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PHASE-OUT AND DISPOSAL ISSUES OF OBSOLETE INVENTORY ITEMS IN RETAIL STORES

A Dissertation Presented
by

Nizar Zafer Zaarour

to
The Department of Mechanical and Industrial Engineering

In partial fulfillment of the requirements
for the degree of

Doctor of Philosophy

In the field of
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Preface and Acknowledgments

At the end of this journey, one might think that this is the time of reflection and remembering all the long nights, the hardship, the people that doubted and questioned my ability of making it through. But everything has an end, and when it comes, it is only the beginning of something else.

Life is a function of time, and this is the *time* to be thankful to all the people that have impacted this journey in a positive way and to look forward to the next adventure. I want to dedicate this to the person that has been my most important supporter as well as being my most influential inspiration, my mom. She has been the only constant among all the variables of life.

I would like to thank the usual suspects, my dad, my sister, the rest of the family and friends, and my committee members for their support and help in the last few years. Starting with my advisor, professor Melachrinoudis for encouraging me to get into the PhD program and for putting up with me for all these years, to professor Solomon, for being a major positive impact not only on my academic advancement, but on my professional one as well, and to professor kamarthi, for agreeing to join the committee and supporting the push towards the finish line. I also want to send special thanks to Allan Barr for supplying the field data, and to Alexandra for volunteering to edit all the grammatical errors without realizing what she was getting herself into. I also want to send a special shut-out to all the ex-huskies that have accompanied me along the way, to the undergrad gang: Bill, Rick, Todd, Mike, Ed, Chad, Matt, and Mark; and to my grad crew: Victor, Sameer, Chris, and Gilan.

"No man can reveal to you aught but that which already lies half asleep in the dawning of our knowledge. The teacher who walks in the shadow of the temple, among his followers, gives

not of his wisdom but rather of his faith and his lovingness. If he is indeed wise he does not bid you enter the house of wisdom, but rather leads you to the threshold of your own mind.

The astronomer may speak to you of his understanding of space, but he cannot give you his understanding. The musician may sing to you of the rhythm which is in all space, but he cannot give you the ear which arrests the rhythm nor the voice that echoes it. And he who is versed in the science of numbers can tell of the regions of weight and measure, but he cannot conduct you thither. For the vision of one man lends not its wings to another man. And even as each one of you stands alone in God's knowledge, so must each one of you be alone in his knowledge of God and in his understanding of the earth" (Gibran, 1923).

Education, in other words, is a way of life and not a personal goal. It is not a discrete variable that gets measured by getting degrees, but a continuous variable that lasts for the duration of one's life.

Z.A.F.

Abstract

Logistics is the management of the flow of goods, information and other resources between the point of production and the point of consumption in order to meet the requirements of consumers. Logistics involves mainly the integration of information, transportation, and inventory.

This dissertation addresses two important issues of the multifaceted area of logistics. The first pertains to inventory management and focuses on the problems of when and by how much to discount products that are being phased-out due to non-sales or the manufacturer's / distributor's decision. The second issue tackled is the transportation aspect of the reverse logistics problem which will aim to handle the remaining products returned by the consumer to the distributor or the manufacturer.

Often times, items in retail stores are phased-out due to the introduction of replacement items from the distributor. In order to sell out these items within a certain time horizon, retail stores need to develop markdown strategies. In the first phase of this dissertation, an optimal markdown strategy is developed as a primary step using a multi-period nonlinear programming model. Based on price elasticity of demand, the model maximizes revenue from the discontinued items. The mathematical properties of the model are established and a closed form optimal solution of the model is found. Furthermore, this model is tested with real data provided by a retailer. In the second step of this phase, a linear model is developed to address the issues of when and for how long to apply pre-determined markdown strategies during the phase-out period.

The second phase of the dissertation deals with the remaining inventory, in the case that not all items are sold during the phase-out period. A mixed integer nonlinear programming model that aims to manage product returns from individual retail stores (customers) under capacity constraints and service requirements is developed. Given the complexity of this model, a linear transformation of the non-linear objective function is presented. Through computational experiments, it is shown that the linearization produces better quality solutions and enables the handling of larger-sized data problems. Closed form solutions are obtained for special structures of the problem.

Key words: product returns, closed-loop supply chains, linear transformation, phase-out, elasticity of demand, nonlinear, markdown strategies.

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Chapter 1

Introduction

This chapter provides an introduction to the dissertation. Section 1.1 presents an overview of the main topics to be discussed, and the important logistic problems that this dissertation aims to resolve. Section 1.2 presents a description into the motivation behind the work and the research performed in this field. Section 1.3 provides a general scope of the problem and the contribution that this dissertation aims to make. Lastly, Section 1.4 outlines the research and breaks down the main objectives.

1.1 Overview

"The higher the price, the less you will buy" is one of the most famous concepts in economics. To predict consumer behavior, economists use well-defined techniques, evaluating consumers' sensitivity to changes in price; the most commonly used measure is the "price elasticity of demand." Elasticity of demand is the ratio of the percentage of the change in demand with respect to the percentage of the change in price as shown in Figure 1: E_d = (% change in quantity demanded / % change in price) = $\frac{\Delta Q / Q}{\Delta P / P}$

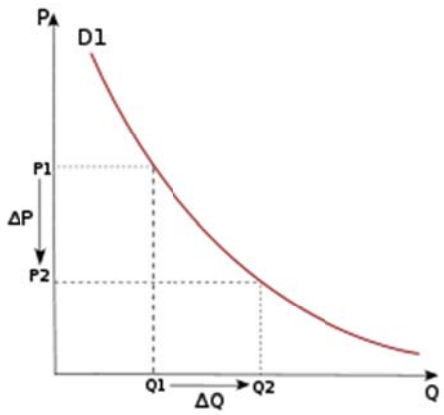


Figure 1: Elasticity of demand

Demand for a product can be said to be very inelastic if consumers will pay almost any price for the product. Inelastic demand allows a producer to raise prices without much affecting demand for the product; elastic demand exists when consumers are sensitive to the price at which a product is sold and will not buy if the price rises by what they consider excessive.

Generally speaking, the demand curve has a negative slope (Figure 2), and if there is a large decrease in the quantity demanded with a small increase in price, the demand curve looks flatter (as shown below in Figure 3), or more horizontal. This flatter curve means that the good or service in question is elastic.

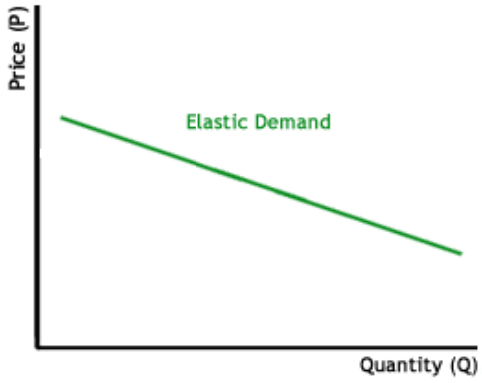


Figure 2: Elastic demand vs. price

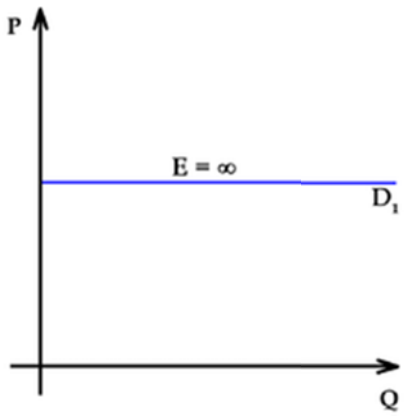


Figure 3: Perfect elastic demand

Meanwhile, inelastic demand is represented with a much more upright curve (Figure 4 and 5) as quantity changes little with a large movement in price.

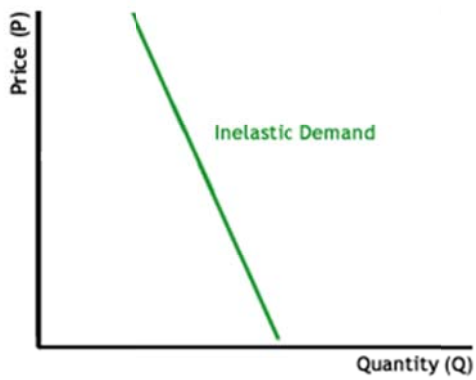


Figure 4: Inelastic demand vs. price

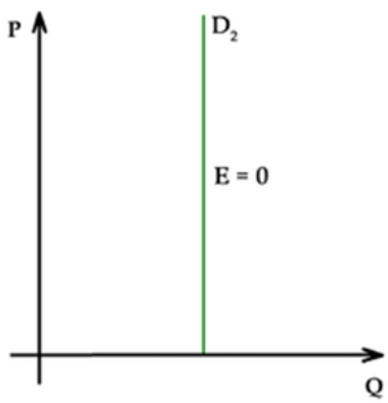


Figure 5: Perfect inelastic demand

In the first stage of this dissertation, changes in demand for items that are being phased-out will be addressed. Once a retailer learns that an item is being discontinued, the question becomes: What are the best prices during the different phase-out periods in order to deplete the given initial inventory on time? The problem of "when" and "by how much" to markdown is always present. Making wrong decisions could result in unsatisfactory consequences for retailers, for example a surplus of unnecessary inventory or worse, a loss in profits. In addition, the problems of the phase-out process will be looked at from a different perspective, where the markdown prices will be fixed and the questions of "when" and for "how long" to discount at each pre-determined price will be resolved. Figure 6 below depicts the phase-out process, with an initial inventory to be depleted at the beginning of the phase-out period, and either no inventory or a remaining inventory at the end of the fixed phase-out period.

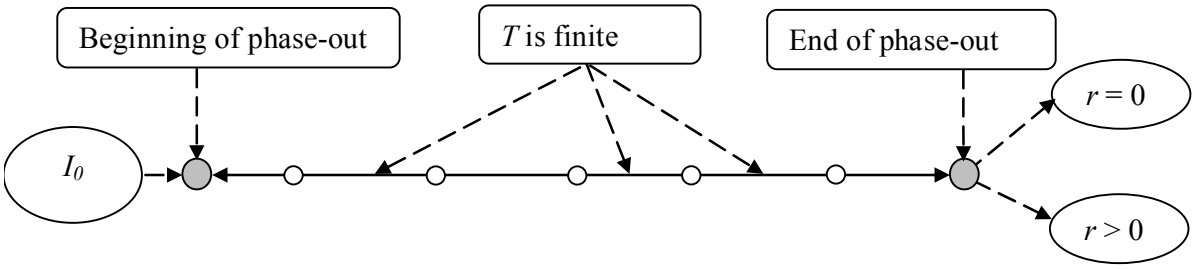


Figure 6: Phase-out process

T represents that phase-out period, I_0 is the initial inventory and r is the final remaining inventory.

Real data from a retailer is used as primary information for this research. In order to address the key problem at hand, consideration is given to the life of a particular product over a period of 54 weeks and its behavior through price changes. Subsequently, the product life period

is narrowed down by using a clustering analysis method. The clustered data are fit into a nonlinear regression model. As a result, this model type is the same used to illustrate the price elasticity of demand. Based on that model, the behavior of an obsolete inventory item over a certain phase-out period can be predicted. Given an initial inventory to be depleted, the best price for each period can be determined in order to maximize revenue. In the case that the whole initial inventory is unable to be entirely depleted, and the option, to sell the remaining inventory to a third party retailer with a particular salvage price is present, the model is modified to determine that remaining amount of inventory, and to determine the new optimal prices for each period in order to maximum revenue. The last approach deals with the problem of when and for how long to discount if the markdown prices are pre-determined. A new model is developed to address at what stage in the phase out period a particular discount should be applied and do all the different values of discounts get used when trying to obtain the objective function of maximizing our profit.

The second phase of the dissertation focuses on the management of the remaining products that have neither been sold during the phase-out period nor have been sold to a third party. This is an area of reverse logistics. Returned products come in all different sizes, shapes, and conditions and they are more difficult and costly to handle than original products. In fact, the logistics of handling returned products accounts for nearly 1% of the total U.S. gross domestic product (Gecker, 2007). To elaborate further, a study conducted by the Reverse Logistics Executive Council reported that U.S. companies spent more than $35 billion annually on the handling, transportation, and processing of returned products (Meyer, 1999). This estimate does not even include disposition management, administration time, and the cost of converting impaired materials into productive assets. In some industries such as e-tailing, apparel, and book

sales where return rates are usually high, the company's capability to manage its returned products may dictate its competitiveness. Though less dramatic, other industries in which return rates are nominal can suffer significantly from poor return management (see Table 1). For example, in the industrial equipment sector where return rates typically run 4-8%, its total revenue can be adversely affected with a potential loss of $52 – 104 billion annually, in just the U.S. alone (Norman and Sumner, 2006). In the computers and network equipment industry, where the average return rate is 8-20%, the potential revenue loss due to poor return management is estimated to be astounding $39 – 97 billion of the total revenue of $486 billion per year (Norman and Sumner, 2006). Hewlett-Packard discovered that the total costs of consumer product returns for North America exceeded 2% of total outbound revenue (Ferguson et al., 2006).

Table 1: The impact of product returns on the industry-wide revenue

Industry Sector	% of products returned w/in 1st warranty period	% of revenues spent on reverse logistics costs	% of initial value recaptured from returned products
Best-in-class	**5.7%**	**9%**	**64%**
Consumer Goods	11%	10%	31%
High-Tech.	6%	8%	28%
Telecom / Utilities	8%	8%	28%
Aerospace & Defense	5%	11%	10%
Medical Device Mfg.	11%	15%	22%
Industrial Mfg.	12%	13%	22%

Sources: Gecker, R. 2007, "Industry best practices in reverse logistics," Unpublished White Paper, *Aberdeen Group.*

1.2 Motivation

There were various sources of motivation behind this research. Mainly, it was the result of a project work, involving manufacturer Y and pharmacy X, which had the objective of addressing issues regarding inventory management.

There are over 6,000 pharmacy X's stores in the U.S. This research's focus was on cosmetic products, which are presenting the most difficult inventory issues. There are two annual reviews conducted by P&G, where products are selected to be phased-out. The "phase-out" decision can either be a soft phase-out, where there is only a change in the packaging, or a hard phase-out. This decision taken by P&G could either be based on product sales and profit, or it could be a decision based on product life cycle, as part of a product update, introduction of a successor product, or a totally new replacement product.

Many reasons can contribute to excess inventory. Some of these issues can be related to maintaining a minimum fixture presentation, seeking different goals by different groups, disorganization and promotional disconnects. Initial recommendations included having fewer items in the fixtures, and / or to follow the "Net Requirement System" (i.e. if the forecasted demand is one hundred items in a certain week and the store only has eighty items, order twenty items). Thus, investigation began regarding the interplay between demand and space, the fixed allocated space, and the discounts (promotions) policies.

After the initial observation, together with the retailers, an action plan was developed, which involved an intensive analysis of the collected inventory data, formulation of a model describing the behavior of the inventory system, and the identification of an optimal inventory

policy to deplete that initial inventory. The reports showed that there was around $10 million of surplus inventory total in all pharmacy X stores.

Pharmacy X owns its inventory and needs to sell it to recover its cost. P&G does not have any system in place to purchase anything back, since their products are targeted as markdown / sell through. Therefore the assumption is that they must sell through and there is not a reverse return / storage policy. P&G uses GMROI (Gross Margin Return on Inventory) as its performance index. GMROI is a financial metric that includes gross margin (profit) and inventory (investment). It measures the profit return as it relates to inventory investment. It is a measure of inventory productivity that expresses the relationship between the total sales, the gross profit margin earned on those sales, and the number of dollars invested in inventory. GMROI is expressed as a percentage or a dollar multiple, indicating how many times the original inventory investment was returned during one year. Furthermore, cost of sales and overhead are also important factors in determining profitability, as are subjective factors such as personal preferences and knowledge of your customers.

Gross margin and inventory are considered in order to contextualize the gross margin levers, opportunities and recommendations. Gross margin lever examples include, price equals impact of markdowns, sales price and regular price, cost equals fixed and variable cost, and volume equals rate of sales per event, per item, or per store. On the other hand, inventory lever examples include phase-out, promotional execution, SKU management, forecast accuracy, order and inventory policy, and supply chain network and inventory policy. Thus the focus will be on depleting the initial inventory at the beginning of the phase-out period and a buy back policy to deplete the remaining inventory, which could be a combination of selling the inventory to a third party with a salvage price, and / or returning whatever is left to the manufacturer / distributor.

The solution to the inventory problem could have the following potential path: to improve forecasting, based on previous demand and sales data; to inspect and change the physical properties of the fixtures in the retail stores; to examine and modify the phase-out process and policies; and to consider and run the reverse logistics model to verify if a return policy is beneficial.

Motivation for this research also derived from a personal interest in addressing issues in the ever-growing field of product returns and reverse logistics. Despite increasing attempts to reduce return rates, product returns have become a necessary evil. For instance, nearly 60 percent of Americans receive unwanted gifts during the holidays. During the holiday season of 2006, an estimated $13.2 billion in holiday gifts were returned to retailers – more than a third of the $36 billion reverse logistics market in the U.S. Especially the emergence of online sales poses many reverse logistics challenges for e-retailers (McCullough et al., 1999). As a matter of fact, return rates for online sales are substantially higher than traditional "bricks-and-mortar" retail sales, reaching 20 to 30% in certain categories of items (ReturnBuy, 2000). In general, product returns stemmed from two phenomena: (1) consumer returns of products to the retailer due to defects, damages, and inaccurate order fulfillments; (2) vendor returns of overstocked or unsold items to the manufacturer as part of the "buyback" policy.

Product returns are daily routines for many companies as evidenced by annual spending of $100 billion for managing product returns in the United States. Though easily overlooked, product returns often adversely affect the company's bottom line and then divert the company's primary focus of selling and distributing its products. In addition, poor management of returned products can increase customer angst and thus hurt customer services. Considering the seriousness of return management to business success, a growing number of companies have

attempted to streamline the process of collecting, handling, storing, and transporting returned products. One of those attempts include: (1) the determination of the optimal number and location of centralized return centers where returned products from customers are collected, sorted, and consolidated into a large shipment destined for manufacturer's repair facilities; (2) the estimation of the optimal holding time at the initial collection points that yields the best tradeoff between inventory carrying costs and shipping costs.

1.3 Research Scope and Contribution

The scope of this research covers issues that deal with inventory management, end of life cycle products, pricing strategies, data analysis for predicting price elasticity of demand, product returns, and transportation management and distribution issues.

In terms of the contribution provided by this dissertation, research determined the optimal prices during a phase-out time period to deplete an initial inventory, given a particular price – demand relationship. Research also concluded the optimal prices for each time period when remaining inventory at the end of the phase-out period is sold at a certain salvage price. Furthermore, studying the behavioral pattern by comparing different SKUs, allowed the identification of the best regression models in order to describe the demand as a function of the changing price. In addition, research was successful in concluding when to apply particular markdown strategies and for how long to achieve maximum revenue. These contributions will serve as a recommendation not only to the firm that supplied the real data, but to any retail chain with similar phase-out inventory problems.

Furthermore, the developed mixed integer linear programming model that has capacity restrictions and service requirements will serve as a tool to solve large-sized reverse logistics network design problems for product returns. The objective function explicitly considers different types of costs, including facility establishment/maintenance costs, inventory carrying costs, handling costs, and shipping costs with potential distance and shipping quantity discount opportunities. An additional contribution is the linearization of the model which eased computational complexity and thus enabled to find the optimal solution for larger customer bases, broader geographical service areas and varying shipping volumes between different collection points, while also predicting where and when the optimal solution is going to occur. The research also leads to the determination of a functional relationship between the daily return rate and the optimal collection period. Furthermore, analysis was performed as to when the optimal solution will be found by examining closed form solutions obtained from special structures designed for both discrete and continuous collection periods.

1.4 Dissertation Organization

This dissertation is organized as follows: in chapter 2, a breakdown of the literature review that covers the demand and pricing strategies, the end of life products and clustering analysis, and the reverse logistics and products returns. In chapter 3, proposed research is presented that includes the deliverables of the phase-out models and the return product models. It also includes the research objectives. Chapter 4 provides the proposed solutions to all the models described in chapter 3, whereas chapter 5 deals with the special problem structures: the markdown strategies analysis and the impact of the collection period in the reverse logistics

model. Chapter 6 provides detailed computational results analysis and chapter 7 discusses the proposed future research and includes some concluding remarks. The following is an outline of the proposed research. To begin, the outline details the type of products that are used in research, and addresses the issue of end-of-cycle products. To follow, this author will present and address the problems and the proven solutions in order to derive a strategy to deplete an inventory of this product type, within a particular phase-out period. This will involve a complete analysis of the work performed to break down the data and find the best strategies to achieve the optimal results of maximizing revenue.

In the next major phase of this research, this dissertation will examine the concerns and complexities as a result of incorporating a reverse logistics model to handle the remaining products and the costs and benefits of returning them to the distributor or the manufacturer. All the different costs associated with this process will be considered and also, what is needed to minimize them to better achieve the desired maximum revenue.

The research of this dissertation aims to achieve a smooth transition throughout all the different phases of the project through careful consideration of all the major and minor issues that arise from such challenges. Throughout the process, alternative scenarios with different models, along with their closed form solutions, theorems, corollaries, and all the necessary computational results are presented.

Chapter 2

Literature Review

In this chapter, the review of relevant literature is broken down into 3 categories. The first category pertains to inventory management, the different types of demand functions and consumer behavior, excess inventory problems and the dynamic pricing strategies that attempt to address these problems. The second category attends to the pricing strategies for end of life products, the elasticity of demand approach and the different clustering procedures. The third category reflects on the issues behind the reverse logistics choices, the difficulties of implementing these choices, and ultimately their benefits to the bottom line of the firms and companies.

2.1 Demand and Pricing Strategies

There has been an increasing adoption of dynamic pricing strategies and their further development in retail and other industries (Coy, 2000). Three factors contributed to this phenomenon:

- An increased availability of demand data
- An ease of changing prices due to new technologies, and
- An availability of decision-support tools for analyzing demand data and for dynamic pricing.

Companies must be aware of their own operating costs and availability of supply, and they must have a good understanding of the customer's reservation price as well as the projection

of future demand. Past research tried to address inventory problems such as Whitin (1955) who was one of the first to highlight the fundamental connection between price theory and inventory control, Scarf (1960) who addressed optimal policies for multi-echelon inventory, and Porteus (1971) who examined a standard inventory model with a concave increasing ordering cost function.

However, today, new technologies allow retailers to collect information not only about the sales, but also about demographic data and customer preferences (Elmaghraby, Keskinocak 2003). Despite significant improvements in reducing supply chain costs via improved inventory management, a large portion of retailers still lose millions annually as a result of lost sales and excess inventory.

According to Elmaghraby and Keskinocak, there are three main characteristics of a market environment that influence the type of dynamic pricing problem a retailer faces:

1- Replenishment vs. no replenishment of inventory (R / NR): inventory decisions are affected by whether inventory replenishment is possible during the price planning horizon.
2- Dependent vs. independent demand over time (D / I): demand is dependent over time for durable goods, and independent for most nondurable goods.
3- Myopic vs. strategic customers (M / S): a myopic customer is one who makes a purchase if the price is below his/her reservation price without taking into consideration future prices.

There are many other factors, of course, that influence dynamic pricing policies.

Based on different combinations of the 3 above mentioned characteristics, different categories can be formed. The first category focuses on market environments where there is no

replenishment and demand is independent over time (NR-I). The NR-I markets reflect a short cycle horizon or when products are at the end of their life cycles. The second category is a market environment where the seller replenishes inventory, demand is independent over time, and customers behave myopically (R-I-M).

Analytical models by Lazear (1986), Zhao and Zheng (2000), and Smith and Achabal (1998) that study how pricing decisions should be made in NR-I markets have the following common assumptions:

- The firm operates in a market with imperfect competition
- The selling horizon T is finite
- The firm has a finite stock of n items and no replenishment option
- Investment made in inventory is sunk cost
- Demand decreases in price P
- Unsold items have a salvage value

Pricing decisions in such markets are mainly influenced by demand, and how it changes when prices change along with other factors (*Elasticity of demand*). In particular, pricing decisions need to look at the arrival process of customers and the changes in the customer's willingness to pay over time.

When demand is deterministic, the optimal price can be computed and the direct correlation between the reservation prices and the optimal prices is shown. In this dissertation, the demand is deterministic which allows to compute the optimal price; however, the reservation prices are not taken into consideration. If demand is stochastic, only bounds on the optimal price can be obtained, and therefore on the optimal revenue.

According to most of the initial research literature written in this area of pricing-inventory, when demand is modeled directly, it typically belongs to the following class of functions:

$D_t(p) = \gamma_t(p)\xi_t + \delta_t(p)$, where:

D_t is the random demand at time t

$\gamma_t(.)$ and $\delta_t(.)$ are non-increasing functions of price p

ξ_t is a random variable

Lazear introduced a model where N customers arrive in each period with a reservation price V, where N is known to the seller, and V is unknown but drawn from a known distribution. Gallego and Van Ryzin (1994), and Feng and Gallego (1995) introduced models where the demand is a homogeneous (time-invariant) Poisson process with intensity $\lambda(p)$, where $\lambda(p)$ is non-increasing in p. In the three above mentioned papers, the reservation prices or their distribution remain constant over time. Feng and Chen (2003) considered a joint pricing and inventory control problem with setup costs and uncertain demand. Specifically, they developed an infinite horizon model that integrated pricing and inventory replenishment in a distribution environment, where they allowed for dynamically varying prices in response to changes in inventory levels by taking advantage of price-sensitive demand.

In contrast, Bitran et al. (1998), Bitran and Mondschein (1997), and Zhao and Zheng (2000) modeled the demand as a non-homogenous Poisson process with rate λ_t and allowed the probability distribution of the reservation price ($F_t(p)$) to change over time. That means that the probability that the customer will buy the item offered at price p is given by the function: $u(p,t) = 1 - F_t(p)$ and the demand rate at t is: $\lambda_t u(p,t)$.

Smith and Achabal (1998) incorporated the impact of the inventory level on demand in addition to the impact of price and time. They used a deterministic continuous demand model where demand at time t is given by: $x(p, I, t) = K(t)y(I)e^{-\gamma p}$, where $K(t)$ is the seasonal demand at time t, $y(I)$ is the inventory effect when inventory level is I, and $e^{-\gamma p}$ is the sensitivity of demand to price p. Smith and Achabal also found that the optimal price at a given time t should compensate any reduction in sales and that the retailer should set the terminal price to clear the entire inventory. Polatoglu and Sahin (2000) studied a periodic-review inventory model where, in addition to the procurement quantity, price is also a decision variable. They developed a model where demand in each period is a random variable having a price and, possibly, period-dependent probability distribution, with the expected demand decreasing in price.

