear partial differential equation to be satisfied is given by

$$\frac{\partial}{\partial x}\left(\nu \frac{\partial A}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu \frac{\partial A}{\partial y}\right) = -J \qquad (1)$$

where ν is the reluctivity or the reciprocal of the magnetic permeability of the ferromagnetic material. The nonlinear magnetic energy functional, the minimization of which yields the desired solution by satisfying Eq. (1), can be expressed as[1]:

$$F = \iint_R \left[\int_0^B \nu b \, db - \int_0^A J \, dA\right] dx \, dy \qquad (2)$$

in which the magnetic flux-density B is given by

$$B = \left[\left(\frac{\partial A}{\partial x}\right)^2 + \left(\frac{\partial A}{\partial y}\right)^2\right]^{1/2} \qquad (3)$$

and R is the surface area of definition of the problem.

The above functional can be interpreted as the difference between the stored energy and the input or applied energy in the system under consideration. In the energy formulation, it should be pointed out that the natural Dirichlet and Neumann boundary conditions are implicitly satisfied in view of the fact that a closed line integral is set equal to zero[2]:

$$\oint_C A \cdot \frac{\partial A}{\partial n} \, d\ell = 0 \qquad (4)$$

Assumptions that are usually made are listed below:

All the materials are taken to be isotropic and homogeneous. Hysteresis effects are neglected, thereby making the magnetization characteristics single-valued. The dielectric effects or the displacement currents are neglected, and the material regions are considered as being void of volume charge density.

For discretizing the field region, first-order triangular finite elements are usually chosen such that the potential solution is defined in terms of the shape functions of the triangular geometry and the nodal values of potential[3].

$$A = \sum_k \varepsilon_k A_k \tag{5}$$

where the index of summation ranges over the vertices of the triangle. The minimization of the energy functional is achieved by setting its first derivative with respect to each of the vertex values of the potential to zero, so that

$$\frac{\partial F}{\partial A_k} = 0 \tag{6}$$

When Eq. (6) is applied for each triangle in the region of integration and the corresponding terms are grouped together, a single matrix equation of the form given below is obtained for the entire field region:

$$[S] \cdot [A] = [T] \tag{7}$$

from which the unknown vector potentials are to be determined. Eq. (7) represents a set of nonlinear algebraic equations of the nonlinear continuum field problem. The sparse and band-structured nature of the coefficient symmetric square matrix S should be taken advantage of in storing inside the computer as well as in any solution method adopted.

As for the representation of the single-valued nonlinear magnetization characteristic, while Frölich approximation, polynomial approximations, and representation by a sum of exponentials are used by some researchers, a piecewise linear interpolation method with a sufficient number of intercepts to ensure slope continuity, or a piecewise cubic-spline interpolation method appears to be more flexible.

SOLUTION METHODS

The nonlinear set of equations (7) may be quasi-linearized through the application of a Newton-Raphson algorithm and the resulting set of equations may be solved either through the Gaussian elimination technique[4], or by a successive point over-relaxation technique[5], while the reluctivities are updated in each iteration with respect to the modeled magnetization characteristic. Alternatively, an iterative solution method consisting essentially of two steps with the use of optimum over- and under-relaxation factors can be employed. In each iteration, the vector potentials at each of the nodal points are calculated using the reluctivities computed during the previous iteration by successive point over-relaxation method; later, the reluctivities are recomputed using the newly calculated vector potentials and are under-relaxed[6,7]. The newly proposed method[8] by this author of computing an appropriate optimum relaxation factor for the reluctivity at each nodal point depending on its location on the magnetization characteristic may be implemented with advantage. Block-relaxation method based on Ampere's law may also be employed to accelerate the convergence of the iterative solution[9]. One can also go to other methods of solution such as the nonlinear successive over-relaxation used by some researchers[10]. The Newton-Raphson process may also be applied to the functional formulation as in Reference [11] instead of to the residual vector as in Reference [4].

