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COMPUTING MODELS FOR THE MAGNETIC FIELD CALCULATION OF SUPERCONDUCTING ELECTRIC MACHINES WITH THE HELP OF SCALAR MAGNETIC POTENTIAL

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ABSTRACT

The method of scalar magnetic potential using imaginary magnetic charges is employed effectively for the calculation of the electromagnetic fields in superconducting electric machines. The computing model consists of several single layers of magnetic charges. Finite-difference numerical techniques are utilized for the computation. The cylindrical coordinate system has been used for the calculation of the electromagnetic losses inside the electromagnetic shield of the rotor winding. The tangential and axial components of eddy currents have been taken into account. Linear magnetic charges replace rotor and stator currents; and surface magnetic charges replace the eddy currents inside the electromagnetic shield of the rotor winding. Various systems of partial differential equations have been formulated for the computer-aided analysis of this problem.

INTRODUCTION

Parameters of superconducting machines are calculated these days under the supposition of two-dimensional electromagnetic field; so the cross-section of the machine in the plane that is perpendicular to the axis is, as a rule, considered. The influence of the end-zone even for conventional electric machines is very important. For superconducting machines the influence of the end-zone is much more because of the relatively slight role of ferromagnetic bodies situated in the electromagnetic fields created by windings of the machine and because of the construction of superconducting machines. The consideration of the endzone of the machine demands the definition of a threedimensional electromagnetic field. Numerical methods have to be applied for the calculation of parameters with high accuracy. Some numerical methods and algorithms used for the conventional electric machine calculations can be applied to the calculation of the electromagnetic fields in superconducting machines. There are many papers devoted to this problem and published in different countries^{1,2,3,4}. All the

difficulties of the numerical solution are connected with the necessity of the calculation of three-dimensional steady and time-varying fields because of the increasing number of unknown potentials.

Particular construction of superconducting machines allows the acceptance of some suppositions simplifying the problem. The scalar potential method and the idea of periodicity of electric currents and fields along one coordinate will be used for the development of the computing algorithms.

GEOMETRY OF THE MACHINE AND EQUATIONS OF THE FIELD

The problem is to calculate the parameters of the machine when the end region is taken into account. The geometry of the superconducting machine is relatively simpler than the geometry of conventional machines because of the absence of the ferromagnetic core and slots. It is possible to consider the ferromagnetic shield as an unsaturated one for the steady state operation so that the problem can be considered as a linear one. The other feature of superconducting machines is the presence of the electromagnetic shield which can serve as thermal shield simultaneously.

The idea of periodicity of electric currents along the peripheral coordinate used for the magnetic field calculation of conventional machines can be effectively applied for the superconducting machines. It will be considered that the density of electric currents of the windings is given by

$$J_{k} = \sum_{v} J'_{vmk} \cos v \phi + J''_{vmk} \sin v \phi \quad (k=r,\phi,z) \quad (1)$$

Such a representation of electric currents allows the possibility of using similar formula for the magnetic fields and for the scalar magnetic potential:

 $u = \sum_{v} U'_{vm} \cos v \phi + U''_{vm} \sin v \phi \qquad (2)$

Taking into account that for any harmonic component of the potential the second derivative with respect to the angular coordinate can be written as

$$\frac{1}{r^2} \frac{\partial^2 u}{\partial \phi^2} = -\frac{v^2}{r^2} U'_{vm} \cos v \phi - \frac{v^2}{r^2} U''_{vm} \sin v \phi \qquad (3)$$

the Laplace equation for the scalar potential is:

 $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U_{\nu m}}{\partial r}\right) - \frac{\nu^2}{r^2}U_{\nu m}'(") + \frac{\partial^2 U_{\nu m}'(")}{\partial z^2} = 0 \qquad (4)$

The equation is valid for (') and (") components of the potential. This equation is written for the amplitudes of the scalar potential and it needs to be solved over the two surfaces (') and ("), which are 90 electrical degrees apart. It is enough to calculate the scalar potential at any point if the problem is a linear one. So the equation becomes two-dimensional and the complete solution is the sum of the harmonic components. The number of harmonics to be calculated depends on the problem. It is enough to calculate just the field of the first harmonic sometimes, as for example in the calculation of the coeffi -cients of self and mutual inductances.

The boundary conditions for the scalar potential have to be satisfied along with this equation. One can write appropriate boundary conditions for the scalar potential if the electric currents of field and armature windings are considered as several thin current sheets. Such a replacement of real currents with thin sheets means loosing the accuracy of the calculation especially in the vicinity of the windings. The difficulties can be overcome and the scalar magnetic potential can be used for the solution of the steady magnetic field problem and time-varying problem with the help of the scalar potential method 4,5,6,7.