Since this dissertation focuses on short life cycle items, it is worth mentioning that there are usually two types of markdowns: temporary and permanent. However, all the research mentioned above does not properly address the following topics: multiple products and stores, salvage value, competitor's pricing strategies, initial inventory, and strategic customers. This dissertation focuses on these topics with the exception of looking into the effects on one product onto the other in terms of demand, and the case of strategic customers, which will be an interest of future research.

When dealing with inventory replenishment, an eye must be kept on the effects of setting the price too low or too high. If it is too low, it could risk stock-outs and lost sales while waiting for replenishment. And if set too high, it could lead to excess inventory and high holding costs.

Other literature takes into consideration that the price is a decision variable and could vary from period to period. Some of these papers include Federgruen and Heching (1999), Thowsen (1975), and Zabel (1970) who consider uncertain demand, convex production, holding

and ordering costs, and unlimited production capacity. Thomas (1970), Chen and Simchi-levi (2002 - 2004), and Chan et al. (2001, 2002) extend the previous research to include a fixed ordering cost and limited production capacity. Biller et al. (2002) and Rajan et al. (1992) focus on models where the seller faces a deterministic demand. The goal of production has been usually considered as cost minimization, and its functions try to optimize its own goal without full consideration of other functions, leading to conflicts.

Eliashberg and Steinberg (1993) addressed the conflicts between production and other functions and the effects on the overall performance at the corporate level. Moreover, Kimes (1989) presented tools for capacity-constrained service firms who use the yield management approach and focused on the need to simulate practical and theoretical research in this area. Yield management allows service firms, such as airlines, to handle their fixed capacity in the most profitable manner possible by providing different prices to different customers.

Technology and the use of software plays a big role in implementing all the recent research in this field especially when it comes to changing product's prices and interpreting large amounts of sales data.

2.2 End of Life Products and Clustering Analysis

Proceeding on to the clearance phase of the price planning horizon, the clearance period is defined as the period bounded by the first markdown and the "outdate" when all remaining inventory is salvaged and new items arrive to replace the old ones on the store shelves (Zhao and Zheng, 2000). In the case where the rate of sale is sensitive to the inventory level, edging towards the end of the selling season, markdowns become deeper. According to Smith and

Achabal (1998), when it comes to clearance pricing, they observe some differences that are worth mentioning: clearance markdowns are permanent; demand tends to decrease at the end of the clearance period due to incomplete assortments and reduced merchandise selection, and clearance period is usually short enough that there is little time to correct pricing errors due to improvement in sales. In their model described before, Smith and Achabal presented two parts of the mathematical formulation for their optimization problem; one that allows inventory transfers and another that does not.

Gupta et al (2004) proposed discrete-time models to deal with the problem of setting prices for clearing retail inventories of fashion goods. They discussed the difficulties that are exacerbated by the fact that markdowns enacted near the end of the selling season have a smaller impact on demand. They showed that optimal prices decline when reservation prices decrease in the case of deterministic demand. On the other hand, when demand is stochastic and arbitrarily correlated across planning periods, they obtained bounds on the optimal expected revenue and on optimal prices. Petruzzi and Dada (1999) examined an extension of the newsvendor problem in which stocking quantity and selling price are set simultaneously. They provided a comprehensive review that synthesizes existing results for the single period problem and developed additional results to enrich the existing knowledge base.

Whenever the demand function being related to price is discussed, and in this case, to markdown prices, the factors of both the flow of customers coming into a store and their reservation to pay for a product, must be taken into consideration. However, markdowns near the end of the selling period have less of an impact on the demand. The main focus as time is increasing and moving closer towards the end of the selling season, is twofold: (1) timing the

markdown, in other words, when to apply the different markdown strategies, and (2) by how much to discount at every decision period.

In the case the products do not sell well, retailers tend to use aggressive markdowns to try to salvage and maximize on the return of whatever is left in the inventory. That is why, most of the research performed considers a single store and independent products, because the problem becomes far more complicated for retail chains especially if they need to coordinate their inventories and prices. In addition, the geographical dispersion of the stores adds another element of difficulty. There are different methods of inventory management for retail chains. Sethi and Cheng (1997) presented a Markovian demand model in the case when unsatisfied demands are lost.

The method, on which this research was conducted, can be characterized by a central warehouse distribution to the stores on a periodic basis. Moreover, retail chains usually manage their prices centrally.

It's important to reflect on the simplicity of the methods behind clustering analysis, but also to highlight the effectiveness of such procedures. Many clustering methods employed are based on data mining methods used to preprocess data. In addition, clustering aims to identify a structure in a collection of unlabeled data. In this research, clustering methods are used in two different occasions: (1) to cluster the different prices into more structured price ranges in order to be able to obtain a better elasticity of demand model, and (2) to cluster all the collection centers when applying the reverse logistics model for the product returns. The similarity criterion is usually distance; whether it is a geometrical distance, or a value distance. Another kind of clustering is called conceptual clustering, where two or more objects belong to the same cluster, if it defines a concept common to all the objects. Clustering algorithms can be applied in many

fields, such as marketing, biology, classification of plants and animals given their features, libraries, insurance, city-planning, etc.

The main requirements that a clustering algorithm should satisfy are: scalability, dealing with different types of attributes, discovering clusters with arbitrary shape; insensitivity to order of input records, high dimensionality, and interpretability and usability. The most commonly used clustering algorithm is the K-means algorithm, which is an exclusive clustering algorithm. An important component of a clustering algorithm is the distance measure between data points. If the components of the data instance vectors are all in the same physical units, then it is possible that the simple Euclidean distance metric is sufficient to successfully group similar data instances. Figure 7, shown below, illustrates this with an example of the width and height measurements of an object. Despite both measurements being taken in the same physical units, an informed decision has to be made as to the relative scaling. As the figure shows, different scaling can lead to different clustering.

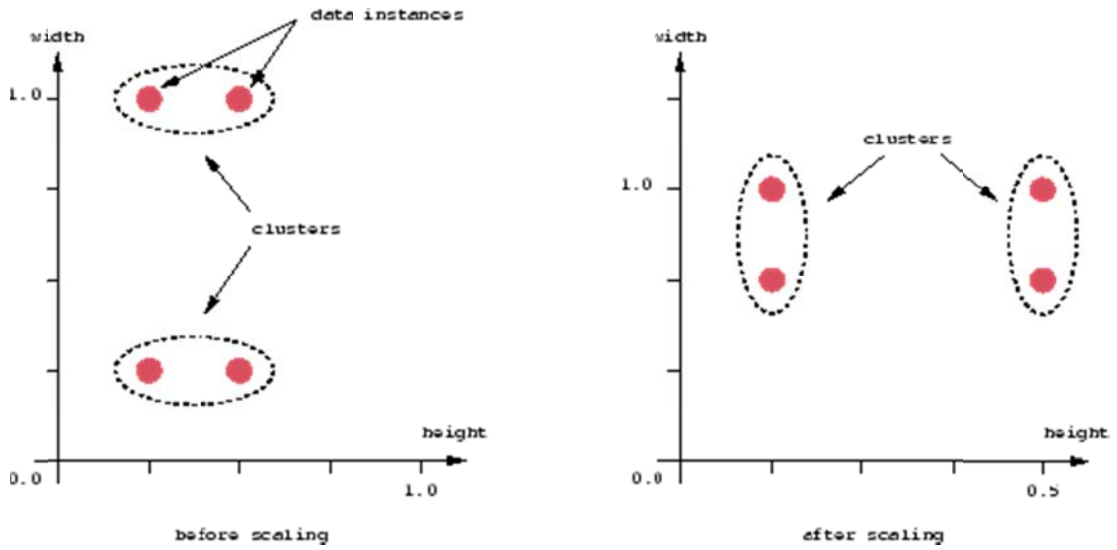


Figure 7: Different scaling to the same type of data

Notice however that this is not only a graphic issue: the problem arises from the mathematical formula used to combine the distances between the single components of the data feature vectors into a unique distance measure that can be used for clustering purposes.

For higher dimensional data, a popular measure is the Minkowski metric:

$$d_p(x_i, x_j) = (\sum_{K=1}^{d} | x_{i,k} - x_{j,k} |^p)^{1/p}$$

Where d is the dimensionality of the data. The *Euclidean* distance is a special case where $p = 2$. However, there are no general theoretical guidelines for selecting a measure for any given application.

2.3 Reverse Logistics and Product Returns

Reverse logistics is concerned with the distribution activities involving product returns, warehousing, source reduction/conservation, recycling, substitution, reuse, disposal, refurbishment, repair and remanufacturing (e.g., Shear et al., 2003; Stock, 1992; Guide et al., 2003; Van Wassenhove and Guide, 2003; Min et al. 2006a). There exists plentiful literature dealing with reverse logistics. For a thorough and detailed review of reverse logistics models, the interested reader should refer to Fleischmann et al. (2000) and Fleischmann (2003). Whereas the majority of the existing reverse logistics literature (e.g., Melachrinoudis et al., 1995; Barrros et al., 1998; Krikke et al., 1999; Jayaraman et al., 1999; Schultmann et al., 2003; Schultmann et al., 2005) focused on the environmental ("green") logistics aspect (e.g., recycling, remanufacturing, waste treatment) of the closed-loop supply chain, studies dealing with the reverse logistics network design involving product returns are still rare. Some of these earlier

studies on product returns worth noting include Min (1989) and Del Castillo and Cochran (1996).

To elaborate, Min (1989) developed a multiple objective mixed integer program that was designed to select the most desirable shipping options (direct versus consolidated) and transportation modes for product recall. Although he considered a tradeoff between transportation time and cost associated with reverse logistics, his problem scenario did not factor in-transit inventory carrying cost and consolidation holding time into his model. Del Castillo and Cochran (1996) presented a pair of linear programs (one aggregated and another disaggregated) and a simulation model to optimally configure the reverse logistics network involving the return of reusable containers so that the number of reusable containers was maximized. However, they did not take into account freight consolidation and transshipment issues related to reverse logistics.

More recently, Min et al. (2006a, b) presented a nonlinear integer program for solving the multi-echelon reverse logistics problem involving product returns. To overcome inherent computational complexity involved in the non-linear program structure, they utilized genetic algorithm (GA). Their contributions include the consideration of freight consolidation possibilities across geographical areas and time. Especially, they explored a possibility that customers will return their products to Initial Collection Points (ICPs) and then after a few days of waiting for an accumulation of sufficient volume those returned products will be transshipped from the ICPs to Centralized Return Centers (CRCs) for consolidation, asset recovery, remanufacturing or disposal. Their models determined the locations of ICPs and CRCs from sets of candidate locations, the collection period at the ICPs, and the shipping volumes from the ICPs to the CRCs. However, their proposed models and GA-based solution procedures were limited

to smaller-sized problems. Srivastava (2007) further conceptualized a product return process within the reverse logistics network that consists of collection centers and two types of rework facilities set up by original equipment manufacturers (OEMs) or their consortia for a few categories of product returns under various strategic, operational and customer service constraints. McCullough et al (1997) discussed the availability of good logistic service providers that have knowledge of handling and sorting, and broke down the weekly orders to small, medium and large, in order to decide whether to keep the transportation operation in house or outsource to a third party.

Despite merits, none of these prior studies is intended to solve large reverse logistics problems involving product returns and is designed to handle the full dynamics of tradeoffs among inventory, transportation, and consolidation costs. More importantly, none of them examined the dynamic interplay between shipping volume (i.e., return rates) and the reverse logistics decision regarding the collection period. To overcome these shortcomings of the prior studies, our research proposes the linearization of the nonlinear-mixed integer model developed by Min et al. (2006), while increasing the geographical service areas and shipping volumes between the initial collection points (ICPs) and the centralized return centers (CRCs). This research also performs extensive sensitivity analyses by varying return rates and assessing their impacts on optimal collection periods and total reverse logistics costs. For the special structure consisting of a single ICP and a single CRC, the optimal tradeoff between inventory and shipping costs is determined and a closed form solution is developed.

Chapter 3

Proposed Research

This chapter presents an overview of all the different aspects of the proposed research, including the complete breakdown of the deliverables, a review of the problems, explanation of all the models used, and finally, detailing the full scope of the research objectives.

3.1 Deliverables to the Phase-out models

In the first phase of the research, a large number of SKUs was analyzed from the obsolete inventory items category in a wide number of retail stores (customers) for a combined 54 week period. Assumptions of independence between the sales of the individual SKUs were made. The information collected is based on "Total Sales Amount (TSA) in $ and Total Sales Volume (TSV) in units." Unit price is then calculated by dividing the TSA by the TSV. Moreover, a clustering procedure was adopted to simplify the problem from the original data set of 54 weeks to a more manageable data set, which could vary between different SKUs.

The clustering analysis can be done visually (*k*-means algorithm) and with basic calculation of averages. The *k*-means algorithm assigns each point to the cluster whose center (also called centroid) is nearest.

It was required that multiple types of regression were performed in order to determine which concluded in the best results. For all the SKUs under study, the power regression was a

much better fit than the simple linear regression model. The next step was to standardize the SKUs mentioned above to further study their behavior as the price decreased. This led to the following two different approaches: (1) the first involved an increase in total sales volume, which had an initial price to start, calculation of the TSV based on the regression equation, increase of the TSV by a particular percentage, and then after backtracking, the calculation of the corresponding unit price; (2) the second approach was based on a decrease in the unit price that also had an initial price, calculation the TSV based on the regression equation, decrease of the unit price by a certain percentage, and then after backtracking, the calculation of the corresponding TSV. Various relationships between total volume (V), given unit price (P), were formulated to fit the clustered data. A linear relationship did not provide a good fit. The nonlinear mathematical model $V(p) = \alpha P^{\beta}$ yielded the best fit. Accordingly, β is called price elasticity of demand constant and given by

$$\beta = \frac{n\sum_{j=1}^{n}(\ln P_j)(\ln V_j) - \sum_{j=1}^{n}\ln P_j \sum_{j=1}^{n}\ln V_j}{n\sum_{j=1}^{n}(\ln P_j)^2 - \left(\sum_{j=1}^{n}\ln V_j\right)^2},$$

and α is a positive constant, given by:

$$\ln(\alpha) = \frac{\sum_{j=1}^{n}(\ln V_j) - \beta\sum_{j=1}^{n}(\ln P_j)}{n},$$

where n is the number of data points (P_j, V_j) found by cluster analysis.

In this mathematical model, $\beta = 0 \Rightarrow V = \alpha$ (perfectly inelastic), and if $\alpha = 1 \Rightarrow V = P^{\beta}$ (standardized power function).

The following table provides an interpretation of the values of the price elasticity coefficient (β).

Table 2: Interpretation of the price elasticity coefficient (β)

Value	Descriptive Terms
$\beta = 0$	Perfectly inelastic demand
$-1 < \beta < 0$	Inelastic or relatively inelastic demand
$\beta = -1$	Unit elastic, unit elasticity, unitary elasticity, or unitarily elastic demand
$-\infty < \beta < -1$	Elastic or relatively elastic demand
$\beta = -\infty$	Perfectly elastic demand

Price elasticity of demand coefficient "β" yields a negative value, due to the inverse nature of the relationship between price and quantity demanded. This behavior is evidently depicted in the below graph, where the effects of β are clearly shown, when all the other factors stay unchanged.

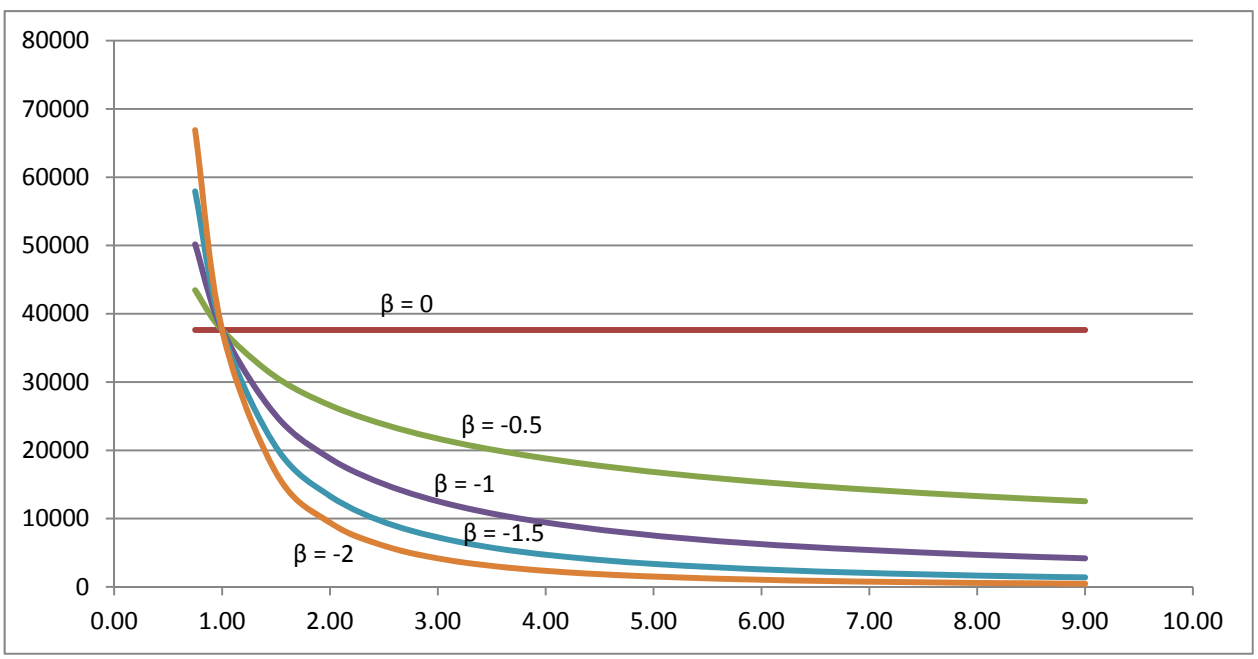


Figure 8: Effect of different values for the price elasticity of demand

The revenue (R) is determined by multiplying the unit price by the demand function, or $R = V * P = \alpha P^{\beta+1}$. The major optimization problem that is proposed here is to determine the

unit price $P_{i,t}$ to charge each SKU (i) in each period t, so that the resulting total revenue is maximized:

Maximize $$\sum_i \sum_t \alpha_i P_{i,t}^{\beta_i+1}$$

Subject to: $$\sum_t \alpha_i P_{i,t}^{\beta_i} = I_i$$

$$P_{i,t} \leq P_{i,t-1} \qquad \forall i, t = 2,...,T$$

Where I_i = Initial inventory to be depleted and T is the total number of periods (i = 1, …, T). Since the SKUs are independent and there are no bundle constraints and no restrictions of one SKU on to another, the case of one SKU (say SKU 1) – and multiple periods (general form) was considered:

Maximize $$\sum_{t=1}^{T} \alpha_1 P_{1,t}^{\beta_1+1}$$

Subject to: $$\sum_{t=1}^{T} \alpha_1 P_{1,t}^{\beta_1} = I_1$$

$$P_{1,t} \leq P_{1,t-1}, \; t = 2,...,T$$

In case the initial inventory is greater than the total units sold at the end of T periods, other options might be considered to handle the remaining inventory (r). One option is to try to sell the remaining inventory to a third party wholesaler. Let r_i be the remaining inventory of SKU (i) and its salvage price C_i. The salvage price associated with handling this remainder is

now added to the objective function. Then, the revenue from the remaining inventory becomes part of the constraint function and the mathematical formulation becomes:

Maximize $\sum_i \sum_t \alpha_i P_{i,t}^{\beta_i+1} + \sum_i C_i r_i$

Subject to: $\sum_t \alpha_i P_{i,t}^{\beta_i} + r_i = I_i$

$P_{i,t} \leq P_{i,t-1}$

Next, a decision was made to evaluate the problem from a different perspective. If prices are pre-determined, and it is known by how much to discount, the main question becomes: when should these different discounts be used. In other words, at what stage in the phase-out period should a particular discount be applied and are all the different values of discounts used when trying to obtain the objective function of maximizing our profit?

The single item (SKU) new model is formulated as follows:

Maximize $\sum_{i=1}^{n} R_i x_i + Cr$

Subject to: $\sum_{i=1}^{n} V_i x_i + r = I$

$\sum_{i=1}^{n} x_i = T,$

where

C is the salvage price associated with the remaining inventory after the phase-out period,

r is the remaining inventory after the phase-out period, where $r \geq 0$,

x_i is the decision variable that represents the duration of the particular discounted price P_i, where x_i are integers and $x_i \geq 0,\ i = 1,...,n$,

P_i is discounted price throughout the different phase-out periods (using the policy of the firm under study, P_i is decreasing throughout the phase-out period in a constant pre-determined manner, where: $P_1 > P_2 > ... > P_n > C$).

$V_i = \alpha P_i^{\beta}$ is the volume associated price P_i in a period, where $V_1 < V_2 < ... < V_n$ due to $\alpha > 0$ and $\beta < -1$, and

$R_i = \alpha P_i^{\beta+1}$ is the revenue associated price P_i in a period, where $R_1 < R_2 < ... < R_n$.

3.2 Deliverables to the Return Products model

As of 2000, product returns averaged approximately 6% of sales (Stock, 2001). The rate of product returns is usually higher for books, magazines, apparel, greeting cards, CD-ROMs and electronics. In particular, mail catalogue or on-line sales are more vulnerable to product returns. Typical reasons for product returns may include: defects, in transit damage, trade-ins, product upgrades, and exchanges for other products, refunds, repair, recalls, and order errors. In this dissertation, the focus is on the products that are getting returned due to the phase-out process by the distributor. Regardless of the reasons for the returns, firms either absorb the cost of return shipment or offer a money back guarantee for returned products, making product returns a major cost center. To control the cost of handling returns, a growing number of firms and their third party logistics providers have begun to examine ways to improve the efficiency of product returns. Examples of such ways are: (1) the reduction of return shipping costs by taking advantage of economies of scale. A number of separate consolidation points such as centralized return centers can be established to aggregate small shipments into a large shipment, (2) the enhancement of customer convenience for product returns. A number of initial collection points

near to the customer population center can help customers reduce their travel time to the collection points for returns, and (3) the reduction in transit inventory carrying costs associated with product returns. Since in transit inventory carrying costs are proportionately related to transit time of transportation modes that are used for return shipment, one should consider the fastest mode of transportation while weighing its freight rate.

Although many customers prefer to return computers directly to original equipment manufacturers (OEMs), direct shipment is far more costly than indirect shipment due to frequent, small volume shipment that often requires a premium mode of transportation. In addition, many customers do not want to deal with the hassle of making shipping arrangements for returns through regional postal services. Even though these collection points will not incur fixed costs such as land purchase, lease, and property tax, they will however incur variable costs associated with renting limited space designated for the non selling returned products. Given the limited storage space of the initial collection points, returned products at the collection points should be quickly transshipped to centralized return centers where returned products are inspected for quality failure, sorted for potential repair or refurbishment, stored long enough to create volume for freight consolidation, and shipped to original manufacturers. From centralized return centers, some returned products, which are found to be defect or damage free, may be re-distributed to customers after repackaging or re-labeling. Centralized return centers are dedicated to return handling and processing. On the other hand, centralized return centers may play a critical role in linking the initial collection points to manufacturing or repair facilities within the reverse logistics network. One can bypass the centralized return center for returning products to manufacturers, if the initial collection points are closer to the location of given manufacturers than that of centralized return centers since consolidation at the centralized return center

considerably delays the return process. With the above situations in mind, the main issues to be addressed are:

1. Location of initial collection points (ICP) in such ways that travel time (or distance) from existing and potential customers to the collection points is minimized.
2. Location of centralized return centers (CRC) in such a way that costs of transshipment between the initial collection points and the centralized return centers are minimized, while freight consolidation opportunities are maximized.
3. How to build the reverse logistics network in such a way that the collection period is minimized.
4. How many initial collection points and centralized return centers are needed to minimize the customer hassles associated with product returns while minimizing the costs of handling returns?

Prior to developing a model that built upon the nonlinear-mixed integer model developed by Min et al. (2006a) described above and transforming it to a linear form, the following assumptions and simplifications are stated: (1) the possibility of direct shipment from customers to a centralized return center is ruled out due to insufficient volume; (2) given a small volume of individual returns from customers, an initial collection point has sufficient capacity to hold returned products during the collection period; (3) the transportation costs between customers and their nearest collection points are negligible given short distances between the two parties; (4) the location/allocation plan covers a planning horizon in which no substantial changes are incurred in customer demands and in the transportation infrastructure, and (5) all customer locations are known and fixed a priori.

The model is used for the return of the remaining inventory to the manufacturer or distributor by going through initial collection points. The ultimate goal was to minimize the total reverse logistics costs associated with this process, which are comprised of five annual cost components: the cost of renting the initial collection points, the handling costs at those points, the cost of establishing and maintaining the centralized return centers, the inventory carrying cost, and most importantly, the transportation cost.

After the linearization of the problem, closed form solutions are offered for specific cases where a relationship between the collection period and the transportation cost is found.

3.3 Research Objectives

The following is a brief description of all the objectives to be accomplished from the mentioned above models. Considering a particular product that is being discontinued in a retail store, design models are developed to (1) deplete the initial inventory; (2) determine the prices for the different time periods throughout the phase-out period; (3) determine when and for how long to implement the markdown prices; (4) determine the optimal collection period for the remaining inventory; and (5) maximize the total revenue of the whole process.

Chapter 4

Solution Methodology

This chapter presents the solution methodology which includes the models, the assumptions, and the results, for all the different models proposed in the previous chapter, pertaining to the phase-out case, as well as to the reverse logistics aspect of the dissertation. During this process, the key issues are addressed and the necessary suitable steps are undertaken.

4.1 Solution to the Phase-out models

This section will focus on the phase-out problem, including its multi-faceted dimensions. The problem involves a particular product at the end of its life, after a decision has been made to phase it out within a fixed time period. Starting with an initial inventory, the model designed to determine the best prices to use during the different periods to maximize revenue. The data for the total units sold and the total volume were provided by a U.S. retailer for a 54 week period. To obtain better results, a clustering analysis of the data has been adopted to facilitate a better understanding of the behavior of demand with respect to changes in price. The most effective clustering method was based on calculating averages of prices and narrowing down the set of data from a 54 point set to an approximate 9 point set, where each average of price represents the cluster of prices around that particular price. That method is also known as the *k*-means algorithm where the steps include, choosing the number of clusters, *k*, then randomly generate

those k clusters and determine the cluster centers, or directly generate k random points as cluster centers. The next step is to assign each point to the nearest cluster center, where "nearest" is defined with respect to one of the distance measures discussed in the previous chapter, re-compute the new cluster centers, and repeat the two previous steps until a convergence criterion is met.