A popular method of solving simultaneous linear equations is to use a banded matrix solution technique which has the advantages of speed and of using minimum computer core, thereby making effective use of the sparsity of the coefficient matrix. In order to take the maximum advantage, the designer must make sure that the matrix bandwidth is as narrow as possible; and to achieve this, one must number the nodal points of the field problem in a particular manner. The ability to number nodes automatically is of particularly great importance in connection with mesh alternations. Computer implementation of the finite element method has been studied by George[12]. While the Cuthill-McKee[13] node numbering algorithm or its modified versions have been used by a few researchers in machine analyses, this author in his recent consultation activities has found bandwidth reduction by automatic renumbering proposed by Collins[14] to be most useful in a number of respects.

In analyzing the field problems associated with electric machinery on load, one has to consider the periodicity condition. Such a boundary condition is handled by Chari[4] with the aid of a connection matrix, while the functional formulation of the Newton process including the periodicity condition is indicated by Silvester et al[11]. This author has been looking into a new method of initially numbering the nodes in the first half-polepitch region to reduce the bandwidth, then simulating the mirror image in the second half-polepitch region, and later combining the corresponding vector-potential coefficients in order to satisfy the periodicity condition. The results of such a method of implementation will be published as and when they are available in the near future.

To cite a few typical applications of the finite element method of analysis, flux distributions in the cross-sections of a turbine generator[4,15], direct current generator[4], salient pole alternator[5], transformer[6,16], inhomogeneous anisotropic reluctance machine rotor[17], accelerator magnet[18,19], end zone of rotating electrical apparatus[20,21,22] have been obtained. In a few cases, analyses have been carried out in polar coordinates.

THREE-DIMENSIONAL NONLINEAR MAGNETOSTATIC FIELD COMPUTATION

The fundamental equation of the vector potential of the magnetostatic field is given by

$$\nabla \times [\nu(\nabla \times \bar{A})] - \bar{J} = 0 \tag{8}$$

It can be shown[23] that Eq. (8) is the Euler equation of the functional defined by either of the following equations, while satisfying homogeneous Dirichlet and Neumann-type boundary conditions if no others are specified:

$$F = \iiint_V \int_0^B \nu b \, db \, dv - \iiint_V (\bar{J} \cdot \bar{A}) \, dv \tag{9}$$

or

$$F = \frac{1}{2} \iiint_V (W - 2\bar{J} \cdot \bar{A}) \, dv \tag{10}$$

in which W is related to ν by the following relationship:

$$\frac{dW}{d[(\nabla \times \bar{A})\cdot(\nabla \times \bar{A})]} = \nu \tag{11}$$

and v is a closed simply-connected volume region.

The energy functional may be chosen in a modified form given below

$$F = \frac{1}{2} \iiint_V (W - 2\overline{J} \cdot \overline{A})\, dv + \oiint_S (\overline{C} \cdot \overline{A})\, ds \tag{12}$$

so as to satisfy a more general boundary condition of the type

$$[\nu(\nabla \times \overline{A})] \times \overline{n} + \overline{C} = 0 \qquad \text{on } S \tag{13}$$

Finite element equations approximating the variational principle using Eq. (10) can then be developed. Choosing the elements as triangular prisms, one can obtain a set of nonlinear algebraic equations of the matrix form of Eq. (7), which in turn may be solved by any of the solution methods discussed above.

TWO-DIMENSIONAL EDDY-CURRENT FIELD PROBLEMS

In terms of the magnetic vector potential, the equation to be satisfied is given by

$$\frac{\partial}{\partial x}\left(\nu \frac{\partial A}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu \frac{\partial A}{\partial y}\right) = -a \frac{\partial A}{\partial t} + J_e \tag{14}$$

where J_e is the external excitation or source current density. Assuming A to be a periodic function of time and adding a term $[\frac{1}{2} j\omega a \iint A^2\, dx\, dy]$ to the functional of Eq. (2), Chari[24] has analyzed the linear eddy-current field problem. Demerdash et al introduced the concept of effective permeability and computed eddy currents in nonlinear ferromagnetics[25] and induced current distributions in the solid rotors of turboalternators[26]. Other researchers who have worked on the eddy-current problems include Donea et al[27], Foggia et al[28], Carpenter[29], Csendes[30], and Konrad[31].

CONCLUDING REMARKS

A comprehensive gist of developments in finite-element method of analysis is presented in this paper for calculating electromagnetic fields in electrical machines.

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