MODELS FOR THE MAGNETIC FIELD CALCULATION

Models for the steady magnetic field calculation were considered in references 4 and 7. The problem of time-varying magnetic field calculation will be considered here. Asynchronous components of the field of the armature winding can induce eddy currents in the conducting shield of the rotor; the interaction between the eddy currents and field of the rotor current can produce the shield vibrations, and also the penetration of an asynchronous field through the shield can increase the losses and heat in the zone of the superconducting windings. All that is to be accounted for in the calculation of time-varying magnetic fields in superconducting winding. One step is to calculate the efficiency of the conducting (or both conducting and thermal) shields and the losses inside the shield. The important supposition for this problem is that there are just two components of the eddy current in the conducting shield. It allows the formulation and

solution of the problem in a rather simple way.

Electric currents of the armature winding can be replaced by magnetic charges as shown in reference 7. The distribution of the armature current along the winding and in any cross-section of the winding will be considered to be known.

Choosing the direction of vector dl as $dl = jrd\phi$ and supposing that there are just two components of eddy currents flowing inside the conducting shield, one can conclude that the field \tilde{H}_2 consists of one component:

$$\tilde{H}_2 = -\tilde{i} H_{2r} = -\tilde{i} \int J_z r d\phi$$
 (5)

The distribution of the eddy current J is unknown; so there is the necessity to solve the equations for the scalar magnetic potential u and to find the sources of the magnetic field. Let the density of the armature current be represented as

 $j\omega\phi_J$, $j\omega\psi_u$, $j\omega\psi_H$, $J = J_z e$ so one has u = ue and $\dot{H}_2 = H_2 e$. By calculating the r-projection of curl of the left and right side of the equation curl $\dot{H} = \dot{J}$, one gets

$$\operatorname{curl}_{r}\operatorname{curl}\overset{\tilde{H}}{\underline{H}}_{2} = -\frac{\partial^{2}\dot{H}_{r2}}{\partial z^{2}} - \frac{1}{r^{2}}\frac{\partial^{2}\dot{H}_{r2}}{\partial \phi^{2}} \qquad (6)$$

$$\operatorname{curl}_{r}\gamma \vec{E} = -j\omega\mu\gamma (\dot{H}_{2r} - \frac{\partial \dot{u}}{\partial r}) + (\operatorname{grad}\gamma + \vec{E})_{r}$$
 (7)

Inside the shield γ is a constant and (grad γ) is 0; then one has the system of equations

$$\frac{\partial^{2}\dot{H}}{\partial z^{2}} + \frac{1}{r^{2}} \frac{\partial^{2}\dot{H}}{\partial \phi^{2}} - j\omega\mu\gamma\dot{H}_{r2} = -j\omega\mu\gamma\frac{\partial\dot{u}}{\partial r}$$
(8a)

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 u}{\partial \phi^2} + \frac{\partial^2 u}{\partial z^2} = -\frac{\rho_{aw}}{\mu} + div\bar{H}_2 \qquad (8b)$$

Because \dot{J}_{z} is zero along the edges of the shield, the boundary conditions for the first equation is that \dot{H}_{r2} is zero at the edge points of the shield. Boundary conditions for the scalar potential can be written to satisfy the normal and tangential components, \dot{H}_{r} and \dot{H}_{t} , over some boundaries. Because of the periodicity of the scalar potential u and the scalar \dot{H}_{2} along the angular coordinate, equations for \dot{H}_{r2} and \dot{u} can be written for the amplitudes H_{rv2m} and H_{vm} :

 $\frac{\partial^{2}\dot{H}_{r2\num}^{\prime}(")}{\partial z^{2}} - [j\omega\mu\gamma + (\frac{\nu}{r})^{2}]\dot{H}_{r2\num}^{\prime}(") = -j\omega\mu\gamma \frac{\partial\dot{U}_{\num}^{\prime}}{\partial r} \qquad (9a)$ $\frac{1}{r} \frac{\partial}{\partial r} (r \frac{\partial\dot{U}_{\num}^{\prime}}{\partial r}) + \frac{\partial^{2}\dot{U}_{\num}}{\partial z^{2}} - (\frac{\nu}{r})^{2}\dot{U}_{\num}^{\prime} = -\frac{\dot{\rho}_{aw}^{\prime}}{\mu} + \frac{1}{r} H_{r\nu2m}^{\prime} + \frac{\partial\dot{H}_{r\nu2m}^{\prime}}{\partial r} \qquad (9b)$

Eq. 9(a) should be solved just inside the conducting shield and Eq. 9 (1) should be solved over the two surfaces (') and (") in rz-planes.

CONCLUDING REMARKS

The use of developed models for the electromagnetic field calculations of superconducting electric machines shows that the scalar magnetic potential method can be very effective. Compared to the vector potential approach, the computing time is decreased considerably with the scalar potential method. All the required parameters of superconducting machine can be calculated with high accuracy by means of the scalar potential.

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