The next major step is to find which of the regression models provides the best fit to relate the demand to the changes in price. Simple linear regression analysis was the initial type of regression to be considered: $y = a + bx$, where y is the total sales volume dependent variable, and x is the unit price independent variable. The coefficients (a,b) can be calculated by solving the following normal equations for polynomial regression:

$$\sum y = ka + b\sum x$$

$$\sum xy = a\sum x + b\sum x^2$$

The power regression approach was analyzed next to see if a better fit to the data can be obtained:

$y = ax^b$, where

$$b = \frac{n\sum_{i=1}^{n}(\ln x_i)(\ln y_i) - \sum_{i=1}^{n}\ln x_i \sum_{i=1}^{n}\ln y_i}{n\sum_{i=1}^{n}(\ln x_i)^2 - \left(\sum_{i=1}^{n}\ln y_i\right)^2}, \qquad e^a = \frac{\sum_{i=1}^{n}(\ln y_i) - b\sum_{i=1}^{n}(\ln x_i)}{n}$$

The same procedure was implemented for all multiple items of the same product and it was found that the power regression formula was similar to the price elasticity of demand equation. After implementing the clustering procedure, the newly obtained reduced number of data sets was used as the phase-out period, where every point represents a period. The following is the

model developed with aim to maximize the revenue based on determining what would be the best prices for each period during the entire phase-out process:

Maximize $$\sum_{t=1}^{T} \alpha_1 P_{1,t}^{\beta_1+1} \tag{4.1}$$

Subject to: $$\sum_{t=1}^{T} \alpha_1 P_{1,t}^{\beta_1} = I_1, \tag{4.2}$$

where (t = 1, 2 …, T) are the different time periods and I_1 is the initial inventory to be depleted. Lagrangean multipliers were used to obtain a solution to the model. Let $\lambda_1 \geq 0$ be the Lagrangean multiplier to the single constraint above. Then, the Lagrangean function is

$$L(P_{1,1}, P_{1,2}, ..., P_{1,T}, \lambda) = f(P_{1,1}, P_{1,2}, ..., P_{1,T}) - \lambda_1 (g(P_{1,1}, P_{1,2}, ..., P_{1,T}) - I_1)$$

$$= \alpha_1 P_{1,1}^{\beta_1+1} + \alpha_1 P_{1,2}^{\beta_1+1} + ... + \alpha_1 P_{1,T}^{\beta_1+1} - \lambda_1 (\alpha_1 P_{1,1}^{\beta_1} + \alpha_1 P_{1,2}^{\beta_1} + ... + \alpha_1 P_{1,T}^{\beta_1} - I_1).$$

By setting $\frac{\partial L}{\partial P_t} = 0$, t = 1, 2 …, T and $\frac{\partial L}{\partial \lambda} = 0$, the following is obtained

$$\frac{dL}{dP_{1,t}} = \alpha_1 (\beta_1 + 1) P_{1,t}^{\beta_1} - \lambda_1 \alpha_1 \beta_1 P_{1,t}^{\beta_1 - 1} = 0 \tag{4.3}$$

for t = 1, …, T

and $$\frac{dL}{d\lambda_1} = \alpha_1 P_{1,1}^{\beta_1} + \alpha_1 P_{1,2}^{\beta_1} + ... + \alpha_1 P_{1,T}^{\beta_1} - I_1 = 0. \tag{4.4}$$

The solution of the system of the above two equations (4.3) and (4.4), is:

$$P_{1,1} = P_{1,2} = ... = P_{1,T} = \frac{\lambda_1 \beta_1}{\beta_1 + 1}. \tag{4.5}$$

Replacing this result in equation (4.4), the following results are obtained:

$$P_{1,1} = P_{1,2} = ... = P_{1,T} = \left(\frac{I_1}{T \alpha_1} \right)^{1/\beta_1} \tag{4.6}$$

and

$$R_{Max} = \frac{(I_1)^{\frac{\beta_1+1}{\beta_1}}}{(T\alpha_1)^{1/\beta_1}}. \tag{4.7}$$

For a particular SKU (n) and a particular number of periods / prices (T): $P_{n,T} = \left(\frac{I_n}{T\alpha_n}\right)^{1/\beta_n}$,

where $i = 1,...,n$ and $t = 1,...,T$

$$\Rightarrow R_{Max} = \sum_{t=1}^{T}\sum_{i=1}^{n}\frac{(I_i)^{\frac{\beta_i+1}{\beta_i}}}{(t_i\alpha_i)^{1/\beta_i}} \tag{4.8}$$

To ensure that the stationary solutions attained above would result in the maximum revenue, further analysis of the model was conducted, by using the Lagrangean method.

The Lagrangean function for the nonlinear optimization problem {max f(X) | g(X) = 0} is defined as $L(X,\lambda) = f(X) - \lambda g(X)$

The equations, $\frac{\partial L}{\partial X} = 0$ and $\frac{\partial L}{\partial \lambda} = 0$, yield the necessary conditions, given above, and hence the Lagrangean function can be used directly to generate the necessary conditions.

Define $H^B = \left(\frac{0 \mid P}{P^T \mid Q}\right)_{(m+n)\times(m+n)}$ where $P = \begin{pmatrix} \nabla g_1(X) \\ \vdots \\ \nabla g_m(X) \end{pmatrix}_{m\times n}$ and $Q = \left\| \frac{\partial^2 L(X,\lambda)}{\partial x_i \partial x_j} \right\|_{n\times n}$, for all i and j.

In this case, there is only one constraint g(X).

Simplifying the model to two periods:

$$\sum_{t=1}^{2}\alpha P_t^{\beta+1} = \alpha P_1^{\beta+1} + \alpha P_2^{\beta+1}$$

Subject to the constraint:

$$\sum_{t=1}^{2} \alpha P_t^{\beta} = I .$$

As shown above, $P_1 = \dfrac{\lambda\beta}{\beta+1} = P_2$ and $P_1 = P_2 = \left(\dfrac{I}{2\alpha}\right)^{1/\beta}, \lambda = \dfrac{\beta+1}{\beta}\left(\dfrac{I}{2\alpha}\right)^{1/\beta}$

The matrix H^B is identified as the bordered Hessian matrix. Given the stationary point (X_0, λ_0) for the Lagrangean function and the bordered Hessian matrix evaluated at that point, then X_0 is a maximum point if, starting with the principal major determinant of order ($2m$ + 1), and the last ($n - m$) principal minor determinants of H^B form an alternating sign pattern starting at $(-1)^{m+1}$.

$$H^B = \begin{vmatrix} 0 & (\alpha\beta)P_1^{\beta-1} & (\alpha\beta)P_2^{\beta-1} \\ (\alpha\beta)P_1^{\beta-1} & \alpha(\beta+1)P_1^{\beta-2}[\beta P_1 - (\beta-1)(I/2\alpha)^{1/\beta}] & 0 \\ (\alpha\beta)P_2^{\beta-1} & 0 & \alpha(\beta+1)P_2^{\beta-2}[\beta P_2 - (\beta-1)(I/2\alpha)^{1/\beta}] \end{vmatrix}$$

The principal minor determinant is of order ($2m$ + 1) = 3, since m = 1. The last ($n - m$) principal determinant is equal to 1 since n = 2, which forms an alternating sign pattern starting at $(-1)^{m+1}$ = 1. In order for these prices to yield the maximum revenue, the last T - 1 principal minor determinants of the *Bordered Hessian matrix* must have an alternating sign pattern starting at (+) with the principal minor determinant of order 3. The principal minor determinant of order k + 2, k = 1, 2 …, T - 1, is

$|M_{k+2}| = -\alpha^{k+2}\beta^2(\beta+1)^k\left(\prod_{t=1}^{k+1} P_t\right)^{\beta-1}\left(\sum_{t=1}^{k+1} P_t^{\beta-1}\right)$, and it has an alternating pattern starting at (+) with $|M_3|$ because β < -1.

To prove that $\alpha P_1^{\beta} + \alpha P_2^{\beta} \le I$ defines a convex set, it suffices to prove that

$$(\lambda x_1 + (1-\lambda)x_2)^{\beta} + (\lambda y_1 + (1-\lambda)y_2)^{\beta} \le \lambda x_1^{\beta} + (1-\lambda)x_2^{\beta} + \lambda y_1^{\beta} + (1-\lambda)y_2^{\beta}$$

for any $0 \leq \lambda \leq 1$. (x_1, y_1) and (x_2, y_2) are two points in the convex set, i.e., the coordinates of each point are associated with P_1 and P_2.

The right hand side of the inequality is less than I/α (Initial volume to be depleted / coefficient of the power function of the Volume as a function of price):

$$\lambda x_1^{\beta} + (1-\lambda)x_2^{\beta} + \lambda y_1^{\beta} + (1-\lambda)y_2^{\beta} \leq \frac{I}{\alpha}.$$

This is proven by using the constraints to the objective function for two random points in the convex set: $\alpha x_1^{\beta} + \alpha y_1^{\beta} \leq I$ and $\alpha x_2^{\beta} + \alpha y_2^{\beta} \leq I$, and then adding the two equations together after multiplying the first constraint by λ and the second by (1-λ), respectively.

Assuming that $\beta \leq -1$ and $x_1, x_2, y_1, y_2 > 1$,

$(\lambda x_1 + (1-\lambda)x_2) \geq \lambda x_1$ and/or $(\lambda x_1 + (1-\lambda)x_2) \geq (1-\lambda)x_2$

$\Rightarrow (\lambda x_1 + (1-\lambda)x_2)^{\beta} \leq \lambda x_1^{\beta}$ and/or $(\lambda x_1 + (1-\lambda)x_2)^{\beta} \leq (1-\lambda)x_2^{\beta}$

$\Rightarrow (\lambda x_1 + (1-\lambda)x_2)^{\beta} \leq \lambda x_1^{\beta} + (1-\lambda)x_2^{\beta}$.

By the same deduction, $(\lambda y_1 + (1-\lambda)y_2)^{\beta} \leq \lambda y_1^{\beta} + (1-\lambda)y_2^{\beta}$.

Adding the above two inequalities results in,

$$(\lambda x_1 + (1-\lambda)x_2)^{\beta} + (\lambda y_1 + (1-\lambda)y_2)^{\beta} \leq \lambda x_1^{\beta} + (1-\lambda)x_2^{\beta} + \lambda y_1^{\beta} + (1-\lambda)y_2^{\beta}.$$

Therefore, for $0 \leq \lambda \leq 1$,

$$(\lambda x_1 + (1-\lambda)x_2)^{\beta} + (\lambda y_1 + (1-\lambda)y_2)^{\beta} \leq \lambda x_1^{\beta} + (1-\lambda)x_1^{\beta} + \lambda y_1^{\beta} + (1-\lambda)y_1^{\beta}.$$

This signifies that every point (price) on the line segment defined by the two analyzed above points is also within the set.

In the case that the initial inventory is greater than the total units sold at the end of T periods, other options might be considered to manage the remaining inventory (r). One option is to try to sell the remaining inventory to a third party wholesaler.

The salvage price associated with handling this remainder is now added to the objective function. Furthermore, the remainder value becomes a part of the constraint function. For T periods and several different items, the optimization problem becomes:

Maximize $$\sum_i \sum_t \alpha_i P_{i,t}^{\beta_i+1} + \sum_i C_i r_i \tag{4.9}$$

Subject to: $$\sum_t \alpha_i P_{i,t}^{\beta_i} + r_i = I_i \tag{4.10}$$

where C_i is the salvage price, r_i is the remaining inventory r, and I_i is the initial inventory to be depleted for item i, respectively.

The above problem can be decomposed to simpler form for the individual items. Considering one item, say (i = 1), and denoting the salvage price by C_{T+1} (price for the T+1 period), the following is obtained:

Maximize $$\sum_{t=1}^{T} \alpha_1 P_{1,t}^{\beta_1+1} + C_{T+1} r_1 \tag{4.11}$$

Subject to: $$\sum_{t=1}^{T} \alpha_1 P_{1,t}^{\beta_1} + r_1 = I_1 \tag{4.12}$$

Where (t = 1, 2 …, T) are the different time periods.

The Lagrangean function of the above optimization problem is:

$$L(P_{1,1}, P_{1,2}, ..., P_{1,T}, r, \lambda) = f(P_{1,1}, P_{1,2}, ..., P_{1,T}, r) - \lambda(g(P_{1,1}, P_{1,2}, ..., P_{1,T}, r) - I_1)$$

$$= \alpha_1 P_{1,1}^{\beta_1+1} + \alpha_1 P_{1,2}^{\beta_1+1} + ... + \alpha_1 P_{1,T}^{\beta_1+1} + C_{T+1} r_1 - \lambda_1(\alpha_1 P_{1,1}^{\beta_1} + \alpha_1 P_{1,2}^{\beta_1} + ... + \alpha_1 P_{1,T}^{\beta_1} + r_1 - I_1).$$

The necessary optimality conditions become

$$\frac{dL}{dP_{1,t}} = \alpha_1(\beta_1 + 1)P_{1,t}^{\beta_1} - \lambda_1\alpha_1\beta_1 P_{1,t}^{\beta_1 - 1} = 0, \tag{4.13}$$

for $t = 1, \ldots, T$

$$\frac{dL}{dr_1} = C_{T+1} - \lambda_1 = 0, \tag{4.14}$$

$$\text{and}\,\frac{dL}{d\lambda_1} = \alpha_1 P_{1,1}^{\beta_1} + \alpha_1 P_{1,2}^{\beta_1} + \ldots + \alpha_1 P_{1,T}^{\beta_1} + r_1 - I_1 = 0. \tag{4.15}$$

Solving equation (4.14), the following result is obtained: $\lambda_1 = C_{T+1}$.

Furthermore, solving system of equations (4.13) simultaneously yields

$$P_{1,1} = P_{1,2} = \ldots = P_{1,T} = \frac{\lambda_1\beta_1}{\beta_1 + 1} = \frac{C_{T+1}\beta_1}{\beta_1 + 1} \text{ and } r_1 = I_1 - T\alpha_1\left(\frac{\beta_1 C_{T+1}}{\beta_1 + 1}\right)^{\beta_1}.$$

Since r_1 needs to be non-negative, imposing $r_1 \geq 0$ above implies that

$$I_1 \geq T\alpha_1\left(\frac{\beta_1 C_{T+1}}{\beta_1 + 1}\right)^{\beta_1} \Rightarrow C_{T+1} \geq \left(\frac{\beta_1 + 1}{\beta_1}\right)\left(\frac{I_1}{T\alpha_1}\right)^{1/\beta_1}.$$

This is a remarkable result because it provides the minimum salvage price above of which it is better for the company to leave inventory at the end of the phase-out period. Thus, it would be more beneficial not to lower the prices further but instead selling the remaining inventory (r_1) to a third party wholesaler.

The above mentioned results were verified using Lingo. How much to sell, how much remains, and the price by which to sell the item during the phase-out period, can all be determined if the following factors are known: (1) the duration of the phase-out period; (2) the "alpha" and "beta" from the cluster analysis power regression model; and (3) the initial volume to be depleted.

There are many different types of nonlinear programming problems, depending on the characteristics of the objective function ***f(x)*** and the constraint functions g_i***(x)***. Different algorithms are used for the different types. The general constrained problem uses the Karush-Kuhn-Tucker (KKT) conditions for optimality. Assume that f(x), g_1(x), g_2(x), …, g_m(x) are differentiable functions, then $x^* = (x_1^*, x_2^*, ..., x_n^*)$ can be an optimal solution for the nonlinear programming problem only if there exist m numbers $\lambda_1, \lambda_2, ..., \lambda_m$ such that all the following *KKT* conditions are satisfied:

1. $\dfrac{\partial f}{\partial x_j} - \sum_{i=1}^{m} \lambda_i \dfrac{\partial g_i}{\partial x_j} \leq 0$

2. $x_j^* \left(\dfrac{\partial f}{\partial x_j} - \sum_{i=1}^{m} \lambda_i \dfrac{\partial g_i}{\partial x_j} \right) = 0$ At x = x*, for j = 1, 2… n.

3. $g_i(x^*) - b_i \leq 0$

4. $\lambda_i (g_i(x^*) - b_i) = 0$ For i = 1, 2…m.

5. $x_j^* \geq 0$ For j = 1, 2…n.

6. $\lambda_i \geq 0$ For i = 1, 2…m.

Applying these conditions to our model, the following is considered:

Maximize $\sum_{t=1}^{T} \alpha P_t^{\beta+1} + Cr$

Subject to: $\sum_{t=1}^{T} \alpha P_t^{\beta} + r \leq I$

And $r \geq 0$, $P_t \geq 0$ where: t = 1 … T

An equivalent form with all constraints being "≤ 0" is:

Maximize $$\sum_{t=1}^{T} \alpha P_t^{\beta+1} + Cr \tag{4.16}$$

Subject to: $$\sum_{t=1}^{T} \alpha P_t^{\beta} + r \le I \tag{4.17}$$

$$-P_t \le 0,\ t = 1 \ldots T \tag{4.18}$$

$$-r \le 0 \tag{4.19}$$

Let u, v_t (t = 1 … T) and w be the Lagrangean multipliers associated with constraints (4.17), (4.18) and (4.19), respectively. The *KKT* necessary conditions for optimality at $(P_1,...,P_t,...,P_T,r)$ are:

$$\alpha(\beta+1)P_t^{\beta} - u\alpha\beta P_t^{\beta-1} + v_t = 0,\ t = 1 \ldots T \tag{4.20}$$

$$C - u + w = 0,\ u \ge 0,\ w \ge 0,\ v_t \ge 0,\ \mathrm{t} = 1 \ldots T \tag{4.21}$$

$$u\left(\sum_{t=1}^{T} \alpha P_t^{\beta} + r - I\right) = 0 \tag{4.22}$$

$$P_t v_t = 0 \tag{4.23}$$

$$rw = 0 \tag{4.24}$$

$$\sum_{t=1}^{T} \alpha P_t^{\beta} + r \le I \tag{4.25}$$

$$P_t \ge 0,\ t = 1 \ldots T \tag{4.26}$$

$$r \ge 0 \tag{4.27}$$

Constraints (4.20) and (4.21) are the optimality conditions, constraints (4.22) – (4.24) are the complementary slackness conditions, and constraints (4.25) – (4.27) are the feasibility conditions.

It is assumed that $u = 0$. Then (4.20) and (4.26) imply that $v_t = -\alpha(\beta+1)P_t^{\beta} > 0$ for t = 1 … T, since $\beta < -1$. This contradicts (4.21). Furthermore, for $P_t = 0$, t = 1 … T, the $\lim_{P_t \to 0} \alpha P_t^{\beta} = \infty$, and constraint (4.25) is violated. Therefore $u > 0$ and (4.22) implies:

$$\sum_{t=1}^{T} \alpha P_t^{\beta} + r = I \tag{4.28}$$

In addition, $P_t > 0$, t = 1 … T, which implies from (4.23) that $v_t = 0$, t = 1 … T and (4.20) yields

$P_t = \left(\frac{\beta}{\beta+1}\right)u$, t = 1 … T.

Finally, r can be positive or zero. If $r > 0$, then $w = 0$, and (4.21) implies that $u = C$. Therefore,

$P_t = \left(\frac{\beta}{\beta+1}\right)C$, t = 1 … T and (4.28) yields $r = I - \sum_{t=1}^{T} \alpha \left(\frac{\beta}{\beta+1}\right)^{\beta} C^{\beta}$.

If $r = 0$, (4.28) yields $P_t = \left(\frac{I}{T\alpha}\right)^{1/\beta}$, t = 1 … T and u becomes: $u = \left(\frac{\beta+1}{\beta}\right)P_t = \left(\frac{\beta+1}{\beta}\right)\left(\frac{I}{T\alpha}\right)^{1/\beta}$

In addition, since $r = 0$, (4.24) implies that $w \geq 0$, and combining that with (4.21), $w = u - C \geq 0$ is obtained. Substituting u from above, results in the following inequality for C: $C \leq \left(\frac{\beta+1}{\beta}\right)\left(\frac{I}{T\alpha}\right)^{1/\beta}$.

The following theorem and corollary follow the above *KKT* conditions derivation.

Theorem 1. *The optimal solution of the maximizing revenue problem with an initial inventory to be depleted over a finite time horizon, where the demand is a nonlinear function of price is*

solved by finding the optimal price for every period $t, P_t = \left(\frac{\beta}{\beta+1}\right)C$, $t = 1 \ldots T$ *and the remaining inventory,* $r = I - \sum_{t=1}^{T} \alpha \left(\frac{\beta}{\beta+1}\right)^{\beta} C^{\beta}$.

Corollary 1. *If the salvage price is lower than a certain threshold,* $C \leq \left(\frac{\beta+1}{\beta}\right)\left(\frac{I}{T\alpha}\right)^{1/\beta}$ *then items should not be left at the end of the time horizon T.*

The single period process was used for a dynamic pricing policy, where the model ran for the first period, and the remaining inventory was used at the end of that period as the new initial inventory for the new period. The same process was repeated until the end of the phase-out period. By updating the inventory at the beginning of every period, it was found that the prices remain the same throughout the phase-out periods. In addition, the resulting values are exactly the same as the prices obtained, when the model ran for the entire phase-out period as a whole.

4.2 Solution to the Return Products model

Moving forward from the results of the previous section, if it was decided to consider returning the remaining inventory to the manufacturer or the distributor, there is a need to implement a reverse logistics model to help minimize all the costs associated with this operation, such as the renting and handling costs at the initial collection points, establishing and maintaining the centralized return centers, inventory carrying costs, and transportation costs. Some of the decisions to be made are which initial collection points and centralized return centers to be established, which customer is allocated to which initial collection point, and the volume of products to be returned from an initial collection point to a centralized return center.

In addition, freight discount rates based on quantity and distance should be explicitly considered. The complete mathematical model is presented next with symbol definitions and mathematical formulation.

Indices:

i = index for customers; $i \in I$

j = index for initial collection points; $j \in J$

k = index for centralized return centers; $k \in K$

Decision Variables:

X_{jk} = volume of products returned from initial collection point j to centralized return center k

$$Y_{ij} = \begin{cases} 1, & \text{if customer } i \text{ is allocated to initial collection point } j \\ 0, & \text{otherwise} \end{cases}$$

$$Z_j = \begin{cases} 1, & \text{if an initial collection point is established at site } j \\ 0, & \text{otherwise} \end{cases}$$

T = length of a collection period (in days) at each initial collection point

$$G_k = \begin{cases} 1, & \text{if a centralized return center is established at site } k \ (k \in K) \\ 0, & \text{otherwise} \end{cases}$$

Model Parameters:

a_j = annual cost of renting initial collection point j

b = daily inventory carrying cost per unit

w = annual working days

r_i = volume of products returned by customer i per day

h_j = handling cost of unit product at initial collection point j

c_k = annual cost of establishing and maintaining centralized return center k

m_k = maximum processing capacity of centralized return center k in new returns per day

d_{ij} = distance from customer i to initial collection point j

d_{jk} = distance from collection point j to centralized return center k

l = maximum allowable distance from a given customer to an initial collection point

$\overline{T}$ = maximum length of a collection period (in days) at an ICP. This upper bound on the length of collection days is necessary to assure that return lead time is not too long for the customers

$C_i = \{j \mid d_{ij} \leq l\}$ set of initial collection points that are within distance l from customer i

$D_j = \{i \mid d_{ij} \leq l\}$ set of customers that are within distance l from initial collection point j

$f(X_{jk}, d_{jk}) = E\alpha_{jk}\beta_{jk}$ unit transportation cost between collection point j and return center k

where E is the standard freight rate (\$/unit), α_{jk} is the freight discount rate according to the volume of shipment between initial collection point j and centralized return center k, and β_{jk} is the penalty rate applied for the distance between initial collection point j and centralized return center k.

$$\alpha_{jk} = \begin{cases} 1 & for \quad X_{jk} \leq P_1 \\ \alpha_1 & for \quad P_1 < X_{jk} \leq P_2 \\ \alpha_2 & for \quad X_{jk} > P_2 \end{cases} \qquad \beta_{jk} = \begin{cases} 1 & for \quad d_{jk} \leq Q_1 \\ \beta_1 & for \quad Q_1 < d_{jk} \leq Q_2 \\ \beta_2 & for \quad d_{jk} > Q_2 \end{cases}$$

Figure 9 shows the benefits from the economies of scale (i.e., freight discounts) and/or penalties due to distance for a certain shipment X_{jk} between ICP j and CRC k that are d_{jk} distance away. Although only two breakpoints are specified for volume shipment (P_1 and P_2) and distance (Q_1

and Q_2), any number of points can be accommodated by the proposed model which can mimic class rates in practice.

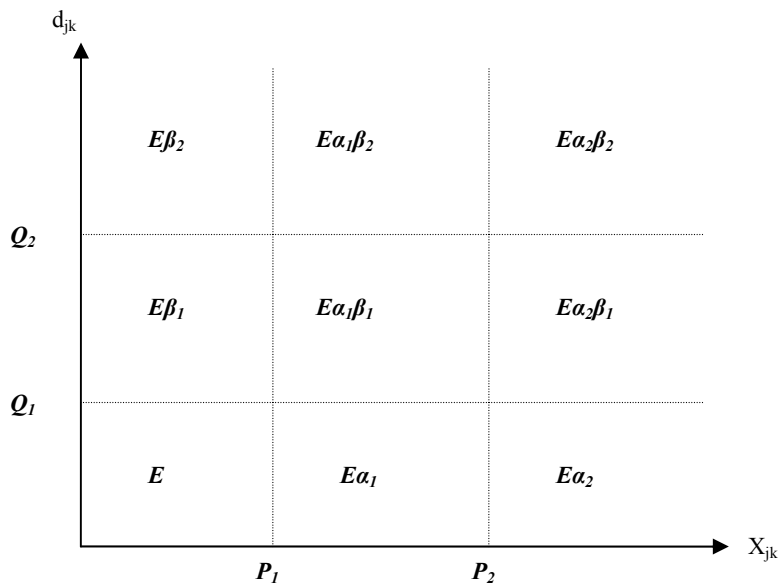


Figure 9: Unit transportation cost function

Mathematical Formulation:

Minimize

$$\sum_{j\in J} a_j Z_j + bw\left(\frac{T+1}{2}\right)\sum_{i\in I} r_i + w\sum_{i\in I}\sum_{j\in C_i} r_i h_j Y_{ij} + \sum_{k\in K} c_k G_k + \frac{w}{T}\sum_{k\in K}\left(\sum_{j\in J} X_{jk} f(X_{jk}, d_{jk})\right) \tag{4.29}$$

Subject to

$$\sum_{j\in C_i} Y_{ij} = 1 \qquad \forall i \in I \tag{4.30}$$

$$\sum_{i\in D_j} Y_{ij} \le M\ Z_j \qquad \forall j \in J \tag{4.31}$$

$$T\sum_{i\in D_j} r_i Y_{ij} = \sum_{k} X_{jk} \qquad \forall j \in J \tag{4.32}$$

$$\sum_{j} X_{jk} \le T m_k G_k \qquad \forall k \in K \tag{4.33}$$

$$X_{jk} \geq 0, \qquad \forall j \in J \qquad \forall k \in K \tag{4.34}$$

$$Y_{ij} \in \{0,1\} \qquad \forall i \in I \qquad \forall j \in J \tag{4.35}$$

$$Z_j \in \{0,1\} \qquad \forall_j \in J \tag{4.36}$$

$$G_k \in \{0,1\} \qquad \forall_k \in K \tag{4.37}$$

The objective function (4.29) minimizes the total reverse logistics costs, which are comprised of five annual cost components: the cost of renting the ICPs, the cost of establishing and maintaining the CRCs, the handling costs at the ICPs, the inventory carrying cost, and the transportation cost.

Constraint (4.30) assures that a customer is assigned to a single initial collection point. Constraint (4.31) prevents any return flows from customers to be collected at a closed ICP (*M* is an arbitrarily set big number). Constraint (4.32) makes the incoming flow equal to the outgoing flow at each initial collection point. Constraint (4.33) ensures that the total volume of products shipped from initial collection points to a centralized return center does not exceed the maximum capacity of the centralized return center. Constraint (4.34) preserves the non-negativity of decision variables X_{jk}. Constraint sets (4.35) – (4.37) declare decision variables Y_{ij}, Z_j and G_k as binary.

4.2.1 Linearization of the Model

The first contribution is to linearize the model. The non-linearity of the last term of the objective function $\frac{w}{T}\sum_{k\in K}\left(\sum_{j\in J} X_{jk} f(X_{jk}, d_{jk})\right)$ can be transformed to a linear term $X_{jk} f(X_{jk}, d_{jk})$ as follows:

For any given (*j,k*) pair, β_{jk} can be easily determined by the known value of d_{jk}. Parameter α_{jk}, however, depends on the value of decision variable X_{jk} and therefore cannot be determined in advance. The following transformation is used to linearize the term $X_{jk} f(X_{jk}, d_{jk})$. Let U_{jk}^1, U_{jk}^2 and U_{jk}^3 be continuous variables associated with ranges $[0, P_1]$, $(P_1, P_2]$ and (P_2, ∞), respectively, such that

$$X_{jk} = U_{jk}^1 + U_{jk}^2 + U_{jk}^3 \tag{4.38}$$

and

$$0 \le \frac{U_{jk}^3}{M} \le W_{jk}^2 \le \frac{U_{jk}^2}{P_2 - P_1} \le W_{jk}^1 \le \frac{U_{jk}^1}{P_1} \le 1, \tag{4.39}$$

where W_{jk}^1 and W_{jk}^2 are binary variables and *M* is a big number. Accordingly, when $W_{jk}^1 = W_{jk}^2 =$ 0, then $X_{jk} = U_{jk}^1 \le P_1$ is in the first range; when W_{jk}^1 = 1 and W_{jk}^2 = 0, then $U_{jk}^1 = P_1$, $U_{jk}^2 \le P_2 - P_1$, U_{jk}^3 = 0 and X_{jk} is in the second range. Finally, when both W_{jk}'s take the value of 1, then X_{jk} is in the third range (See Figure 10). The term $X_{jk} f(X_{jk}, d_{jk})$ can now be mathematically expressed as a linear function of U_{jk}'s and W_{jk}'s:

$$X_{jk} f(X_{jk}, d_{jk}) = r_{jk}^1 U_{jk}^1 + r_{jk}^2 U_{jk}^2 - (r_{jk}^1 - r_{jk}^2) P_1 W_{jk}^1 + r_{jk}^3 U_{jk}^3 - (r_{jk}^2 - r_{jk}^3) P_2 W_{jk}^2,$$

where $r_{jk}^1 = E\beta_{jk}$, $r_{jk}^2 = E\alpha_1\beta_{jk}$ and $r_{jk}^3 = E\alpha_2\beta_{jk}$ are the slopes of function $X_{jk} f(X_{jk}, d_{jk})$, as shown in Figure 10:

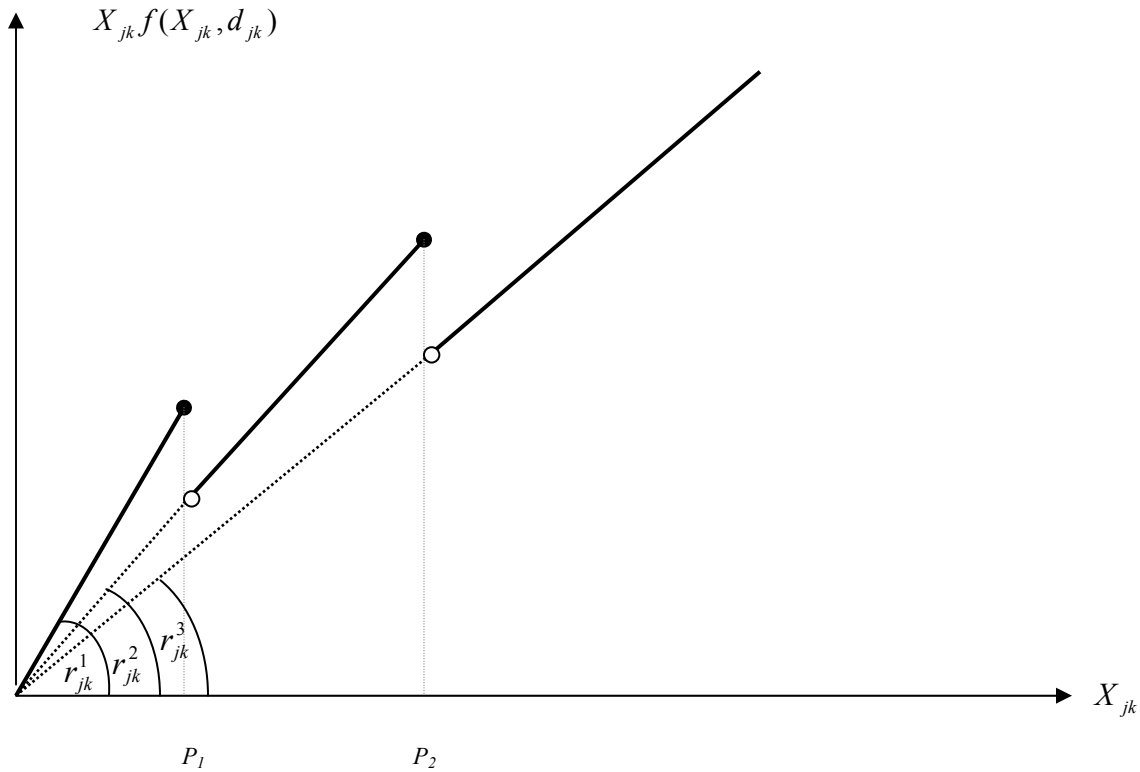


Figure 10: Transportation cost function

Each continuous variable X_{jk} is replaced by $(n+1)$ $|J||K|$ continuous variables U_{jk}'s and n $|J||K|$ binary variables W_{jk}'s, where n is the number of breakpoints (P_l) in shipment volumes. Two breakpoints were used above ($n = 2$) to illustrate discounts in shipment costs. Moreover, the problem has additional constraint (1) and 3 + 2n additional constraints (2) for each original variable X_{jk}. Although the new problem has more variables and constraints, it is still a linear mixed integer program (MIP) for a fixed value of T and can be solved optimally by readily available commercial software such as Lingo Version 11 (2008). By solving sequentially the MIP for the possible values of $T = 1, \ldots, \overline{T}$, the optimal solution to the original problem can be obtained.

Chapter 5

Special Problem Structures

This chapter presents the special structures of the models discussed earlier in Chapter 4, and further dissects the solutions obtained in order to handle the specific cases where theorems and corollaries are derived. This work aims to set the foundation for the next chapter where the computational results are detailed and sensitivity analysis performed. During this process, the following section will discuss the different issues that were encountered during research, along with the steps taken to resolve them. In summary, this chapter will present the assumptions made for the special structures and the closed form solutions.

5.1 Markdown Strategies Analysis

A special case arises in the phase-out problem, where the prices are pre-determined, i.e., it is known by how much to discount; therefore, the main question becomes: when these different discounts should be applied and for how long? In addition, should all the different discounts be used or not when trying maximize revenue?

The mathematical model for this case can be formulated as follows.

Maximize $$\sum_{i=1}^{n} R_i x_i + Cr \tag{5.1}$$

Subject to: $$\sum_{i=1}^{n} V_i x_i + r = I \tag{5.2}$$

$$\sum_{i=1}^{n} x_i = T \tag{5.3}$$

where C is the salvage price associated with the remaining inventory after the phase-out period.

r is the remaining inventory after the phase-out period, where $r \geq 0$

x_i is the decision variable that represents the duration of the particular discounted price P_i, where x_i are integers and $x_i \geq 0,\ i = 1,...,n$.

P_i is the ith. Using the policy of the firm under study, P_i is decreasing throughout the phase-out time horizon in a constant pre-determined manner, i.e., $P_1 > P_2 > ... > P_n > C$.

$V_i = \alpha P_i^{\beta}$ is the volume sold in one time period when discounted price P_i is applied. It follows that $V_1 < V_2 < ... < V_n$ due to $\alpha > 0$ and $\beta < -1$.

$R_i = \alpha P_i^{\beta+1}$ is the revenue in one time period associated with discounted price P_i. It follows from above that $R_1 < R_2 < ... < R_n$.

The problem defined above by equations (5.1) – (5.3), can be readily solved by MIP software such as Lingo (2008). However, since the number of constraints is only two, the aim is to find a closed form solution of the linear version of the problem, i.e., considering x_i to be continuous variables. The x_i values can then be rounded to the closest integer to obtain an appropriate solution. Given that there are only two constraints in the resulting linear programming problem, there will be only two basic variables in the optimal solution. This will result in two cases:

Case 1: r and x_j are the two basic variables

This set of equations can be denoted by $Bx_B = b$ where the vector of basic variables $x_B = \begin{bmatrix} x_j \\ r \end{bmatrix}$, the basis matrix $B = \begin{bmatrix} V_j & 1 \\ 1 & 0 \end{bmatrix}$, and $b = \begin{bmatrix} I \\ T \end{bmatrix}$. Therefore, the desired solution for the basic variables is $x_B = B^{-1}b \Rightarrow \begin{bmatrix} x_j \\ r \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -V_j \end{bmatrix} \begin{bmatrix} I \\ T \end{bmatrix} \Rightarrow x_j = T \geq 0$ and $r = I - V_j T \geq 0 \Rightarrow V_j \leq \frac{I}{T}$

Applying the optimality test on the reduced costs of the non-basic variables, the following is obtained:

$$[R_j \quad C] \begin{bmatrix} 0 & 1 \\ 1 & -V_j \end{bmatrix} \begin{bmatrix} V_i \\ 1 \end{bmatrix} - R_i \geq 0 \Rightarrow CV_i + R_j - CV_j - R_i \geq 0 \Rightarrow R_i - CV_i \leq R_j - CV_j .$$

The solution is guaranteed if $V_j \leq \frac{I}{T}$, which means that if $\frac{I}{T} < V_j$ for all $j \in \{1,...,n\}$, an infeasible solution results. This infeasibility is due to a lack of enough amount of inventory in this case.

Case 2: x_j and x_k are the two basic variables

Using the same notation for the set of equations, $Bx_B = b$, the vector of basic variables now becomes $x_B = \begin{bmatrix} x_j \\ x_k \end{bmatrix}$, the basis matrix $B = \begin{bmatrix} V_j & V_k \\ 1 & 1 \end{bmatrix}$, and $b = \begin{bmatrix} I \\ T \end{bmatrix}$. Therefore, the solution for the basic variables is

$$x_B = B^{-1}b \Rightarrow \begin{bmatrix} x_j \\ x_k \end{bmatrix} = \begin{bmatrix} \frac{1}{V_j - V_k} & \frac{-V_k}{V_j - V_k} \\ \frac{-1}{V_j - V_k} & \frac{V_j}{V_j - V_k} \end{bmatrix} \begin{bmatrix} I \\ T \end{bmatrix} \Rightarrow x_j = \frac{I - V_k T}{V_j - V_k} \geq 0 \text{ and } x_k = \frac{-I + V_j T}{V_j - V_k} \geq 0 .$$

Without loss of generality, let it be assumed that $V_j \leq V_k$.

Then $I - V_k T \le 0 \Rightarrow \frac{I}{T} \le V_k$ and $-I + V_j T \le 0 \Rightarrow V_j \le \frac{I}{T}$. Therefore, $V_j \le \frac{I}{T} \le V_k$.

Again, consistent with the previous case, if there does not exist a j such that $V_j \le \frac{I}{T}$, there is no feasible solution.

If $\frac{I}{T} > V_k$, for all $k \in \{1,...,n\}$, then this is the case of excessive inventory, and it would be impossible to deplete all of it, by discounted prices; as a result, there will always be remaining inventory at the end of the phase-out period.

Applying the optimality test on the reduced costs of the non-basic variables, the following is obtained:

$$[R_j \quad R_k]\begin{bmatrix} \frac{1}{V_j - V_k} & \frac{-V_k}{V_j - V_k} \\ \frac{-1}{V_j - V_k} & \frac{V_j}{V_j - V_k} \end{bmatrix}\begin{bmatrix} V_i \\ 1 \end{bmatrix} - R_i \ge 0.$$

Again, without loss of generality, assuming that $V_j \le V_k$, implies

$$\frac{R_j V_i - R_k V_i}{V_j - V_k} + \frac{R_k V_j - R_j V_k}{V_j - V_k} - R_i \ge 0 \Rightarrow \frac{R_j V_i - R_k V_i}{V_j - V_k} + \frac{R_k V_j - R_j V_k}{V_j - V_k} \ge 0$$

$$\Rightarrow R_j V_i - R_k V_i + R_k V_j - R_j V_k \le 0 \Rightarrow V_i \le \frac{R_j V_k - R_k V_j}{R_k - R_j}$$

And $[R_j \quad R_k]\begin{bmatrix} \frac{1}{V_j - V_k} & \frac{-V_k}{V_j - V_k} \\ \frac{-1}{V_j - V_k} & \frac{V_j}{V_j - V_k} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} - C \ge 0 \Rightarrow \frac{R_j - R_k}{V_j - V_k} - C \ge 0_j$

$$\Rightarrow C \le \frac{R_j - R_k}{V_j - V_k}$$

Based on this optimality condition, the objective was to prove that V_j and V_k represent the volume of two consecutive prices, respectively, and that any value V_i will either fall below V_j or above V_k. Consider any three consecutive prices, say P_1, P_2, P_3. It is sufficient to show that the following lemma holds.

Lemma 1. *For* $P_1 \geq P_2 \geq P_3$, *where* $V_i = \alpha P_i^{\beta}$, $R_i = \alpha P_i^{\beta+1}$, $i = 1, 2, 3$, *and* $\beta < -1$, *the incremental rate of return is decreasing with i , or* $\frac{R_2 - R_1}{V_2 - V_1} \geq \frac{R_3 - R_2}{V_3 - V_2} \geq 0.$

Proof. Let $\gamma = \frac{\beta + 1}{\beta}$. Since β < -1 $\Rightarrow \beta + 1 < 0$ and $\frac{\beta + 1}{\beta} > 0$

Furthermore, $\beta + 1 > \beta \Rightarrow \frac{\beta + 1}{\beta} < 1$. Therefore, for β < -1 , 0 < γ < 1.

Consider the transformation $Y_i = P_i^{\beta}$, i = 1, 2, 3. Since $P_1 \geq P_2 \geq P_3$, $Y_1 \leq Y_2 \leq Y_3$.

Substituting γ for β and using the transformation, the inequality $\frac{R_2 - R_1}{V_2 - V_1} \geq \frac{R_3 - R_2}{V_3 - V_2} \geq 0$ becomes

$\frac{Y_2^{\gamma} - Y_1^{\gamma}}{Y_2 - Y_1} \geq \frac{Y_3^{\gamma} - Y_2^{\gamma}}{Y_3 - Y_2} \geq 0$, which holds because Y^{γ} is a concave function of Y (γ is a constant).

The results from this optimality condition along with the model and its constraints (after being converted to "≤"), fit into the well known linear multiple choice knapsack problem (LMCK). This problem has been widely investigated (Lin, 1998) and (Kozanidis et al., 2002 & 2004). However, what makes this a unique situation is the absence of dominated variables, i.e., all the points (V_i, R_i), when connected, form a piecewise linear concave function and are part of the non-dominated frontier illustrated in Figure 11. The data and results used to obtain this figure will be shown in chapter 6.

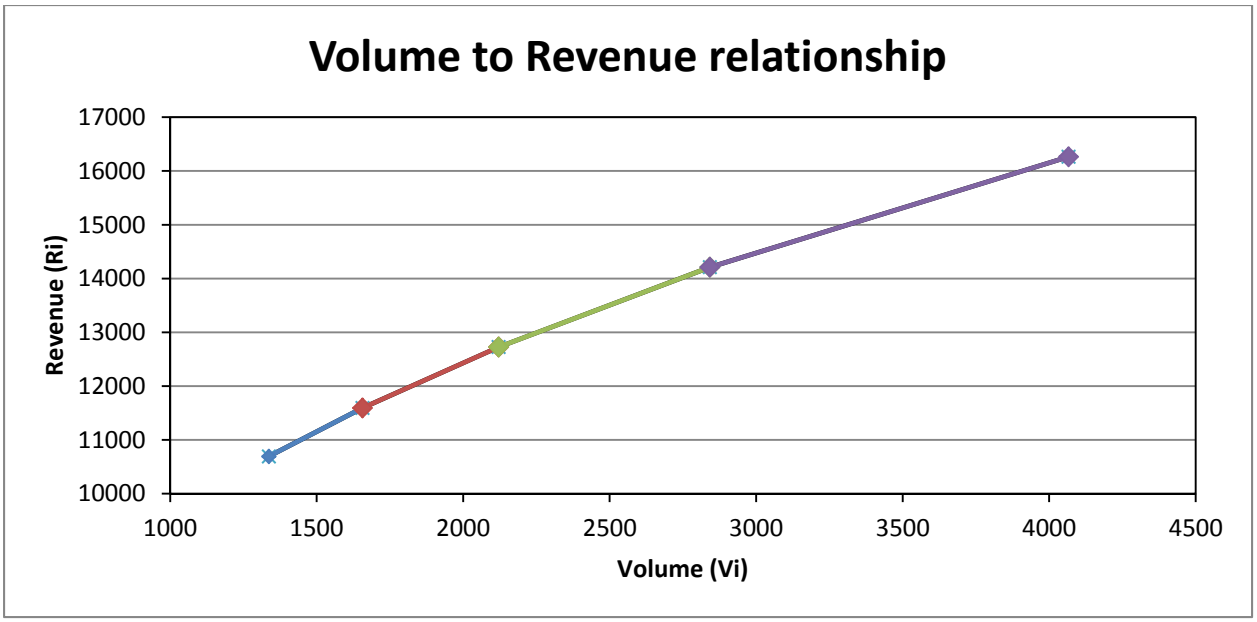


Figure 11: Rate of change of revenue with respect to change in volume

In addition to fit into the LMCK problem, the conversion of equality constraints (5.2) and (5.3) to "≤" type, eliminates the earlier mentioned infeasibilities and generalizes the model for any inventory value, even small enough that may be sold in a fraction of the phase-out period *T*.

In summary, knowing the initial inventory "*I*" to be depleted, the phase-out period "*T*" to deplete it by, and the different discount prices, the following algorithm provides the discount prices to use and for how long (x_i) as well as the remaining inventory to sell at salvage price so that revenue is maximized.

Algorithm 1.

Step 1. *Compute* $s_i = \dfrac{R_{i+1} - R_i}{V_{i+1} - V_i}$, $i = 1, \ldots, n\text{-}1$ (*rate of change of the revenue with respect to the change in volume).* *Let* $\arg\max_i \{V_i \mid V_i \le \dfrac{I}{T}\} = k$ *and* $\arg\max_i \{s_i \mid s_i \ge C\} = j$.

Step 2. If $k = 0$, *set* $x_1 = \dfrac{I}{V_1}$; $x_i = 0,\ i \ne 1, r = I - TV_1$,

else if $1 \le j \le k$, *set* $x_j = T$; $x_i = 0,\ i \ne j, r = I - TV_j$,

else, set $x_k = \dfrac{I - TV_k}{V_{k+1} - V_k}$; $x_{k+1} = \dfrac{TV_{k+1} - I}{V_{k+1} - V_k}$; $x_i = 0,\ i \ne k, k+1; r = 0$.

End if

Experimental results in the next chapter were obtained through lingo and MS Excel that verified the accuracy of the above algorithm.

5.2 Determining the Optimal Collection Period in the Reverse Logistics model

The optimal collection period can be analytically determined by focusing on the two major cost components of the model, which depend on the collection period T: inventory carrying cost and transportation cost. To simplify the analysis, it is assumed that there is no penalty imposed on the travel distance between the ICPs and the CRCs. Equivalently, the penalty rate has already been included into the volume discount rate, α_{jk}, for shipments between ICP j and CRC k. The more general case can be considered when there are n breakpoints for freight volume discounts, $P_1, P_2, \ldots, P_n$ (see Figure 11).

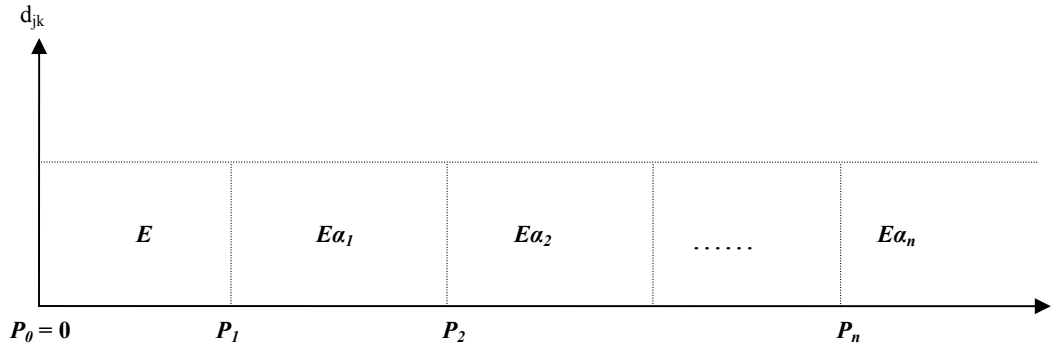


Figure 12: Simplified unit transportation cost function

The inventory carrying cost is a linear function of the number of collection days (T) at the ICPs, $bw\left(\frac{T+1}{2}\right)\sum_{i\in I} r_i$. Therefore, as T increases, the inventory carrying cost difference (slope) is kept constant at $\left(\frac{bw}{2}\right)\sum_{i\in I} r_i$. The transportation cost, on the other hand, is a function of the volume of products shipped from the ICPs to the CRCs, which in turn depends on the daily volume returned from customers to the ICPs and the number of collection days, T. Moreover, depending on the quantity returned from a particular ICP to a CRC, a discount rate may be applied.

Substituting $f(X_{jk}, d_{jk})$ for $E\alpha_{jk}$ in the transportation cost component of the objective function (1), multiplying by $T\sum_{i\in I} r_i$ and dividing by its equal, $\sum_{(j,k)} X_{jk}$, results in the following expression that provides an intuitive interpretation of shipping costs:

$$\frac{T\sum_{i\in I} r_i}{\sum_{j,k} X_{jk}} \frac{w}{T}\sum_{j,k}(E\alpha_{jk})X_{jk} = \left[\frac{\sum_{j,k}(E\alpha_{jk})X_{jk}}{\sum_{j,k} X_{jk}}\right]\left(w\sum_{i\in I} r_i\right) = (WA)_T\left(w\sum_{i\in I} r_i\right)$$

In other words, the annual transportation cost can be expressed as the annual number of returned products multiplied by the effective unit transportation cost, $(WA)_T$, which is the weighted

average of the unit transportation costs ($E\alpha_{jk}$). The weights are the shipment volumes X_{jk}. It is worth noting that although T was cancelled above, $(WA)_T$ depends on T through X_{jk} and eventually α_{jk}. In particular, $(WA)_T$ is a non-increasing function of T, since larger shipments X_{jk} cannot increase the unit transportation cost but they may decrease it whenever X_{jk} moves to a new range of consecutive discount breakpoints.

For two consecutive integer values of T, the difference between the corresponding transportation costs is $[(WA)_{T+1} - (WA)_T](w\sum_{i\in I} r_i)$. This indicates that in order to determine the change in the combined cost, as the collection period increases by one day, it is sufficient to compare the constant slope of the change in inventory cost noted earlier, $\left(\frac{bw}{2}\right)\sum_{i\in I} r_i$, to the above non- positive change in the weighted average of the transportation cost.

5.2.1 Special Structures of the Optimal Collection Period Problem

This section addresses specific scenarios in the reverse logistics model of the dissertation by considering the following: multiple customers are returning products to multiple ICPs and thereafter, products are shipped to multiple CRCs; however, customers are divided into groups (clusters) where each cluster is within the geographical area of a single ICP. In addition, no capacity is imposed on the CRCs and fixed costs at the ICPs and CRCs are considered negligible.

Equivalently, it may be assumed that customers have already been assigned to ICPs and ICPs have been assigned to CRCs. The only decision to be made is to determine the number of collection days at each ICP. Under the above scenario, the problem can be decomposed into

smaller sub-problems, each one involving an individual cluster of customers with one ICP and one CRC. This special structure of a reverse logistics problem for product returns allows a closed form solution for the optimal collection period T to be found. For notational simplicity, the total daily volume of product returns from all customers assigned to an ICP $(\sum_{i \in I} r_i)$, will be denoted by R.

A Discrete Time Collection Period Model

If the total daily volume of product returns is greater than the largest shipping volume breakpoint, $R > P_n$, then the slope of the transportation function is $E\alpha_n$ and the optimal collection period is $T = 1$. If the total daily volume of product returns R falls between two shipping volume breakpoints, i.e. $P_{l-1} < R \leq P_l$ for l = 1, …, n, then the transportation cost will always decrease with an increase in the collection period that will move the shipping volume beyond P_l, realizing more shipping economies of scale. If a change in the transportation cost in absolute value is larger than the change in inventory carrying cost, then the total cost will decrease from time period T to time period T+1. The weighted average of the unit transportation cost becomes less effective as it moves from one quadrant to the other (Figure 11).

Without loss of generality, the total daily return rate R is less than the first shipping volume breakpoint $0 \leq R < P_1$. The following analysis for determining the optimal collection period can be applied for any $R \leq P_n$. The minimum length of the collection period T is one. At $T = 1$, the weighted average is equal to E, the transportation cost is EwR and the inventory carrying cost is bwR. The first time the economies of scale are implemented is at $[P_1 / R]$, where [.] is the ceiling function. For integer values of T such that $1 < T < [\mathrm{P}_1 / R]$, the transportation cost

remains unchanged at EwR while the inventory cost increases linearly with $bwR\left(\frac{T+1}{2}\right)$. Since the total cost is an increasing functions of T, collection periods such that $1 < T < [P_1 / R]$ are not optimal. The next value of interest after $T = 1$ is $T = [P_1 / R]$.

In general, the collection period at $T = [P_l / R]$ is better than collection periods T, such that $[P_l / R] < T < [P_{l+1} / R]$, $l = 1, \ldots, n-1$, and the collection period at $T = [P_n / R]$ is better than the ones for $T > [P_n / R]$ because the total cost is an increasing function of T within each shipping range, $bwR\left(\frac{T+1}{2}\right) + E\alpha_l wR$, $l = 1, \ldots, n-1$. Therefore, the following theorem holds:

Theorem 2. *For the special structure of a single ICP and CRC, the optimal collection period can be either at* 1 *or at* $[P_l / R]$, $l = 1 \ldots n$.

The above theorem suggests a straightforward procedure for finding the optimal collection period by comparing the total cost at only $n + 1$ values of T. Experimental results indicate that the optimal collection period often occurs in the two extreme candidate values of T, i.e., 1 and $[P_n / R]$. The range of values of the daily collection amount R is found below.

Based on Theorem 1, the optimality condition for $T = 1$ is

$EwR + bwR \le \frac{b}{2}([P_l / R] + 1)wR + E\alpha_l wR$, $l = 1, \ldots, n$. After simplification, the above inequality reduces to $[P_l / R] \ge 2E\left(\frac{1-\alpha_l}{b}\right) + 1$, $l = 1, \ldots, n$. Noting that $[P_l / R]$ is positive integer,

$[P_l / R] \geq \left\lceil 2E\left(\frac{1-\alpha_l}{b}\right)+1 \right\rceil$, l = 1, …, n and solving for R to obtain $R \leq R_L$, where

$$R_L = \min_{l=1,\ldots,n} \frac{P_l}{\left\lceil 2E\left(\frac{1-\alpha_l}{b}\right)+1 \right\rceil} \tag{5.4}$$

Similarly, the optimality conditions for $T = [P_n / R]$ are

$\frac{b}{2}([P_n / R]+1)wR + E\alpha_n wR \leq \frac{b}{2}([P_l / R]+1)wR + E\alpha_l wR$, $l = 1, \ldots, n-1$,

and $\frac{b}{2}([P_n / R]+1)wR + E\alpha_n wR \leq EwR + bwR$. These conditions reduce to $R \geq R_U$, where R_U is the maximum value of R satisfying

$$\left.\begin{array}{l} [P_n / R] \leq \left\lfloor 2E\left(\frac{\alpha_l - \alpha_n}{b}\right)\right\rfloor + [P_l / R] \\ R \geq \dfrac{P_n}{\left\lfloor 2E\left(\frac{1-\alpha_n}{b}\right)+1 \right\rfloor} \end{array}\right\} \tag{5.5}$$

The above analysis leads to the following corollary for locating the optimal collection period:

Corollary 2. *For the special structure of a single ICP and a single CRC, the optimal collection*

$$\textit{period is } T^* = \begin{cases} 1, & \textit{if} \quad R \leq R_L \\ [P_n / R] \ , & \textit{if} \quad R \geq R_U \\ [P_l / R] \ , \ \ l = 1,\ldots,n-1 & \textit{if} \quad R_L < R < R_U \end{cases}$$

Where R_L and R_U are given by (12) and (13), respectively. It should be noted that if the collection period has an upper bound $\overline{T}$, as defined earlier, the above corollary can be adjusted by replacing n with $\bar{l}$, where $\frac{P_{\bar{l}}}{R} \leq \overline{T} \leq \frac{P_{\bar{l}+1}}{R}$.

The above corollary provides a straightforward method to check the following: (1) if the shipping economies of scale are not applicable because the total daily volume of product returns R is below the lower critical value (R_L), (2) the advantage of full shipping economies of scale can be taken because R is beyond the upper critical value (R_U), (3) or when partial shipping economies of scale can be used because R is between the lower and the upper critical values. A noteworthy observation is that the two critical values depend on the breakpoints at which freight rate discounts occur (P_l) and on the ratio of the shipping savings rate over the unit inventory carrying cost $\left(\frac{1-\alpha_l}{b}\right)$ and the analogous ratio of the relative shipping rate over the unit inventory carrying cost $\left(\frac{\alpha_l - \alpha_n}{b}\right)$.

A Continuous Time Collection Period Model

The collection period T is now considered to be continuous in the interval $[1, \overline{T}]$ and each customer i returns products to a single ICP at a uniform rate r_i. If Q is the shipment quantity, then $Q = TR$ products are shipped to a single CRC during every collection period T. Let Q fall within range of breakpoint shipment levels, say $P_l < Q \le P_{l+1}$. Then $T \in [1, \overline{T}]$ implies that integer index $l \in [\underline{l}, \overline{l}]$, where $\left\{ \begin{array}{l} \underline{l} = l \mid \frac{P_l}{R} < 1 \le \frac{P_{l+1}}{R} \\ \overline{l} = l \mid \frac{P_l}{R} < \overline{T} \le \frac{P_{l+1}}{R} \end{array} \right\}$ and $P_0 = 0$. The total annual cost (including inventory carrying cost and transportation cost only) is given by $\frac{bwQ}{2} + \frac{QE\alpha_l w}{T}$. After substituting Q in terms of T, the total cost TC reduces to a piece-wise linear function of T with constant slope, $bwR/2$, and discontinuities (jumps) at P_l / R:

$TC = (bwR/2)T + E\alpha_l wR$, $\quad P_l / R < T \le P_{l+1} / R$, $\underline{l} \le l \le \bar{l}$

At $T = 1$, the total cost is $bwR/2 + E\alpha_{\underline{l}} wR$. The lowest value of the total cost within each shipping range is obtained at the leftmost point, $T_l = P_l / R + \varepsilon$, $\underline{l}+1 \le l \le \bar{l}$, where ε is an infinitesimal quantity, practically zero. Therefore, the following theorem holds:

Theorem 3. *For the continuous collection period model with shipping economies of scale, the optimal collection period occurs at either* $T = 1$*, or at one of the critical collection periods* T_l*,* $\underline{l}+1 \le l \le \bar{l}$.

As discussed earlier, the total daily volume of product returns R determines if no shipping discounts are used (T = 1), full shipping discounts are used $(T = T_{\bar{l}})$, or partial shipping discounts are used $(T = T_l, \underline{l}+1 \le l \le \bar{l}-1)$ under the optimal policy. A similar analysis yields the following:

Let $R_L = \min\limits_{\underline{l}+1 \le l \le \bar{l}} \dfrac{P_l}{2E\left(\dfrac{\alpha_{\underline{l}} - \alpha_l}{b}\right)+1}$, and $R_U = \max\left\{ \max\limits_{\underline{l}+1 \le l \le \bar{l}-1} \dfrac{P_{\bar{l}} - P_l}{2E\left(\dfrac{\alpha_l - \alpha_{\bar{l}}}{b}\right)}, \dfrac{P_{\bar{l}}}{2E\left(\dfrac{\alpha_{\underline{l}} - \alpha_{\bar{l}}}{b}\right)+1} \right\}$

Then the following corollary holds:

Corollary 3. *The optimal collection period is 1 for* $R < R_L$ *and* $T_{\bar{l}}$ *for* $R > R_U$.

For $R_L \le R \le R_U$, the optimal collection period can be at T_l, $l = \underline{l}+1, \ldots, \bar{l}-1$.

Chapter 6

Computational Results

This chapter thoroughly analyzes all the computational results attained using the aforementioned models. A complete breakdown, along with an extensive sensitivity analysis was conducted using multiple tools and software. These programs included Microsoft Excel, Lingo 11, MATLAB, SPSS, Minitab, Mathematica, and AutoCAD. The results are divided into four different sections as detailed below. This process starts by finding an optimal markdown strategy to deplete an initial inventory and extends to the identification of the optimal collection period of the remaining returned products.

6.1 Clustering Procedures and Regression Analysis

The first section of this chapter highlights the usefulness and the implementation of the clustering procedure in the different levels of the process. Furthermore, regression analysis methods were used to find the best fit of sales-price data to a demand-price function. A fitting start is to first describe the source and the type of the data. The data are obsolete inventory items in retail stores, such as cosmetic items in the pharmacy X, distributed by manufacturer Y. There are over 650 SKUs in more than 6,000 stores. The total sales volume and the total sales amount were provided for a period of 54 weeks. Therefore, there was a need to sieve through the data and use scientific methods to break it down. The first step taken was to analyze the largest

possible amount of SKUs and then to narrow down the selection to the ones where a "normal" relationship between volume and demand is found. This can be accomplished by breaking down the geographical areas based on the states where the pharmacy X stores are located, which are in 41 states. The next step was to connect the stores from the different states, which share the same distribution centers. After careful analysis and calculation of all excess inventories within these stores, the focus was then shifted to the individual SKUs. This process was thoroughly conducted until the final selection of 6 SKUs. The following table shows the 6 SKUs selected for the detailed analysis.

Table 3: Final selection of SKUs to be analyzed

Pharmacy X SKU Number	ITEM DESCRIPTION
111111	LIP 555
111112	LIP 560
111113	LIP 587
111114	LIP 701
111115	LASH 830
111116	LASH 835

Next, the relationship between the total units sold and the total amount sold for 54 weeks was analyzed for each SKU, by calculating the unit prices and drawing all different types of comparisons between the unit prices and the units sold. Different types of regression were used, and the power regression proved to be the best fit for all the SKUs analyzed. In addition, a clustering algorithm was used to come up with the relevant unit prices to describe all sales. The following is a breakdown of an example SKU and the steps involved in the process just mentioned.

For illustration, the last SKU in Table 3, number "111116", is considered below. Figure 13displays a graph showing the relationship between the unit prices and the total sales volume for the entire 54 week period.

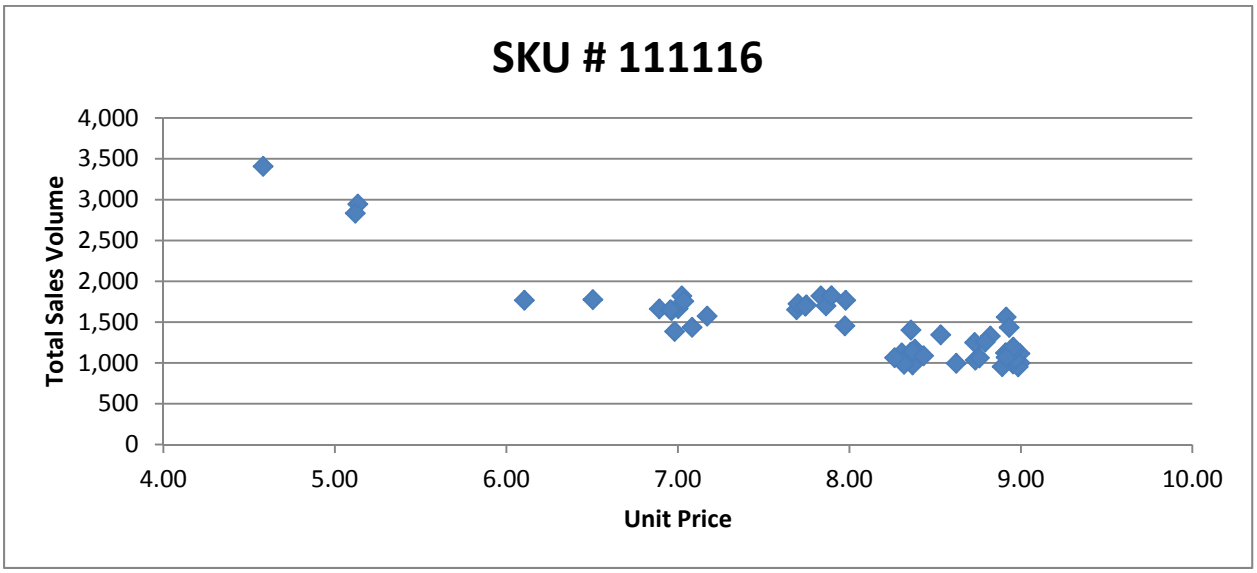


Figure 13: Unit price vs. total sales volume for a particular SKU

As Figure 13 shows, there are very few data points for low unit prices. This was true for other SKUs as well. It was evident therefore that there was a need to cluster the data to determine a better understanding of the relationship between price and demand. Thus, a *k*-means algorithm was used where the number of clusters *k*, had to be chosen first. The cluster centers in this case are represented by the unit prices. Using $k = 9$, the clustered data are shown in Table 4 and Figure 14, respectively, around 9 different unit prices.

Table 4: K-means algorithm results

Cluster Means (Unit Price)	Total Sales Volume
4.58	3,407
5.13	2,890
6.10	1,767
6.50	1,775
7.01	1,621
7.83	1,705
8.37	1,142
8.74	1,155
8.95	1,132

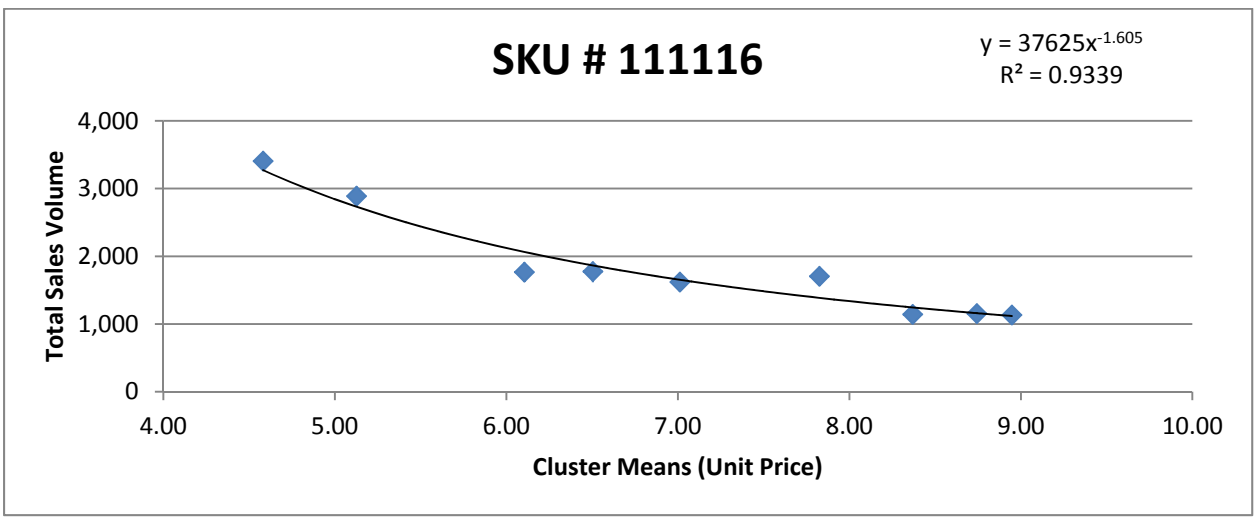


Figure 14: Cluster means (unit price) vs. total sales volume for a particular SKU

The same procedure was followed for all the SKUs under study. The next step was to try to figure out what resulted in the best fit model. As mentioned before, the power regression analysis showed a better fit. The power equation for SKU # 111116 is shown in Figure 14.

Demand as a function of price according to the equations obtained from the regression analysis, fit into the family of elasticity of demand equations. Thus, the general form for the demand function can be written as $V(p) = \alpha P^{\beta}$, where P is the unit price and V is the total sales volume. β is the price elasticity of demand constant and α is a positive constant.

If $\beta = 0$ then $V = \alpha$, this is the case of a perfectly inelastic condition. If $\alpha = 1$ then $V = P^{\beta}$, this will represent the standardized form of the equation. Normalizing the equations, and using them to see how the volume changes by systematically decreasing to 40 percent of its original price, the behavior was observed and compared amongst all the different SKUs. Table 5 and Figure 15 below show the results.

Table 5: As Unit price decreases, volume increases

SKU	111116	111115	111114	111113	111112	111111
UP ↘	TSV ↗	TSV ↗	TSV ↗	TSV ↗	TSV ↗	TSV ↗
1.00	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
0.9	1.18	1.20	1.31	1.29	1.30	1.28
0.8	1.43	1.48	1.78	1.71	1.74	1.68
0.7	1.77	1.88	2.50	2.36	2.43	2.29
0.6	2.27	2.46	3.72	3.42	3.57	3.28
0.5	3.04	3.40	5.95	5.31	5.63	5.00
0.4	4.35	5.03	10.57	9.08	9.82	8.40
	y = x^(-1.605)	**y = x^(-1.764)**	**y = x^(-2.574)**	**y = x^(-2.408)**	**y = x^(-2.493)**	**y = x^(-2.323)**

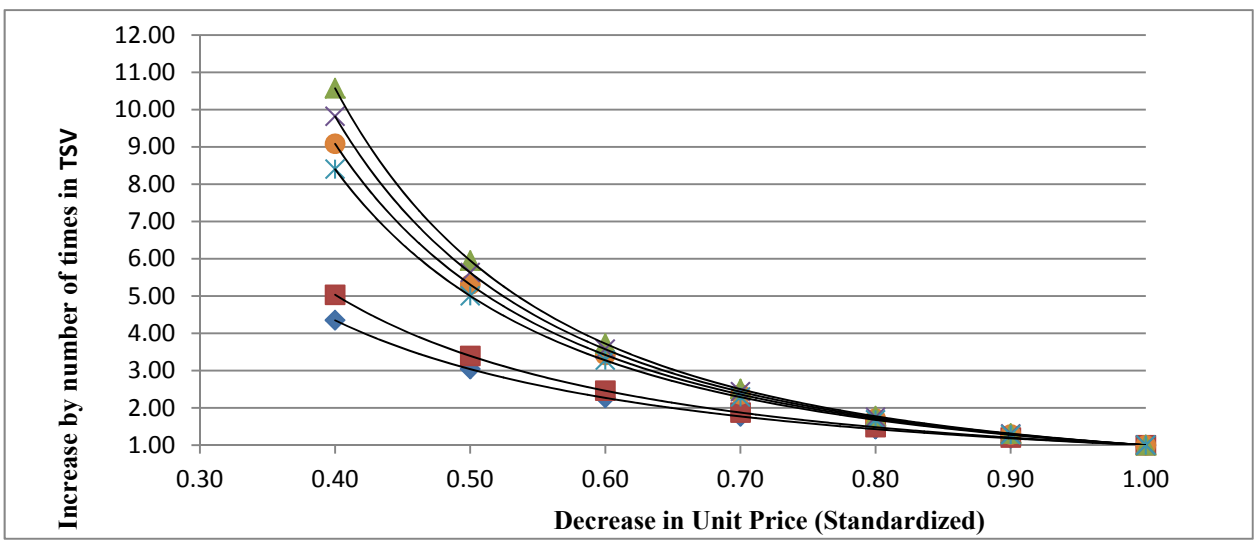


Figure 15: Comparison of the power functions of the different SKUs

Clustering Procedures and Regressions Analysis served as the stepping stone to develop the mathematical models and addresses the various inventory issues.

6.2 Inventory depletion and markdown strategies within a phase-out period

This particular phase of the dissertation focuses on the results obtained from the inventory and markdown strategy models presented in the previous chapters. The first assumption made is that SKU # 111116 has been phased-out due to manufacturer's decision or due to lack of sales, and that the phase-out period is pre-determined. The second assumption is that the phase-out period is 9 weeks. Starting with that initial inventory at the beginning of the phase-out period, the first problem is to determine by how much to discount through every period to completely deplete the entire initial inventory by the end of the nine weeks.

Maximizing the revenue based on the model $\sum_{t=1}^{T} \alpha P_t^{\beta+1}$ that is subject to: $\sum_{t=1}^{T} \alpha P_t^{\beta} = I$

Where t = 1, 2 …, T are the different time periods, "I" is the initial inventory to be depleted, and alpha and beta are determined from the power regression analysis done in the previous section of this chapter. For SKU # 111116, α = 37,625 and β = -1.605, and I = 20742 units.

Solving the model through Lingo, the optimal solution resulted in the same price for every time period throughout the phase-out process. That optimal price is calculated by the following equation: $P_t = \left(\frac{I}{T\alpha}\right)^{1/\beta}$, which will lead to maximum revenue of $R_{Max} = \frac{(I)^{\frac{\beta+1}{\beta}}}{(T\alpha)^{1/\beta}}$.

For SKU # 111116, the price obtained for every time period is $5.70, which will result in total revenue of $118,237. Table 6 shows a summary of the optimal prices and maximum revenues for all 6 SKUs.

Table 6: Optimal prices / maximum revenue at the end of the phase-out period

SKU #	α	β	Initial Inventory	Optimal Price	Max Revenue
111116	37625	-1.605	20,742	5.70	**118,237**
111115	69700	-1.764	28,726	5.75	**165,043**
111114	10325	-2.408	3,205	4.05	**12,975**
111113	16946	-2.574	3,930	4.14	**16,284**
111112	12941	-2.493	3,188	4.24	**13,502**
111111	11362	-2.323	4,058	4.01	**16,279**

These results are attained only in the case of a complete depletion of an initial inventory of a particular SKU over a phase-out period given a price elasticity of demand type of function.

In the case that the initial inventory is greater than the total units sold at the end of *T* periods, other options to handle the remaining inventory (*r*) can be considered. One option is to attempt to sell the remaining inventory at some salvage price to a third party wholesaler. The revenue from selling the remaining inventory is now added to the objective function. Furthermore, the remaining inventory becomes part of the constraint function.

Objective function:

Maximize $$\sum_t \alpha P_t^{\beta+1} + Cr \tag{6.1}$$

Subject to: $$\sum_t \alpha P_t^{\beta} + r = I \tag{6.2}$$

where *C* is the salvage price associated with the remaining inventory *r* for a particular SKU, and *I* is the initial inventory to be depleted, and $P_t \leq P_{t-1}$, $t = 2, \ldots, n$.

The solution now results in a new optimal price, which is still identical for every time period and is calculated according to the following equation: $P_t = \dfrac{C\beta}{\beta+1}$, $t = 2, \ldots, n$.

The solution demonstrates that selling the remaining inventory to a third party wholesaler is only beneficial if the salvage price is greater than a certain value: $C > \left(\frac{\beta+1}{\beta}\right)\left(\frac{I}{T\alpha}\right)^{1/\beta}$.

Table 7 presents the minimum values of the required salvage price for all 6 SKUs:

Table 7: Lowest values of salvage price C

SKU #	Minimum Salvage Price required
111116	2.15
111115	2.49
111114	2.37
111113	2.53
111112	2.54
111111	2.28

Finding the lowest value that the salvage price can take proves to be very crucial for the retailer, since the salvage price at which the remaining inventory is sold can be determined. The optimal price for every time period during the phase-out is a function of the salvage price, which means that as C increases, so does P; naturally, this will also increase the maximum revenue. However, it is not reasonable to have a salvage price higher than the actual price of the item being sold during the phase-out period. Therefore, returning the products to the manufacturer / distributor is the next option to evaluate in the next section of this chapter.

However, before tackling the reverse logistics model for the return products, a mathematical model was developed, which addresses the inventory problem from a different perspective. If the markdown prices are set in advance and they are fixed, the question becomes: when to use them and for how many periods during the phase-out time horizon? With the initial inventory remaining the same and the length of the phase-out period also unchanged, it was important to resolve when to use the pre-determined markdown prices and for how long. By revamping the model, and using the same power regression function, the following was obtained:

Maximize $$\sum_{i=1}^{n} R_i x_i + Cr \tag{6.3}$$

Subject to: $$\sum_{i=1}^{n} V_i x_i + r = I \tag{6.4}$$

$$\sum_{i=1}^{n} x_i = T \tag{6.5}$$

where, C is the salvage price associated with the remaining inventory after the phase-out period,

r is the remaining inventory after the phase-out period, where $r \geq 0$,

x_i is the decision variable that represents the duration of the particular discounted price P_i, where x_i are integers and $x_i \geq 0$, $i = 1,...,n$.

P_i is the discounted price throughout the different phase-out periods. Using the policy of the firm under study, P_i is decreasing throughout the phase-out period in a constant pre-determined manner, where: $P_1 > P_2 > ... > P_n > C$.

Considering the same SKU # 111116, Table 8 shows the pre-determined prices for the markdown strategy:

Table 8: Pre-determined markdown prices

P1	P2	P3	P4	P5
8	7	6	5	4

The two factors that determined where the solution was going to fall within the 9 week phase-out period are the inventory / time period (I/T) ratio, and the salvage price (C), as explained in Chapter 4.

Using the same initial inventory (I = 20742) with a total phase-out time period of 9 weeks, and with a salvage price that varies from 0 to $4, the following table shows the results of where the solution falls, along with the maximum revenue.

Table 9: Determining when and for how long to use the markdown prices

C	x1	x2	x3	x4	x5	Remaining inventory	**Maximum Revenue**
4.00	9	0	0	0	0	8,712	**131,087**
3.00	9	0	0	0	0	8,712	**122,375**
2.50	0	9	0	0	0	5,837	**118,928**
2.00	0	0	6.71	2.29	0	0	**117,937**
1.50	0	0	6.71	2.29	0	0	**117,937**
1.00	0	0	6.71	2.29	0	0	**117,937**
0.50	0	0	6.71	2.29	0	0	**117,937**
0.00	0	0	6.71	2.29	0	0	**117,937**

This analysis was performed for different values of initial inventories, and for all 6 SKUs. The optimal decision depends on the ratio I/T with regard to the total sales volume, and on the salvage price (as shown in Chapter 4). The significance of the solution and its closed form is the ability to predict in which periods the markdown strategy should be implemented, for how long, and whether to keep any remaining inventory. In addition, it guarantees that the solution always falls between two consecutive time periods. The following Table 10 depicts model results for the above analyzed SKU # 393543, and selected initial inventory levels (*I*). The results were obtained by algorithm 1 from chapter 5 and were verified by running the Lingo implementation of the model given equations (5.1) – (5.3).

Table 10: Model results using I/T and C

Phase-out Period (T)				
9				
Price	Volume	Initial Inventory (I)	I / T	Set of Slopes
8	1,337	12,030	1,336.67	8.00
7	1,656	15,000	1,666.67	2.82
6	2,121	20,742	2,304.67	2.43
5	2,842	30,000	3,333.33	2.05
4	4,066	36,595	4,066.11	1.68

Salvage Price	x1	x2	x3	x4	x5	Remaining Inv.	Objective Function
4.0	9.00	0.00	0.00	0.00	0.00	0.17	96,239
3.0	9.00	0.00	0.00	0.00	0.00	0.17	96,239
2.0	9.00	0.00	0.00	0.00	0.00	0.00	96,239
1.0	9.00	0.00	0.00	0.00	0.00	0.00	96,239
0.0	9.00	0.00	0.00	0.00	0.00	0.00	96,239
4.0	9.00	0.00	0.00	0.00	0.00	2970.17	108,119
3.0	9.00	0.00	0.00	0.00	0.00	2970.17	105,149
2.0	0.00	8.80	0.20	0.00	0.00	0.00	104,567
1.0	0.00	8.80	0.20	0.00	0.00	0.00	104,567
0.0	0.00	8.80	0.20	0.00	0.00	0.00	104,567
4.0	9.00	0.00	0.00	0.00	0.00	8712.17	131,087
3.0	9.00	0.00	0.00	0.00	0.00	8712.17	122,375
2.0	0.00	0.00	6.71	2.29	0.00	0.00	117,937
1.0	0.00	0.00	6.71	2.29	0.00	0.00	117,937
0.0	0.00	0.00	6.71	2.29	0.00	0.00	117,937
4.0	9.00	0.00	0.00	0.00	0.00	17970.17	168,119
3.0	9.00	0.00	0.00	0.00	0.00	17970.17	150,149
2.0	0.00	0.00	0.00	9.00	0.00	4421.67	136,735
1.0	0.00	0.00	0.00	5.39	3.61	0.00	135,312
0.0	0.00	0.00	0.00	5.39	3.61	0.00	135,312
4.0	9.00	0.00	0.00	0.00	0.00	24565.17	194,499
3.0	9.00	0.00	0.00	0.00	0.00	24565.17	169,934
2.0	0.00	0.00	0.00	9.00	0.00	11016.67	149,925
1.0	0.00	0.00	0.00	0.00	9.00	0.76	146,378
0.0	0.00	0.00	0.00	0.00	9.00	0.76	146,377

It may be impossible to avoid a remaining inventory when using all the models discussed in this section, thus the need to develop a return product strategy. The following section of this Chapter addresses the issues that result from a reverse logistics model and the outcome attained by solving the developed model.

6.3 Computational Results of the Reverse Logistics Model

After examining the different inventory and markdown strategy models, this section will now address the option of returning the remaining inventory back to the distributor or the manufacturer. When faced with the possibility of being unable to deplete the entire initial inventory of the items selected to be phased-out, the option of selling the remaining inventory to a third party retailer with a salvage price was explored.

This section begins by first presenting the required assumptions and requirements to develop a reverse logistics model. The data analyzed in the previous two sections of this chapter was drawn from over 6,200 stores, from all over the U.S. Therefore, the first assumption was that the model was handling a particular geographical area. Next, every pharmacy X store was treated as a customer, and from there, the remaining inventory was collected and taken to a warehouse (initial collection point). The items would then be returned to the manufacturer (centralized return centers). The geographical location of the customers, initial collection points, and centralized return centers were randomly selected within a particular geographical location. Of course the exact location of the pharmacy X stores and P&G warehouses could have been used, but it was more effective to make the problem more general and not limit it to those specific locations.

The next step was to break down all the different costs associated with the objective function. As it was shown in Chapter 4, the total reverse logistics costs comprised five annual cost components: the cost of renting the initial collection points (ICPs), the cost of establishing and maintaining the centralized return centers (CRCs), the handling costs at the ICPs, the inventory carrying cost, and the transportation cost.

The model also assures that a customer is assigned to a single initial collection point, making the incoming flow equal to the outgoing flow at each initial collection point, and ensuring that the total volume of products shipped from initial collection points to a centralized return center does not exceed the maximum capacity of the centralized return center.

The computational results that are presented here are based on a sample of 30 customers (pharmacy X stores), 10 ICPs, and 5 CRCs. The remainder of the input parameters is presented in Table 11:

Table 11: Input parameters to the reverse logistics model

Parameter	***Symbol***	***value***	***Unit***
Annual Cost of renting an initial collection point	a_j	*200*	$
Daily Inventory carrying cost per unit	b	*0.1*	$ / day-unit
Working days per year	w	*250*	day
Unit handling cost at the collection point	h_j	*0.1*	$ / day-unit
Anualized cost of establishing & maintaining a CRC	c_k	*3000*	$
Capacity of a centralized return center / day	m	*1000*	units / day
Service Coverage	l_{ij}	*25*	miles
Unit standard transportation cost	E	*1*	--
Discount rate with respect to shipping volume between ICPs and CRCs	α_1	0.8	--
	α_2	0.6	--
Shipping Volumes between ICPs and CRCs	P_1	*200*	units
	P_2	*400*	units
Penalty rate with respect to distance between ICPs and CRCs	β_1	1.1	--
	β_2	1.2	--
Lower and upper bounds of the distances between ICPs and CRCs	Q_1	*25*	miles
	Q_2	*60*	miles
Maximum number of collection days at each collection point	T	*7*	days
Minimum number of customers allocated to collection points	Y	*1*	--
Minimum number of the established collection points	Z	*1*	--
Minimum number of the established return centers	G	*1*	--
Minimum Number of collection days at each collection point	T	*1*	days

Based on the geographical location of the customers, the initial collection points, and the centralized return centers, the distance was calculated and all the penalty rates were applied.

Furthermore, the discount rates were also applied on the shipping volumes between the ICPs and the CRCs. Table 12 shows the remaining inventory in each of the 30 stores.

Table 12: Remaining inventory of the SKU in question broken down by store

Customer	Remaining Inventory
1	12
2	43
3	34
4	21
5	19
6	10
7	37
8	22
9	35
10	29
11	22
12	21
13	11
14	27
15	44
16	41
17	46
18	22
19	37
20	45
21	38
22	27
23	29
24	11
25	23
26	10
27	39
28	18
29	44
30	33

The modeling language and optimizer Lingo Version 11 (2008) was used to solve the model. The average computational time it took Lingo to solve the example problem with 30 customers, 10 ICP sites, and 5 CRC sites was 12 seconds. Table 13 compares the results to those obtained

by the nonlinear MIP model developed by Min et al. (2006a) along with the results obtained by a naïve greedy approach using AutoCAD (by assigning each customer to its closest ICP and each ICP to the closest CRC site). The results show an improvement of almost 10 percent over the nonlinear MIP model. At the additional $200 cost of opening one more ICP, the MIP model yielded both inventory and transportation cost savings at the combined amount of $20,745 over the nonlinear MIP model. This discrepancy can be explained by the fact that the MIP model is solved for the exact optimal solution, while the nonlinear MIP model was solved by a genetic algorithm for a local optimum.

Table 13: Cost breakdown and comparison of model results

Costs	AutoCAD	Nonlinear MIP model	MIP model
Total annual cost of renting ICPs	$1,800	$800	$1,000
Total cost of establishing CRCs	$15,000	$6,000	$6,000
Total inventory costs	$31,875	$35,350	$31,875
Total handling costs	$21,250	$21,250	$21,250
Total transportation costs	$169,200	$148,170	$130,900
Total Annual Reverse Logistics Costs	**$239,125**	**$211,570**	**$191,025**

6.4 Sensitivity Analysis

Extensive sensitivity analysis was performed on all levels of the research. Different types of clustering analysis methods were tested for the various inventory models in order to determine which one best fit the data. As for the regression analysis, different types of regression for the different SKUs were run, including but not limited to the simple linear regression. Furthermore, the normality of the data for each one of the six SKUs was studied to see if there was a predictable pattern to the mean and standard deviation (See Appendix H). Additionally, the

application of preference discount factors to the different prices throughout the phase-out period was considered in order to realize the effect of marking down prices in a predicted way.

In order to truly test the capability of the proposed linearized model in the reverse logistics part of the dissertation, showing its capability to solve large-sized problems and to find a relationship between the optimal collection period T and the parameters of the model, the scope of the work was broadened by increasing the geographical service area to 100 by 100 miles. The model was then tested on three problem sizes with 50, 100 and 150 customers. For each customer size, 10 replications were run, where the geographical locations of the customers, initial collection points and centralized return centers were randomly generated every time. The daily return rates for the customers were randomly generated within the range [0, 20]. The number of ICPs and CRCs remained unchanged at 10 and 5, respectively.

As the number of customers was increased, it resulted in the increase of both the daily return rate and the volume of products returned from the ICPs to the CRCs, benefitting from the full effect of the economies of scale from the beginning. On the other hand, computational results showed that the optimal collection period is increasing as the daily return rate range decreases and/or the breakpoints for shipping discounts increase (P_1 and P_2).

Starting with 50 customers, the results showed that in one of the replications, the optimal collection period T obtained was 4 days, in 8 T was 3 days, and in 1 T was 2 days. Next, keeping all data the same, another 50 randomly generated customers were added and the model was run again for 10 replications. The above experiment was repeated by adding 50 more customers. The results showed that by increasing the number of customers from 50, to 100, and then to 150, the optimal collection period stays or moves to a lower value in a similar way (see columns of Table 14).

Table 14: Behavior of T as the number of customers increases

Number of Customers	**Optimal Collection Period *T* (days)**									
50	4	3	3	3	3	3	3	3	3	2
100	2	2	2	2	2	2	2	2	2	2
150	2	1	1	1	1	1	1	1	1	1

To illustrate both discrete and continuous analyses, an example was constructed using parameter values from Table 10. The model was then run with a cluster of customers that had a total daily return volume of R = 75. All customers returned the products to a single ICP and subsequently the products were collected during a period T before they were shipped to a single CRC. Figure 16 displays the total reverse logistics cost for both discrete and continuous values of T. For the discrete case, the cost function points are depicted as square bullets for integer values of T = 1…, 10. The function increases at constant rate, while it takes a dip at points $\left\lceil \frac{P_l}{R} \right\rceil$. The critical values of T were found to be $R_L = 40$ and $R_U = 86$ using equations (5.4) and (5.5), respectively. Since $R_L < R = 75 < R_U$, the optimal collection period occurs at one of the intermediates $\left\lceil \frac{P_l}{R} \right\rceil$, specifically in this case at T = 6 (confirmed by Theorem 1 and Corollary 1). If the same data is used, but the continuous case of T is considered, the continuous line segments have the same slope as the rate of increase in the discrete case. Herein, the exact breaking points can be located and then highlighted with a vertical line that connects a dip in the total cost. Where the breakpoints are exactly going to occur can also be indicated. If all the breakpoints are examined, T = 1, $T = \frac{P_1}{R}$ = 2.67, $T = \frac{P_2}{R}$ = 5.33, and $T = \frac{P_3}{R}$ = 8, the optimal solution occurs at

one of the intermediate breakpoints, since R falls between R_L and R_U, namely T = 5.33 + ε. This is also confirmed by the continuous case theorem.

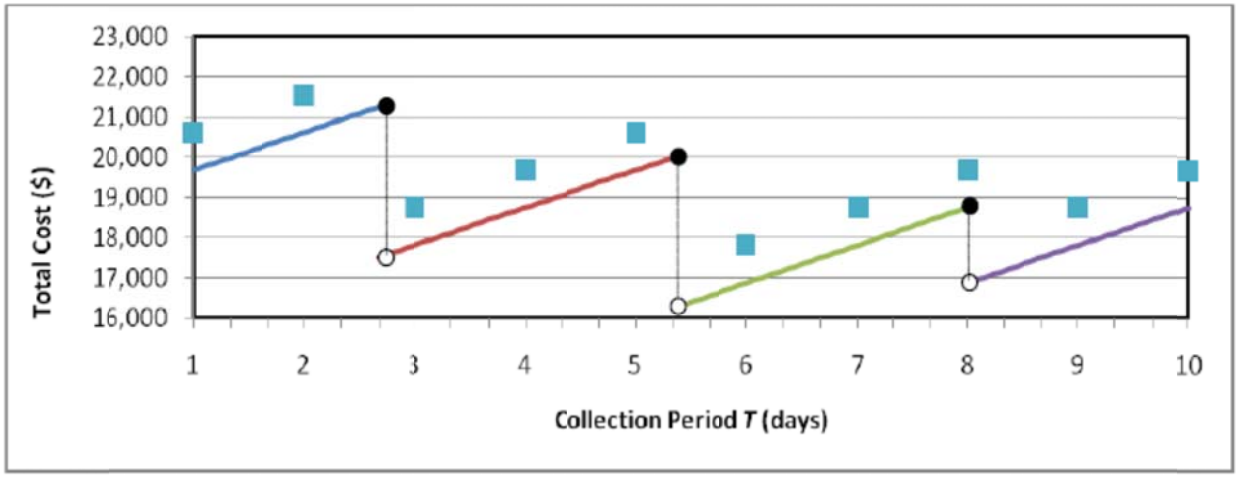


Figure 16: Optimal periods in discrete vs. continuous T

The special structure was based on multiple customers returning products to multiple ICPs and thereafter products are transshipped to multiple CRCs; however, customers were divided into clusters where each cluster is within the geographical area of a single ICP. In addition, no capacity is enforced on the CRCs, while fixed costs at the ICPs and CRCs are considered negligible. This special structure was also based on a given average daily return rate and was compared to the shipping breakpoints during both the discrete and the continuous collection periods. In addition to the aforementioned contributions of the proposed model, it can be further extended to consider the random, unpredictable nature of customer demands and the subsequent return rates.

Chapter 7

Summary and Recommendations for Future Research

In this final chapter of the dissertation, a summary of the problems solved, models developed, and their contributions will be presented, as well as a section recommending future research.

The issues addressed throughout the dissertation dealt with two major aspects of logistics, inventory and reverse logistics.

Using sales records from a retail chain, a model was developed that can determine the best prices to set during a phase-out time horizon so that the total initial inventory is depleted and revenue is maximized. Cluster analysis approach was applied to the data, and then a nonlinear power regression model was used to determine the best-fit regression equation, based on the price elasticity of demand. Furthermore, the different SKUs were compared to one another in order to study the pattern of behavior. The case of the initial inventory being greater than the total units sold at the end of the phase-out period was also addressed to handle the remaining inventory. In addition, the problem was looked at from a different perspective, where the discounted prices were pre-determined, and the question to be answered was related to what

stage in the phase-out period should a particular discount be applied and are all the different values of discounts used?

The second focus was on the products returned at the end of the phase-out period. A model was used to address the issues of reverse logistics. The main contributions from that model are: (1) the linearization of the model which eased computational complexity and thus enabled the identification of the optimal solution for larger customer bases; (2) the handling of the problem with a broader geographical service area and varying shipping volumes between the ICP and the CRC, and the prediction of where and when the optimal solution is going to occur; (3) the determination of a functional relationship between the daily return rate and the optimal collection period; (4) the analysis as to when the optimal solution will be found by examining closed form solutions obtained from special structures designed for both discrete and continuous collection period T.

7.1 Recommendations for Future Research

The following topics can be considered for further research: (1) demand dependency of SKUs on each other using bundled constraints, (2) customers' reactions to fluctuation in prices, (3) the case of stochastic demand, and (4) whether we can apply our models to different types of relationships between price and demand. Furthermore, the model to determine the best prices for each time period in the case of remaining inventory will be extended, given a salvage price. Another future research aspect will be to look into the integer case of the markdown strategy prices.

With regards to an action plan to be presented to pharmacy X and manufacturer Y, the reports show that there was around $10 million in surplus inventory. Pharmacy X owns whatever remaining inventory after the phase-out period. The distributor, P&G, does not have any system in place to purchase back their products, since they are targeted as markdown / sell through; so the assumption is that they are considered and there is no reverse return / storage policy. However, P&G will be graded on GMROI which has an inventory component. Therefore, the following topics could be addressed as recommendations as well as potential future work:

- Minimum fixture presentation and the promotional items that add to the inventory. Thus, one aspect of the problem is to develop and execute a better planogram.
- The replenishment is by SKU and by Store. If one gets sold, it's registered in the system, and then one is ordered. The policy depends on demand for replenishment; but does not depend on demand for the promotional items. There is a different Target Inventory Level for different stores, which is based on the fixture minimum requirements.
- Disorganization disconnect, promotional disconnect.
- "Net Requirement System" (i.e.: Tide - if it's forecasted that one hundred items will be sold and the store has eighty items, only twenty items will be ordered).
- Better forecasting based on previous demand and sales data.

Consequently for the reverse logistics model, future research will be focused on expanding the geographical area, which will increase the number of customers, the number of initial collection points, and the number of the centralized return centers. This expansion will also lead to the increase of the remaining inventory to be handled and transported. Furthermore,

a closer look will be given to the economies of scale policies in order to observe the effects of implementing different approaches.

Appendix A: Reverse Logistics Lingo Model

```
MODEL:

! This is a Mixed Integer Linear Programming Model to deal with the Reverse Logistics Network

for Product Returns;

SETS:

CUSTOMER/1 .. 75/: ri;
ICP/1  .. 10/: a, h, Z;
CRC/1 .. 5/: ck, m, G;
CUSTOMER_ICP(CUSTOMER,ICP): dc_icp, Y;
ICP_CRC(ICP,CRC): dicp_crc, X, U1, U2, U3, W1, W2, rjk1, rjk2, rjk3;

ENDSETS

[OBJFUN] MIN = ICP_RENT_COST + CRC_EST_COST + INV_COST +
HAND_COST + TRANS_COST;
ICP_RENT_COST = @SUM (ICP: a*Z);
CRC_EST_COST =  @SUM(CRC: ck*G);
INV_COST = b*w*((T+1)/2)* @SUM(CUSTOMER: ri);
HAND_COST = w*@SUM(CUSTOMER_ICP(i,j)|dc_icp(i,j) #LE# l:ri(i)*h(j)*Y(i,j));
TRANS_COST = (w/T)* @SUM(ICP_CRC:rjk1*U1 + rjk2*U2 -(rjk1-rjk2)*p1*W1+
rjk3*U3 -(rjk2-rjk3)*p2*W2);
@FOR(ICP_CRC:     [X_U1_U2_U3] X=U1+U2+U3;
                  [U3_W2] U3<= M1*W2;
                  [W2_U2] (p2-p1)*W2 <= U2;
                  [U2_W1] U2 <= (p2-p1)*W1;
                  [W1_U1] p1*W1 <= U1;
                  [U1_1] U1 <= p1;);
@FOR(CUSTOMER(i):[CUS_ASSIGN]@SUM(ICP(j)|dc_icp(i,j) #LE# l:Y(i,j)) = 1);
@FOR(ICP(j):[OPEN_ICP]@SUM(CUSTOMER(i)|dc_icp(i,j) #LE# l:Y(i,j)) <= M1*Z(j));
@FOR(ICP(j):[FLOW_BALANCE] T*@SUM(CUSTOMER(i)|dc_icp(i,j) #LE#
                        l:ri(i)*Y(i,j)) = @SUM(CRC(k):X(j,k)));
@FOR(CRC(k):@SUM(ICP(j):X(j,k)) <= T*m(k)*G(k));
@FOR(ICP_CRC:     [BIN_W1]@BIN(W1);
                  [BIN_W2]@BIN(W2));
@FOR(CUSTOMER_ICP(i,j)|dc_icp(i,j) #LE# l:[BIN_Y]@BIN(Y(i,j)));
@FOR(ICP:[BIN_Z]@BIN(Z));
@FOR(CRC:[BIN_G]@BIN(G));
```

```
DATA:

! Indices;
 i = 30  index for customers;
 j = 10  index for initial collection points;
 k = 5   index for centralized return centers;
!Model Parameters;
b = 0.1;
w = 250;
T = 7;
a = @OLE(E:\Data.xls);
ri = @OLE(E:\Data.xls);
rjk1= @OLE(E:\Data.xls);
rjk2= @OLE(E:\Data.xls);
rjk3= @OLE(E:\Data.xls);
h = @OLE(E:\Data.xls);
ck = @OLE(E:\Data.xls);
m = @OLE(E:\Data.xls);
dc_icp = @OLE(E:\Data.xls);
dicp_crc = @OLE(E:\Data.xls);
l = 25;
M1 = 1000;
E = 1;
alpha1 = 0.8;
alpha2 = 0.6;
p1 = 200;
p2 = 400;
beta1 = 1.1;
beta2 = 1.2;
q1 = 25;
q2 = 60;

ENDDATA

END
```

Appendix B: Cluster Algorithm Lingo Model

```
MODEL:

! This is a Linear Programming Model that deals with the Clustering Algorithm;

SETS:

UP/1 .. 54/:P;
CM/1  .. 9/:X;
UP_CM(UP,CM):Y, Z;

ENDSETS

[OBJFUN] MIN = @SUM (UP_CM(i,j): Z(i,j));
@FOR(UP_CM(i,j):M*(1-Y(i,j)) -(P(i)-X(j)) + Z(i,j) >= 0);
@FOR(UP_CM(i,j):M*(1-Y(i,j)) -(-P(i)+X(j)) + Z(i,j) >= 0);
@FOR(UP(i):@SUM(UP_CM(i,j):Y(i,j)) = 1);
@FOR(UP_CM:[BIN_Y]@BIN(Y));
@FOR(CM(j):X(j) <= @MAX(UP(i):P(i)));
@FOR(CM(j):X(j) >= @MIN(UP(i):P(i)));

DATA:

!Model Parameters;
P = @OLE(C:\Data.xls);
@OLE(C:\Data.xls) = X;
M = 100;

ENDDATA

END
```

Appendix C: Price Elasticity Lingo Model

```
MODEL:

! This is a Non-Linear Programming Model to deal with the Price Elasticity of Demand;

[OBJFUN] MAX =    alpha*(P_1^(beta+1)+ P_2^(beta+1)+ P_3^(beta+1)
                  + P_4^(beta+1)+ P_5^(beta+1)+ P_6^(beta+1)+ P_7^(beta+1)
                  + P_8^(beta+1)+ P_9^(beta+1)) + (S_C*R_1);

alpha*(P_1^(beta)+ P_2^(beta)+ P_3^(beta) + P_4^(beta)+ P_5^(beta)+ P_6^(beta)+ P_7^(beta)
+ P_8^(beta)+ P_9^(beta)) + R_1 = vol_tb_depl;


DATA:

alpha = 37625;
beta = -1.605;
vol_tb_depl = 20742;
S_C = 3.0;

@OLE(C:\Documents and Settings\Daisy\Desktop\4.xls) = P_1;
@OLE(C:\Documents and Settings\Daisy\Desktop\4.xls) = P_2;
@OLE(C:\Documents and Settings\Daisy\Desktop\4.xls) = P_3;
@OLE(C:\Documents and Settings\Daisy\Desktop\4.xls) = P_4;
@OLE(C:\Documents and Settings\Daisy\Desktop\4.xls) = P_5;
@OLE(C:\Documents and Settings\Daisy\Desktop\4.xls) = P_6;
@OLE(C:\Documents and Settings\Daisy\Desktop\4.xls) = P_7;
@OLE(C:\Documents and Settings\Daisy\Desktop\4.xls) = P_8;
@OLE(C:\Documents and Settings\Daisy\Desktop\4.xls) = P_9;
@OLE(C:\Documents and Settings\Daisy\Desktop\4.xls) = R_1;

ENDDATA

END
```

Appendix D: Initial Data Set for the Reverse Logistics Model

Potential Sites for Initial Collection Points	*Site Coordinate*	
	x	y
icp1	29.79	9.00
icp2	53.30	30.96
icp3	58.21	22.61
icp4	52.93	50.01
icp5	54.90	14.92
icp6	9.49	44.46
icp7	38.92	38.55
icp8	42.53	35.52
icp9	23.61	30.13
icp10	26.36	55.96

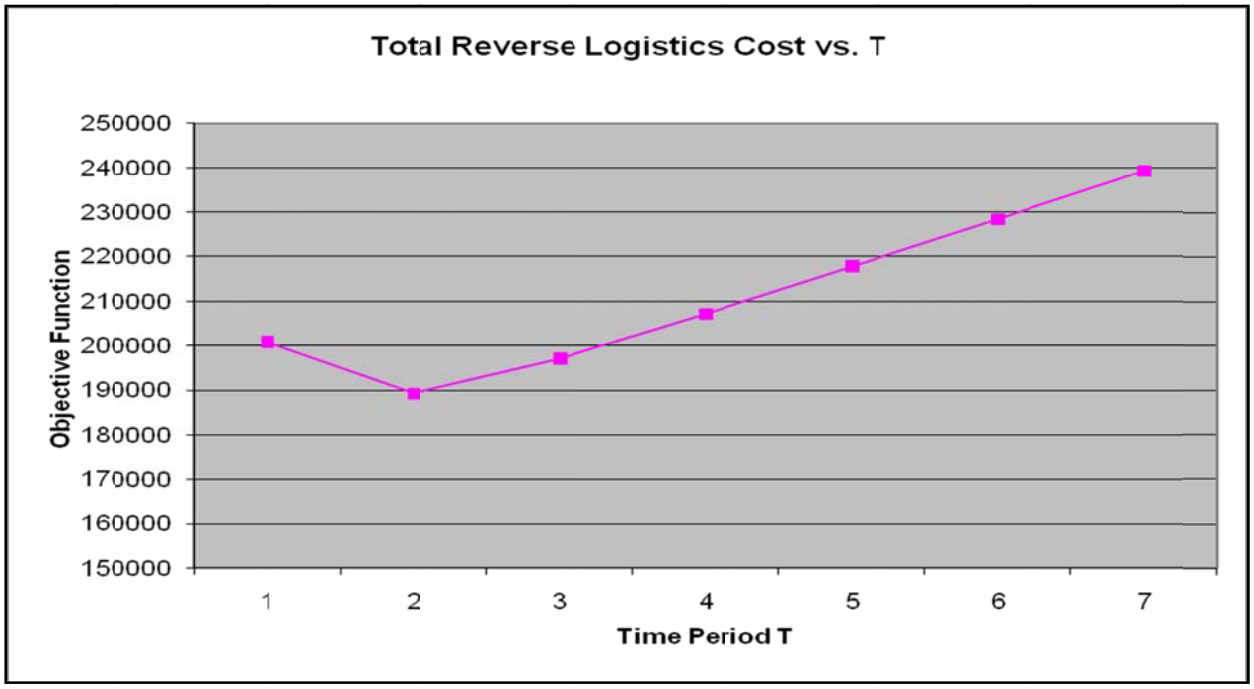
Potential Sites for Centralized Return Centers	*Site Coordinate*	
	x	y
crc1	18.62	15.70
crc2	56.50	26.14
crc3	17.27	10.00
crc4	38.70	20.52
crc5	0.87	56.15

Locations and Daily Demands of Customers	*Coordinates*	
	x	y
1	34.13	41.77
2	51.69	56.33
3	10.79	31.54
4	3.50	25.25
5	14.47	31.31
6	9.84	37.85
7	13.72	37.42
8	42.82	44.66
9	28.61	12.91
10	39.86	20.79

11	32.75	22.64
12	17.53	51.30
13	25.82	20.26
14	5.99	34.49
15	8.80	37.71
16	35.92	52.13
17	10.72	16.35
18	2.09	47.20
19	23.21	43.28
20	43.39	12.44
21	9.10	13.48
22	10.38	54.81
23	50.97	59.94
24	52.46	59.03
25	56.32	27.23
26	48.61	56.77
27	17.88	55.26
28	56.33	1.24
29	30.74	14.99
30	24.09	34.81

Appendix E: Solution Summary for the Reverse Logistics Model

Solution Summary							
# of collection frequency	T = 1	T = 2	T = 3	T = 4	T = 5	T = 6	T = 7
# of established CRC	1	2	1	1	1	1	1
# of established ICP	5	4	4	4	5	5	6
Total annual cost of renting ICP	$1,000	$800	$800	$800	$1,000	$1,000	$1,200
Total cost of establishing CRC	$3,000	$6,000	$3,000	$3,000	$3,000	$3,000	$3,000
Total inventory costs	$21,250	$31,875	$42,500	$53,125	$63,750	$74,375	$85,000
Total handling costs	$21,250	$21,250	$21,250	$21,250	$21,250	$21,250	$21,250
Total Transportation Costs	$154,400	$129,300	$129,510	$129,000	$128,820	$128,820	$128,820
Total Annual Reverse Logistics Costs	$200,900	$189,225	$197,060	$207,175	$217,820	$228,445	$239,270



Appendix F: Initial Data Set for the 6 Cosmetics SKUs

Pharmacy X SKU Number	ITEM DESCRIPTION		
111116	LASH 835		
Fiscal Week Ended	**TSA ($)**	**TSV (Units)**	**Unit Price**
01/02/2010	$15,609.27	3,407	4.58
11/07/2009	$14,507.21	2,834	5.12
09/26/2009	$15,124.42	2,946	5.13
01/16/2010	$10,787.20	1,767	6.10
08/08/2009	$11,545.92	1,775	6.50
09/19/2009	$11,447.55	1,661	6.89
02/20/2010	$11,485.28	1,651	6.96
12/19/2009	$11,427.01	1,641	6.96
01/09/2010	$9,668.62	1,385	6.98
10/03/2009	$11,664.19	1,666	7.00
07/04/2009	$12,782.48	1,820	7.02
06/12/2010	$12,352.01	1,756	7.03
01/23/2010	$10,177.74	1,437	7.08
03/06/2010	$11,279.75	1,573	7.17
09/05/2009	$12,698.95	1,651	7.69
12/05/2009	$13,283.51	1,725	7.70
06/05/2010	$13,125.64	1,695	7.74
03/13/2010	$13,258.78	1,711	7.75
02/13/2010	$14,257.08	1,820	7.83
05/08/2010	$13,352.63	1,698	7.86
05/29/2010	$14,386.27	1,822	7.90
10/17/2009	$11,586.14	1,453	7.97
04/10/2010	$14,105.53	1,768	7.98
08/22/2009	$8,784.66	1,063	8.26
06/27/2009	$9,327.55	1,123	8.31
08/01/2009	$8,186.20	984	8.32
07/18/2009	$9,425.49	1,128	8.36
07/11/2009	$11,719.04	1,402	8.36
07/25/2009	$8,167.94	976	8.37

08/15/2009	$9,557.99	1,142	8.37
06/20/2009	$9,790.75	1,168	8.38
10/31/2009	$9,166.70	1,087	8.43
12/26/2009	$11,476.56	1,345	8.53
09/12/2009	$8,588.42	996	8.62
12/12/2009	$10,921.12	1,251	8.73
08/29/2009	$9,030.98	1,034	8.73
02/27/2010	$9,301.58	1,062	8.76
04/24/2010	$11,033.70	1,255	8.79
10/10/2009	$11,723.85	1,329	8.82
11/28/2009	$8,490.97	955	8.89
05/01/2010	$9,997.68	1,122	8.91
02/06/2010	$13,914.88	1,561	8.91
06/26/2010	$9,520.64	1,068	8.91
05/22/2010	$9,985.00	1,120	8.92
04/03/2010	$12,817.10	1,435	8.93
04/17/2010	$10,481.79	1,171	8.95
11/14/2009	$8,837.76	987	8.95
06/19/2010	$10,692.59	1,194	8.96
05/15/2010	$10,379.26	1,158	8.96
03/27/2010	$9,344.07	1,042	8.97
10/24/2009	$9,885.99	1,102	8.97
01/30/2010	$8,561.41	953	8.98
03/20/2010	$10,028.49	1,115	8.99
11/21/2009	$8,986.48	999	9.00
Grand Total	**$598,039.82**	**77,989**	

Pharmacy X SKU Number	ITEM DESCRIPTION		
111115	LASH 830		
Fiscal Week Ended	**TSA ($)**	**TSV (Units)**	**Unit Price**
01/02/2010	$23,553.62	5,127	4.59
11/07/2009	$19,936.94	3,905	5.11
09/26/2009	$19,159.87	3,745	5.12
01/16/2010	$17,029.94	2,784	6.12
08/08/2009	$14,014.11	2,124	6.60
09/19/2009	$15,044.57	2,174	6.92
01/09/2010	$13,222.29	1,886	7.01
02/20/2010	$15,096.77	2,148	7.03
10/03/2009	$15,302.72	2,170	7.05
06/12/2010	$17,390.09	2,464	7.06
12/19/2009	$14,981.02	2,121	7.06
07/04/2009	$17,301.16	2,448	7.07
01/23/2010	$15,475.69	2,185	7.08
03/06/2010	$15,048.53	2,094	7.19
05/08/2010	$18,176.44	2,344	7.75
09/05/2009	$15,899.30	2,048	7.76
12/05/2009	$18,098.87	2,326	7.78
06/05/2010	$19,227.26	2,466	7.80
03/13/2010	$19,799.86	2,538	7.80
05/29/2010	$19,941.22	2,549	7.82
02/13/2010	$21,785.58	2,769	7.87
04/10/2010	$19,634.06	2,486	7.90
10/17/2009	$16,364.99	2,057	7.96
08/22/2009	$10,693.74	1,289	8.30
08/15/2009	$12,361.76	1,490	8.30
08/01/2009	$9,796.92	1,179	8.31
06/27/2009	$12,666.24	1,524	8.31
06/20/2009	$13,476.01	1,617	8.33
07/25/2009	$11,005.93	1,320	8.34
07/11/2009	$16,019.80	1,920	8.34
07/18/2009	$11,788.64	1,410	8.36
10/31/2009	$12,431.82	1,474	8.43
12/26/2009	$15,703.95	1,840	8.53
09/12/2009	$10,540.29	1,224	8.61
12/12/2009	$14,172.34	1,621	8.74
08/29/2009	$10,014.89	1,140	8.78

02/27/2010	$12,995.10	1,479	8.79
10/10/2009	$14,357.35	1,634	8.79
04/24/2010	$14,138.98	1,604	8.81
06/26/2010	$13,657.12	1,536	8.89
05/01/2010	$12,780.13	1,437	8.89
04/03/2010	$16,381.40	1,840	8.90
11/28/2009	$10,990.18	1,234	8.91
05/22/2010	$14,746.58	1,653	8.92
11/14/2009	$11,706.51	1,311	8.93
02/06/2010	$20,331.77	2,274	8.94
05/15/2010	$14,174.59	1,581	8.97
01/30/2010	$12,823.28	1,430	8.97
03/20/2010	$12,397.06	1,381	8.98
06/19/2010	$14,982.50	1,669	8.98
04/17/2010	$13,798.72	1,537	8.98
03/27/2010	$13,317.15	1,483	8.98
10/24/2009	$12,861.54	1,431	8.99
11/21/2009	$12,529.01	1,392	9.00
Grand Total	**$811,126.20**	**105,912**	

Pharmacy X SKU Number	ITEM DESCRIPTION		
111111	LIP 555		
Fiscal Week Ended	**TSA ($)**	**TSV (Units)**	**Unit Price**
05/15/2010	$3,077.96	985	3.12
07/18/2009	$1,489.20	361	4.13
11/28/2009	$1,206.08	283	4.26
05/08/2010	$2,107.45	456	4.62
03/13/2010	$1,889.09	408	4.63
12/05/2009	$1,389.50	297	4.68
06/05/2010	$1,797.82	377	4.77
07/04/2009	$1,339.77	278	4.82
02/13/2010	$1,708.31	354	4.83
08/08/2009	$1,434.70	295	4.86
10/17/2009	$1,243.47	253	4.91
01/02/2010	$1,035.55	210	4.93
09/26/2009	$1,374.91	274	5.02
10/03/2009	$952.61	164	5.81
08/22/2009	$995.25	171	5.82
04/24/2010	$1,409.81	242	5.83
12/19/2009	$1,171.45	201	5.83
02/20/2010	$1,474.84	253	5.83
06/26/2010	$1,361.66	233	5.84
02/06/2010	$963.46	164	5.87
05/01/2010	$1,272.00	216	5.89
08/29/2009	$1,042.36	177	5.89
07/11/2009	$1,319.80	224	5.89
06/27/2009	$997.61	169	5.90
08/01/2009	$1,098.92	186	5.91
05/29/2010	$1,400.34	237	5.91
09/19/2009	$1,004.63	170	5.91
09/05/2009	$975.19	165	5.91
01/16/2010	$1,111.34	188	5.91
03/06/2010	$1,040.74	176	5.91
05/22/2010	$1,516.07	256	5.92
01/30/2010	$1,078.90	182	5.93
12/12/2009	$889.29	150	5.93
12/26/2009	$1,045.04	176	5.94
04/17/2010	$1,533.23	258	5.94
01/09/2010	$933.05	157	5.94

08/15/2009	$1,106.36	186	5.95
06/12/2010	$1,910.25	321	5.95
04/10/2010	$1,322.79	222	5.96
01/23/2010	$1,002.52	168	5.97
10/10/2009	$990.84	166	5.97
03/20/2010	$1,158.66	194	5.97
09/12/2009	$956.30	160	5.98
10/31/2009	$1,274.47	213	5.98
11/07/2009	$1,107.25	185	5.99
02/27/2010	$1,083.49	181	5.99
11/14/2009	$921.96	154	5.99
10/24/2009	$1,143.49	191	5.99
11/21/2009	$964.09	161	5.99
06/20/2009	$1,054.44	176	5.99
03/27/2010	$1,480.53	247	5.99
06/19/2010	$1,367.82	228	6.00
04/03/2010	$1,428.42	238	6.00
07/25/2009	$988.94	164	6.03
Grand Total	**$68,944.02**	**12,901**	

Pharmacy X SKU Number	ITEM DESCRIPTION		
111112	LIP 560		
Fiscal Week Ended	**TSA ($)**	**TSV (Units)**	**Unit Price**
05/15/2010	$2,632.62	867	3.04
11/28/2009	$951.21	220	4.32
07/18/2009	$1,394.57	320	4.36
07/04/2009	$1,100.40	232	4.74
05/08/2010	$1,669.84	352	4.74
12/05/2009	$1,336.95	280	4.77
06/05/2010	$1,547.22	324	4.78
02/13/2010	$1,200.55	251	4.78
03/13/2010	$1,450.57	298	4.87
08/08/2009	$1,302.51	264	4.93
01/02/2010	$953.78	192	4.97
09/26/2009	$1,175.20	233	5.04
10/17/2009	$1,051.42	208	5.05
02/20/2010	$1,096.89	195	5.63
10/03/2009	$956.28	168	5.69
12/19/2009	$1,037.32	181	5.73
08/22/2009	$781.69	136	5.75
04/24/2010	$993.22	172	5.77
05/01/2010	$1,044.44	180	5.80
09/19/2009	$731.07	125	5.85
07/25/2009	$801.65	137	5.85
01/16/2010	$780.20	133	5.87
11/07/2009	$815.91	139	5.87
08/15/2009	$1,095.87	186	5.89
06/12/2010	$1,522.60	258	5.90
10/10/2009	$814.51	138	5.90
12/26/2009	$838.18	142	5.90
08/01/2009	$891.99	151	5.91
10/24/2009	$851.56	144	5.91
02/06/2010	$727.57	123	5.92
05/29/2010	$996.22	168	5.93
06/26/2010	$960.88	162	5.93
01/23/2010	$635.03	107	5.93
06/20/2009	$973.37	164	5.94
03/06/2010	$837.59	141	5.94
11/21/2009	$827.31	139	5.95

07/11/2009	$1,113.65	187	5.96
02/27/2010	$833.80	140	5.96
09/12/2009	$709.31	119	5.96
06/19/2010	$1,102.76	185	5.96
08/29/2009	$780.89	131	5.96
05/22/2010	$1,169.45	196	5.97
04/10/2010	$1,074.11	180	5.97
09/05/2009	$739.96	124	5.97
06/27/2009	$812.04	136	5.97
01/09/2010	$765.02	128	5.98
11/14/2009	$765.42	128	5.98
04/17/2010	$999.63	167	5.99
04/03/2010	$957.80	160	5.99
01/30/2010	$754.54	126	5.99
10/31/2009	$1,199.80	200	6.00
03/20/2010	$1,002.13	167	6.00
03/27/2010	$1,086.00	180	6.03
12/12/2009	$779.61	129	6.04
Grand Total	**$55,424.11**	**10,413**	

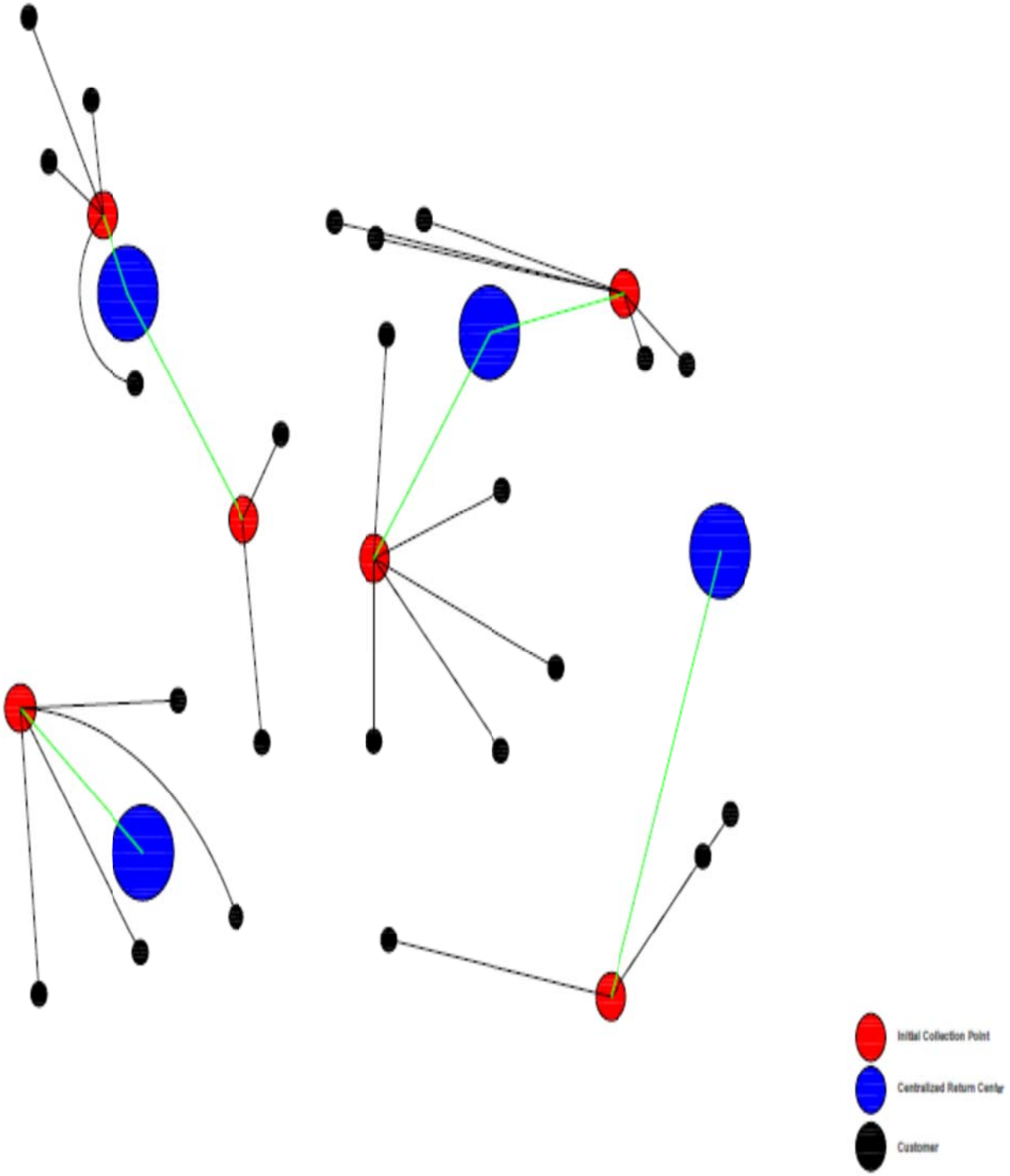
Pharmacy X SKU Number	ITEM DESCRIPTION		
111113	LIP 587		
Fiscal Week Ended	**TSA ($)**	**TSV (Units)**	**Unit Price**
05/15/2010	$3,223.91	988	3.26
07/18/2009	$1,272.48	327	3.89
11/28/2009	$1,117.23	282	3.96
05/08/2010	$1,971.91	429	4.60
03/13/2010	$1,725.76	373	4.63
12/05/2009	$1,699.49	363	4.68
02/13/2010	$1,692.38	358	4.73
08/08/2009	$1,307.54	273	4.79
06/05/2010	$1,657.19	341	4.86
01/02/2010	$1,221.64	251	4.87
07/04/2009	$1,207.92	248	4.87
10/17/2009	$1,450.29	292	4.97
09/26/2009	$2,055.62	409	5.03
10/03/2009	$1,191.22	211	5.65
02/20/2010	$1,060.77	187	5.67
08/01/2009	$906.57	158	5.74
06/26/2010	$878.17	153	5.74
05/29/2010	$1,161.71	202	5.75
08/22/2009	$721.06	125	5.77
12/19/2009	$1,561.26	270	5.78
05/22/2010	$1,125.73	194	5.80
03/06/2010	$1,178.88	203	5.81
06/27/2009	$630.13	108	5.83
08/29/2009	$940.92	161	5.84
09/19/2009	$872.79	149	5.86
07/11/2009	$1,240.03	211	5.88
01/30/2010	$607.09	103	5.89
08/15/2009	$1,031.57	175	5.89
01/09/2010	$737.11	125	5.90
09/12/2009	$772.79	131	5.90
09/05/2009	$957.59	162	5.91
06/20/2009	$786.79	133	5.92
03/27/2010	$1,113.22	188	5.92
04/24/2010	$918.35	155	5.92
12/26/2009	$1,134.49	191	5.94
04/17/2010	$1,035.36	174	5.95

03/20/2010	$898.70	151	5.95
05/01/2010	$999.92	168	5.95
06/12/2010	$1,613.29	271	5.95
06/19/2010	$1,042.41	175	5.96
02/06/2010	$739.26	124	5.96
10/24/2009	$1,103.84	185	5.97
10/10/2009	$1,026.48	172	5.97
11/21/2009	$1,051.34	176	5.97
11/14/2009	$973.77	163	5.97
01/23/2010	$682.36	114	5.99
04/10/2010	$1,047.55	175	5.99
01/16/2010	$933.94	156	5.99
04/03/2010	$1,071.71	179	5.99
11/07/2009	$1,083.99	181	5.99
12/12/2009	$1,138.40	190	5.99
07/25/2009	$815.23	136	5.99
10/31/2009	$1,843.53	307	6.00
02/27/2010	$939.74	156	6.02
Grand Total	**$63,172.42**	**11,982**	

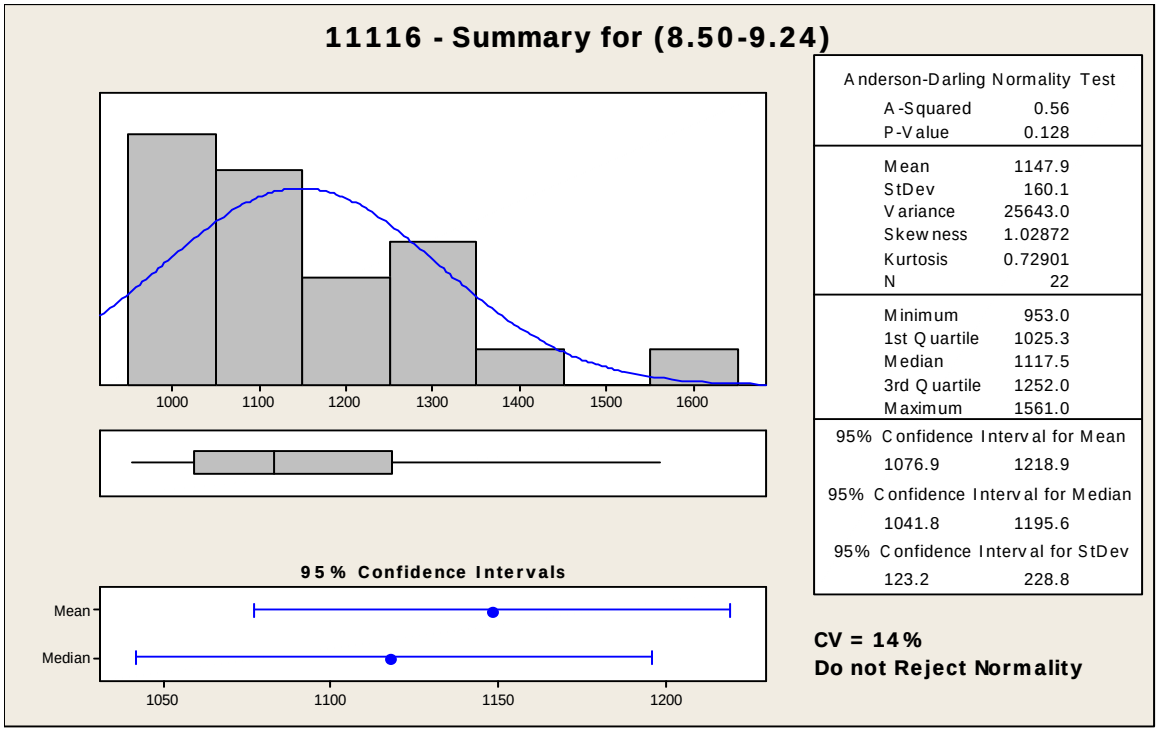
Pharmacy X SKU Number	ITEM DESCRIPTION		
111114	LIP 701		
Fiscal Week Ended	**TSA ($)**	**TSV (Units)**	**Unit Price**
05/15/2010	$2,499.94	791	3.16
11/28/2009	$1,113.44	281	3.96
07/18/2009	$1,075.22	256	4.20
05/08/2010	$1,520.15	335	4.54
08/08/2009	$1,033.22	224	4.61
12/05/2009	$1,427.65	305	4.68
06/05/2010	$1,200.94	256	4.69
01/02/2010	$1,067.15	226	4.72
03/13/2010	$1,547.01	324	4.77
02/13/2010	$1,488.71	310	4.80
10/17/2009	$1,087.18	226	4.81
07/04/2009	$743.68	153	4.86
09/26/2009	$1,412.88	282	5.01
08/22/2009	$643.48	114	5.64
09/12/2009	$662.87	115	5.76
08/29/2009	$720.63	125	5.77
03/06/2010	$872.20	151	5.78
12/26/2009	$867.40	150	5.78
06/26/2010	$848.65	146	5.81
08/15/2009	$761.89	131	5.82
09/19/2009	$855.46	147	5.82
05/01/2010	$925.61	159	5.82
02/06/2010	$751.51	129	5.83
06/27/2009	$496.85	85	5.85
06/19/2010	$707.78	121	5.85
10/24/2009	$974.94	166	5.87
04/24/2010	$955.10	162	5.90
07/11/2009	$856.17	145	5.90
06/12/2010	$1,395.15	236	5.91
12/12/2009	$839.48	142	5.91
05/29/2010	$782.06	132	5.92
10/10/2009	$1,102.43	186	5.93
08/01/2009	$758.82	128	5.93
03/27/2010	$1,068.61	180	5.94
04/03/2010	$1,188.40	200	5.94
04/10/2010	$826.41	139	5.95

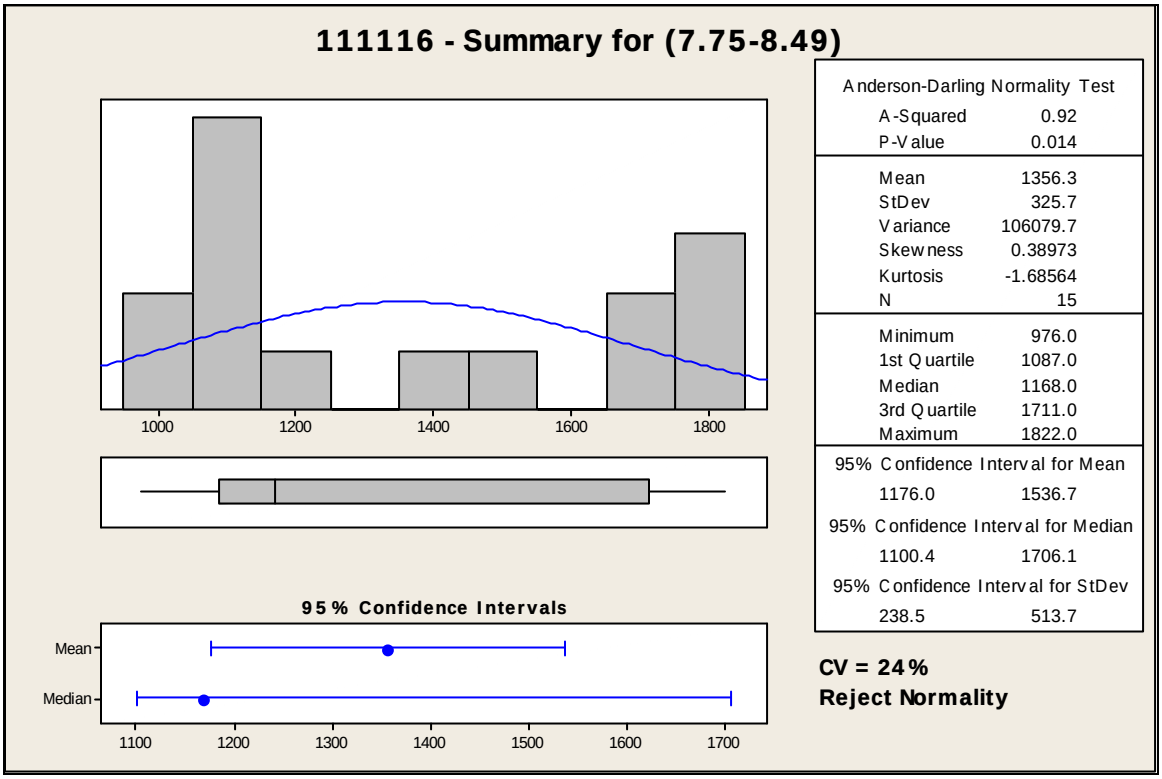
05/22/2010	$922.15	155	5.95
01/09/2010	$804.66	135	5.96
12/19/2009	$971.61	163	5.96
10/03/2009	$882.76	148	5.96
02/20/2010	$794.33	133	5.97
04/17/2010	$944.42	158	5.98
09/05/2009	$664.09	111	5.98
10/31/2009	$1,202.69	201	5.98
11/21/2009	$903.69	151	5.98
02/27/2010	$855.87	143	5.99
11/07/2009	$921.76	154	5.99
01/30/2010	$832.11	139	5.99
01/23/2010	$808.25	135	5.99
03/20/2010	$880.15	147	5.99
06/20/2009	$562.96	94	5.99
11/14/2009	$952.36	159	5.99
01/16/2010	$889.12	148	6.01
07/25/2009	$593.41	98	6.06
Grand Total	**$52,465.46**	**9,930**	

Appendix G: Reverse Logistics Mock-Up

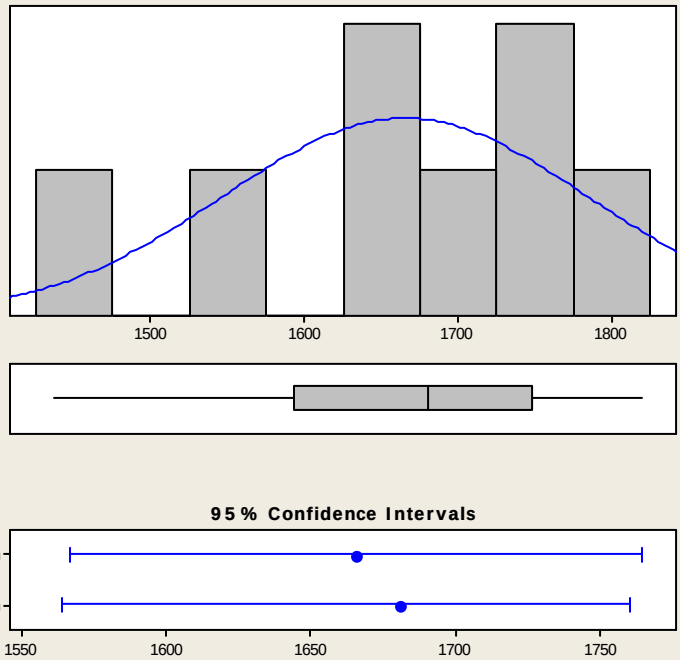


Appendix H: Normality Test for all 6 SKUs





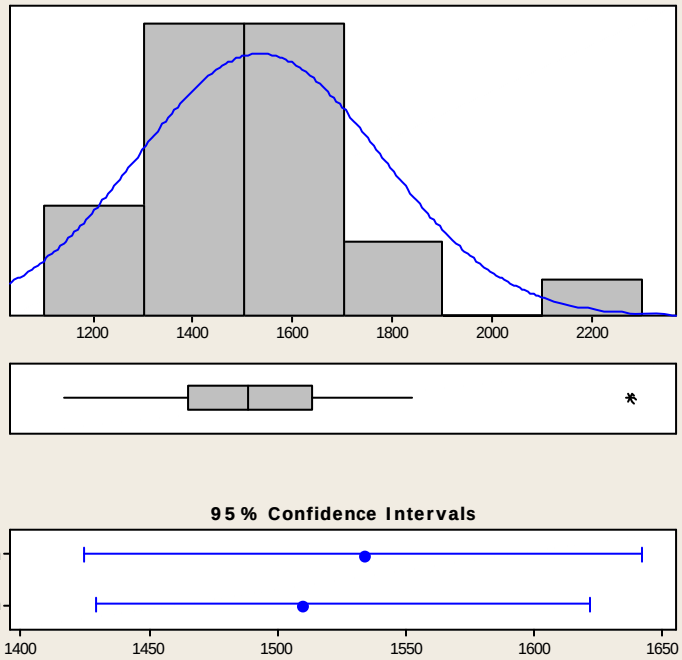
111116 - Summary for (7.00-7.74)



Anderson-Darling Normality Test	
A-Squared	0.26
P-Value	0.613
Mean	1665.4
StDev	118.0
Variance	13921.4
Skewness	-0.93036
Kurtosis	1.19538
N	8
Minimum	1437.0
1st Quartile	1592.5
Median	1680.5
3rd Quartile	1748.3
Maximum	1820.0
95% Confidence Interval for Mean	
1566.7	1764.0
95% Confidence Interval for Median	
1564.2	1760.1
95% Confidence Interval for StDev	
78.0	240.1

CV = 7%
Do not Reject Normality

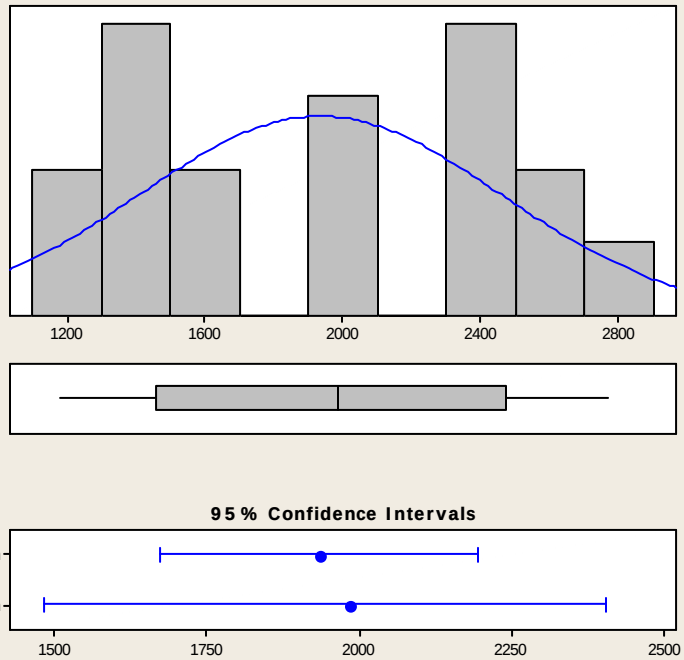
111115 - Summary for (8.50-9.24)_1



Anderson-Darling Normality Test	
A-Squared	0.49
P-Value	0.201
Mean	1533.2
StDev	245.2
Variance	60144.5
Skewness	1.18182
Kurtosis	2.90491
N	22
Minimum	1140.0
1st Quartile	1389.3
Median	1509.5
3rd Quartile	1638.8
Maximum	2274.0
95% Confidence Interval for Mean	
1424.5	1642.0
95% Confidence Interval for Median	
1429.0	1621.4
95% Confidence Interval for StDev	
188.7	350.5

CV = 16%
Do not Reject Normality

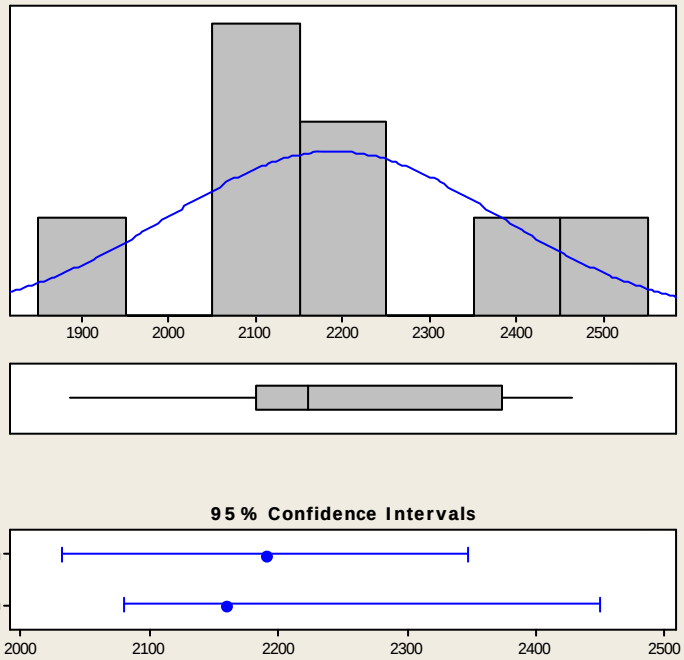
393495 - Summary for (7.75-8.49)_1



Anderson-Darling Normality Test	
A-Squared	0.65
P-Value	0.073
Mean	1933.7
StDev	525.4
Variance	276060.2
Skewness	0.05995
Kurtosis	-1.60497
N	18
Minimum	1179.0
1st Quartile	1458.0
Median	1984.0
3rd Quartile	2471.0
Maximum	2769.0
95% Confidence Interval for Mean	
1672.4	2194.9
95% Confidence Interval for Median	
1482.3	2402.8
95% Confidence Interval for StDev	
394.3	787.7

CV = 27%
Do not Reject Normality

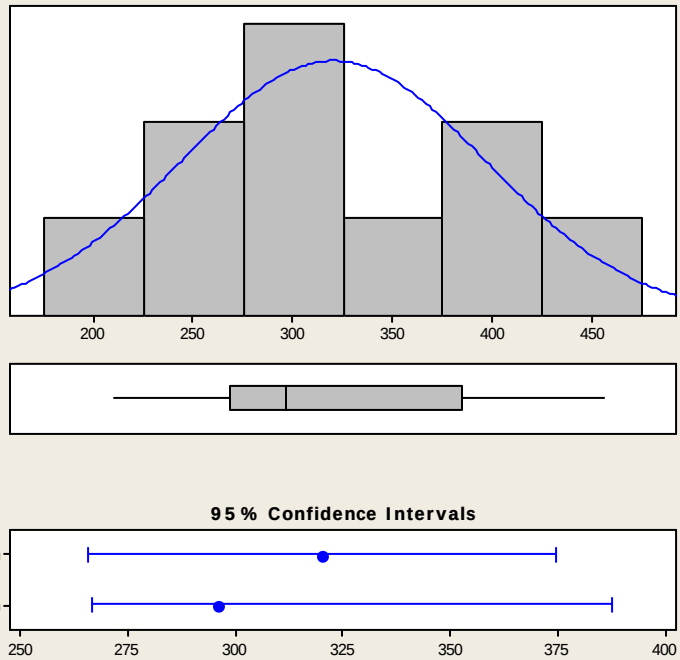
111115 - Summary for (7.00-7.74)_1



Anderson-Darling Normality Test	
A-Squared	0.48
P-Value	0.161
Mean	2189.5
StDev	189.1
Variance	35745.7
Skewness	0.246040
Kurtosis	0.127344
N	8
Minimum	1886.0
1st Quartile	2100.8
Median	2159.0
3rd Quartile	2382.3
Maximum	2464.0
95% Confidence Interval for Mean	
2031.4	2347.6
95% Confidence Interval for Median	
2080.6	2449.0
95% Confidence Interval for StDev	
125.0	384.8

CV = 9%
Do not Reject Normality

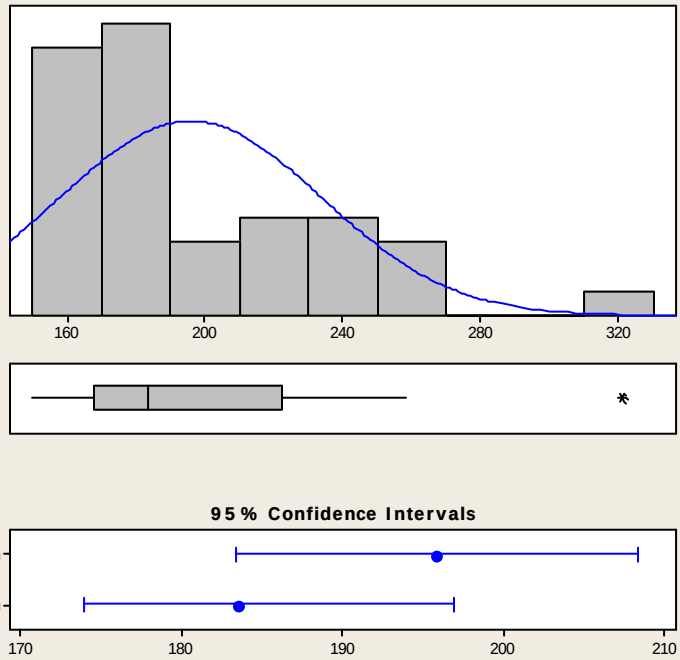
111111 - Summary for (4.50 - 5.24)



Anderson-Darling Normality Test	
A-Squared	0.27
P-Value	0.579
Mean	320.20
StDev	76.21
Variance	5807.51
Skewness	0.489274
Kurtosis	-0.537319
N	10
Minimum	210.00
1st Quartile	268.75
Median	296.00
3rd Quartile	384.75
Maximum	456.00
95% Confidence Interval for Mean	
265.68	374.72
95% Confidence Interval for Median	
266.81	387.61
95% Confidence Interval for StDev	
52.42	139.12

CV = 24%
Do not Reject Normality

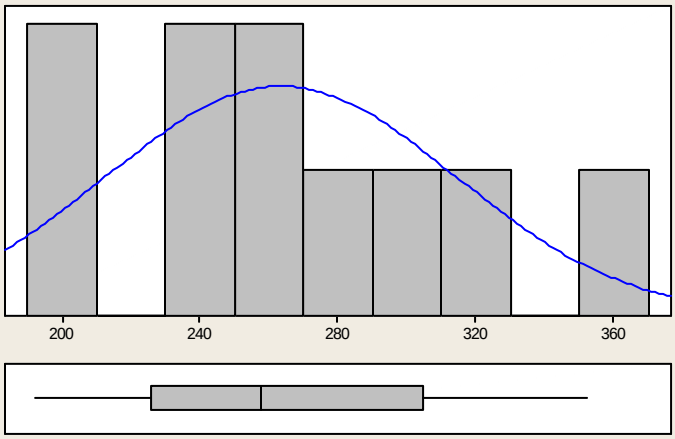
111111 - Summary for (5.25 - 5.99)

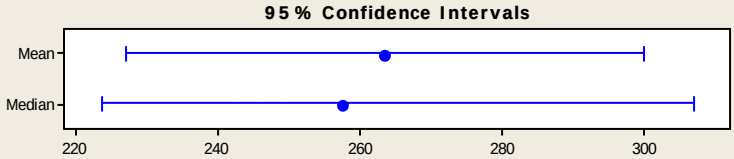


Anderson-Darling Normality Test	
A-Squared	1.58
P-Value <	0.005
Mean	195.79
StDev	37.95
Variance	1440.50
Skewness	1.28889
Kurtosis	1.66430
N	38
Minimum	150.00
1st Quartile	167.50
Median	183.50
3rd Quartile	222.50
Maximum	321.00
95% Confidence Interval for Mean	
183.31	208.26
95% Confidence Interval for Median	
173.90	196.94
95% Confidence Interval for StDev	
30.94	49.10

CV = 19%
Reject Normality

111112 - Summary for (4.50 - 5.24)_1

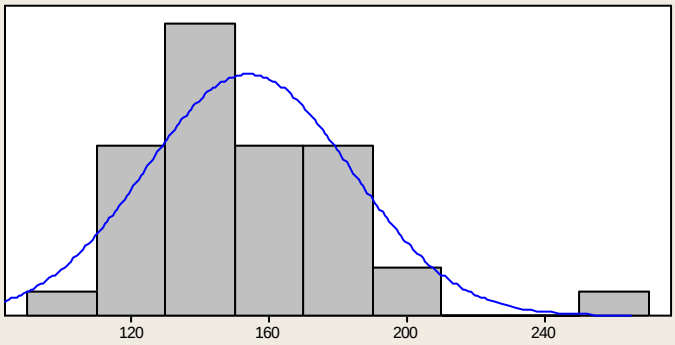


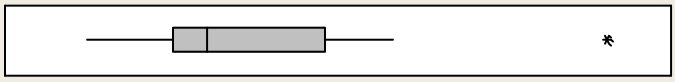


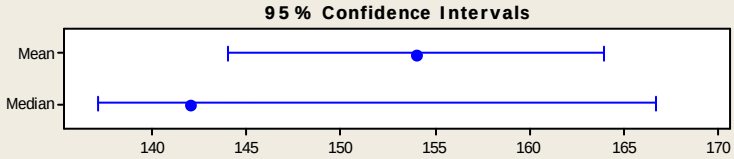
Anderson-Darling Normality Test	
A-Squared	0.15
P-Value	0.939
Mean	263.40
StDev	50.80
Variance	2580.71
Skewness	0.390983
Kurtosis	-0.607510
N	10
Minimum	192.00
1st Quartile	226.00
Median	257.50
3rd Quartile	304.50
Maximum	352.00
95% Confidence Interval for Mean	
227.06	299.74
95% Confidence Interval for Median	
223.78	306.90
95% Confidence Interval for StDev	
34.94	92.74

CV = 19%
Do not Reject Normality

111112 - Summary for (5.25 - 5.99)_1



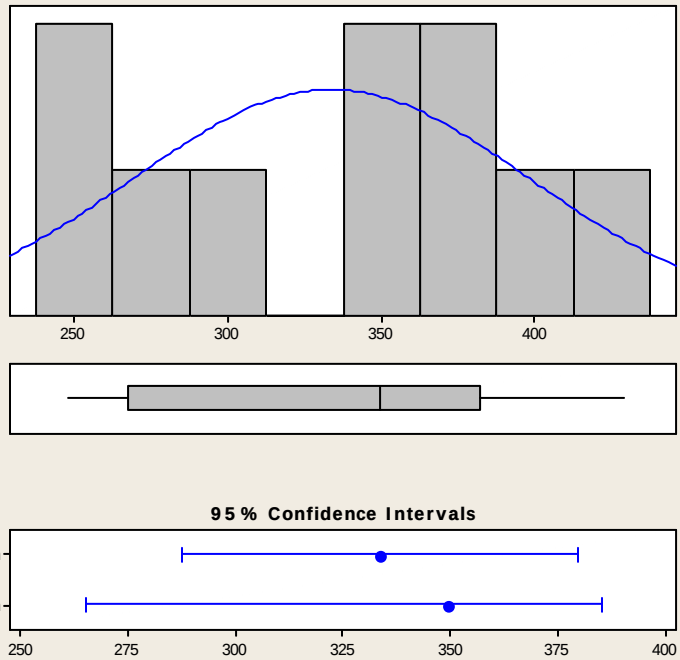




Anderson-Darling Normality Test	
A-Squared	0.95
P-Value	0.015
Mean	153.95
StDev	29.82
Variance	889.00
Skewness	1.21876
Kurtosis	2.52053
N	37
Minimum	107.00
1st Quartile	132.00
Median	142.00
3rd Quartile	176.00
Maximum	258.00
95% Confidence Interval for Mean	
144.00	163.89
95% Confidence Interval for Median	
137.10	166.70
95% Confidence Interval for StDev	
24.25	38.73

CV = 19%
Reject Normality

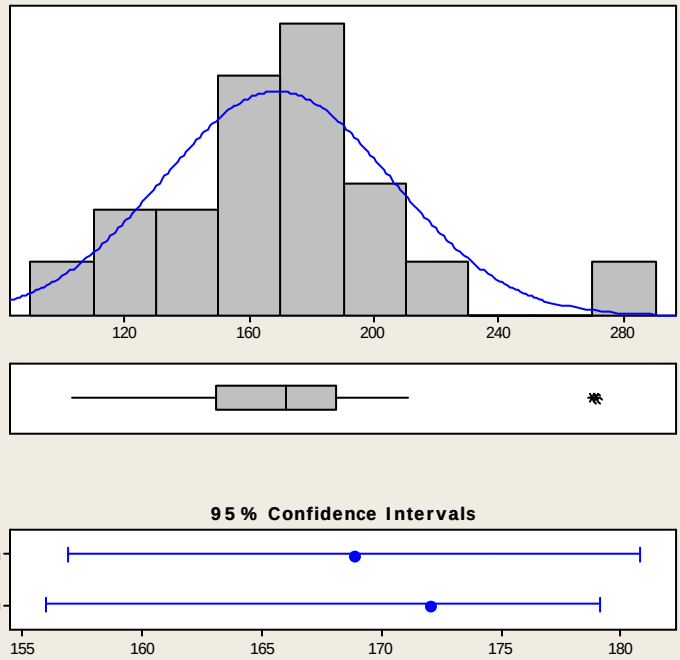
111113 - Summary for (4.50 - 5.24)_2



Anderson-Darling Normality Test	
A-Squared	0.30
P-Value	0.518
Mean	333.70
StDev	64.47
Variance	4156.23
Skewness	-0.05086
Kurtosis	-1.36478
N	10
Minimum	248.00
1st Quartile	267.50
Median	349.50
3rd Quartile	382.00
Maximum	429.00
95% Confidence Interval for Mean	
287.58	379.82
95% Confidence Interval for Median	
265.47	385.32
95% Confidence Interval for StDev	
44.34	117.69

CV = 19%
Do not Reject Normality

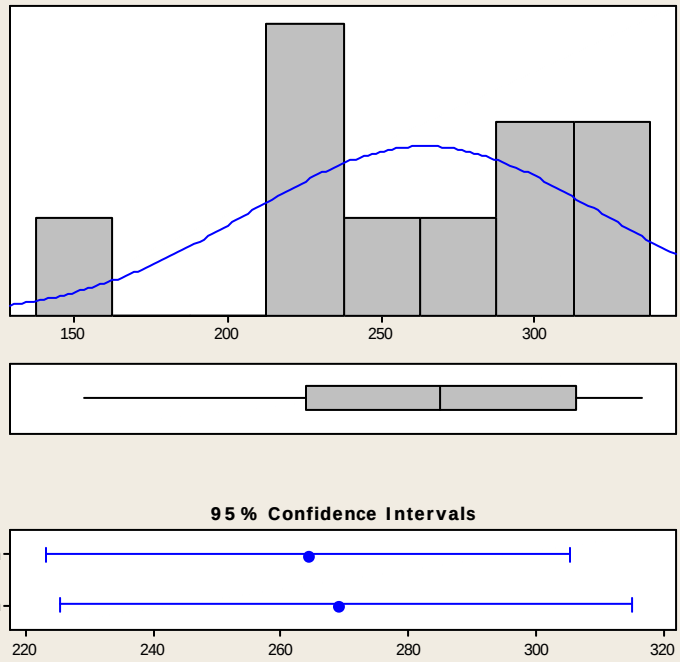
111113 - Summary for (5.25 - 5.99)_2



Anderson-Darling Normality Test	
A-Squared	0.53
P-Value	0.165
Mean	168.85
StDev	36.90
Variance	1361.77
Skewness	0.71201
Kurtosis	1.43348
N	39
Minimum	103.00
1st Quartile	149.00
Median	172.00
3rd Quartile	188.00
Maximum	271.00
95% Confidence Interval for Mean	
156.88	180.81
95% Confidence Interval for Median	
155.94	179.12
95% Confidence Interval for StDev	
30.16	47.56

CV = 22%
Do not Reject Normality

111114 - Summary for (4.50 - 5.24)_3

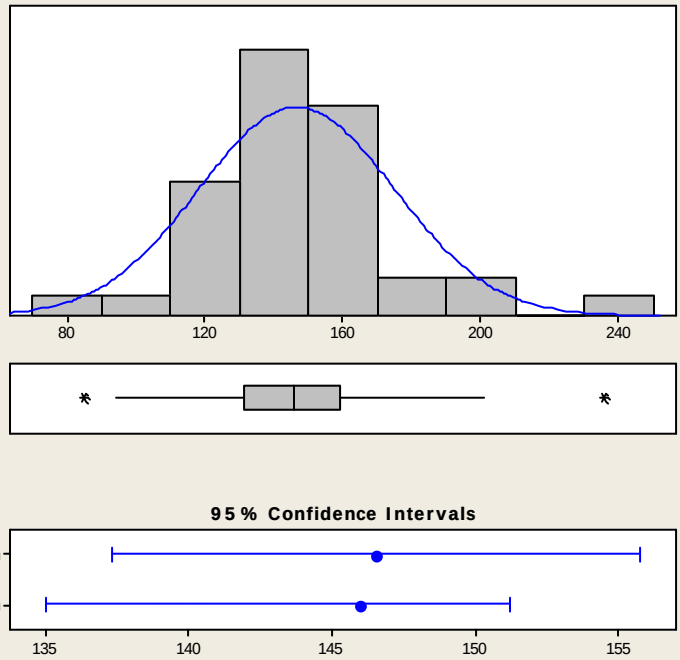


Anderson-Darling Normality Test	
A-Squared	0.33
P-Value	0.448
Mean	264.10
StDev	57.38
Variance	3292.77
Skewness	-0.591564
Kurtosis	-0.219888
N	10
Minimum	153.00
1st Quartile	225.50
Median	269.00
3rd Quartile	313.50
Maximum	335.00
95% Confidence Interval for Mean	
223.05	305.15
95% Confidence Interval for Median	
225.32	314.79
95% Confidence Interval for StDev	
39.47	104.76

CV = 22%

Do not Reject Normality

111114 - Summary for (5.25 - 5.99)_3



Anderson-Darling Normality Test	
A-Squared	0.70
P-Value	0.061
Mean	146.54
StDev	28.42
Variance	807.89
Skewness	0.73786
Kurtosis	2.02451
N	39
Minimum	85.00
1st Quartile	131.00
Median	146.00
3rd Quartile	159.00
Maximum	236.00
95% Confidence Interval for Mean	
137.32	155.75
95% Confidence Interval for Median	
135.00	151.17
95% Confidence Interval for StDev	
23.23	36.63

CV = 19%

Do not Reject Normality

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List of Abbreviated Terms

CRC	Centralized Return Center
D / I	Dependent vs. Independent
GA	Generic Algorithm
GMROI	Gross Margin Return on Inventory
ICP	Initial Collection Point
KKT	Karush – Kuhn – Tucker
LMCK	Linear Multiple Choice Knapsack
MIP	Mixed Integer Program
M / S	Myopic vs. Strategic
OEM	Original Equipment Manufacturers
R / NR	Replenishment vs. No Replenishment
SKU	Stock Keeping Unit
TSA	Total Sales Amount
TSV	Total Sales Volume
UP	Unit Price
WA	Weighted Average

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