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ANALYSIS AND DESIGN CONSIDERATIONS FOR
PERMANENT-MAGNET AEROSPACE-INSTRUMENT DEVICES

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ABSTRACT

The range of permanent-magnet materials for electro-magnetic-device applications has recently been augmented by the commercial development of Samarium Cobalt. Of particular importance is the magnetic characteristic of SmCo_5 in relation to other available hard magnetic materials. The most striking aspects of the material are its coercivity and possible maximum energy level, while the most gratifying aspect is its predictable performance.

In order to make best use of a high quality and relatively expensive magnetic material such as SmCo_5 , one should strive for optimized designs. In view of the complicated geometries involved, significant leakage flux particularly caused by compact design, and nonlinear ferromagnetic materials used in making up the electromagnetic device, it becomes necessary to take resort to numerical solutions to determine the effect of variation of significant parameters involved and to achieve an optimized design for the device.

The paper presents the modeling techniques for simulating permanent magnets in computer-aided analysis, methods to deal with various geometries and inherent symmetry that may be a characteristic of the problem. By numerically analyzing the model, detailed flux-distribution plots have been obtained by means of a high-speed digital computer. The paper discusses typical cases that have been studied.

INTRODUCTION

Magnetically hard materials used for permanent magnets possess, to a varying degree, the ability to retain their magnetization under conditions which experience has shown to be unfavorable for retention. The industrial utilization of permanent magnets has steadily been increasing for the past fifty years. Permanent magnet materials are currently commercially available with a wide-ranged combination of magnetic and physical properties which offer great latitude to the designer.

The rare-earth magnet materials were developed during the late sixties. Samarium is an element of the Rare Earth Group in the periodic table, which when combined with cobalt, produces a hard magnetic material with significant improvements in magnetic properties. Samarium-cobalt powder is compacted in the presence of a

strong magnetic field, then sintered using proven powder metallurgy techniques to optimize the magnetic and mechanical properties. It is clear that, property-wise, they constitute a major step forward in the art of making magnets, much as the alnicos did in the 1930's and the ferrites in the 1950's.² The extremely high coercive force of these materials allows magnetic circuit designs never before possible. Samarium-cobalt magnets far outstrip conventional ceramic or alnico magnet performance, especially in applications involving miniaturization, moving magnets, increased gap tolerance, and high magnetic field strength at the design point with excellent stability.

Of fundamental importance is the magnetic characteristic of SmCo_5 in relation to other hard magnetic materials. The most striking aspects of the material are its coercivity and possible maximum energy level, while the most gratifying aspect is its predictable performance. In view of the lower cost per unit energy, higher residual flux, better stability, better elevated-temperature behavior, improved mechanical properties and better property consistency in production, the rare-earth magnets have already found their applications in electro-mechanical devices (such as FHP motors, a.c. and d.c. generators, actuators, speakers, etc.), repulsive force applications (such as magnetic bearings, low friction linear load suspension, etc.), attractive force applications (such as chucks, separators, snap-action relays, etc.) and torque transmission mechanisms (such as synchronous couplings, eddy current clutches, brakes, etc.)

Besides the acute space limitations intrinsic to aerospace applications, the aircraft-engine environment is quite hostile with vibration levels as high as 250 G's, temperature ranges from -65°F to $+500^\circ\text{F}$, and exposure to potentially life-limiting media such as oil, fuel, and salt water. The devices therefore need to be characterized by design ruggedness, hermiticity, thermal compatibility, and overall reliability. A number of reluctance-type speed-sensing devices, magnetic-particle detectors, d.c. servomotors, and a.c. generators utilize SmCo_5 permanent magnets, thereby achieving optimized designs, size and weight reduction, improved sensitivity and stability, and complete freedom from handling precautions to prevent inadvertent knockdown.

The product of B , the magnetic induction, and H , the magnetic field intensity, for points on the demagnetization curve (the second quadrant B - H curve) may be displayed adjacent to the demagnetization curve from which a maximum, $(BH)_{\text{max}}$, may be readily determined. In working practical engineering problems, one will realize that it is only possible to utilize a small part

of total available energy in a useful way. Thus besides the available energy product, other properties such as the internal or recoil permeability have great significance. It influences the leakage in static air-gap problems, directly affects the conversion of magnetic field energy to other forms in the dynamic application, and determines the stability.

The permanent magnet is not without some disadvantages in its comparison with a wound field. Considerable effort required to regulate or shunt the field of a permanent magnet system, and the stubborn cleaning problem in freeing the exposed members from attracted dirt are some of the disadvantages. Also, SmCo_5 is brittle and has a lousy coefficient of heat transfer; but careful design should be used to overcome the problems; it can be embedded in epoxies and integrated into heat dissipators to allow for the poor characteristics should the design have abnormally high thermal gradients.

NUMERICAL SOLUTIONS

In view of the complicated geometries involved, significant leakage flux particularly caused by compact design, and nonlinear ferromagnetic materials used in making up the electromagnetic device, it becomes necessary to take resort to numerical solutions to determine the effect of variation of significant parameters involved and to achieve an optimized design for the device. For magnetic circuit calculations by other simpler analytical methods, one is referred to references [3], [4], and [5].

Concepts of both scalar and vector magnetic potentials have been used for computing the magnetic fields.⁶ Two distinct numerical approaches are evident in the literature: Finite-Difference Method⁷ and Finite-Element Method^{8,9}. We shall restrict our discussions to the two-dimensional magnetostatic field problems, even though they can be appropriately extended to eddy-current problems as well as three-dimensional field problems with some additional effort.

REPRESENTATION OF THE PERMANENT MAGNET

The magnetomotive properties of the permanent magnet material of Samarium Cobalt can be represented by a linear relationship between the flux density B and the field strength H

$$\vec{B} = \mu_0 (\vec{H} + \vec{H}_c) \quad (1)$$

where μ_0 is the permeability of free space. The magnitude and direction of \vec{H}_c , the coercive strength of the material, define the strength and the direction of permanent magnetization. The relationship of Eq. (1) can

easily be satisfied by replacing the permanent magnet region by that of air, while locating an excitation coil of appropriate ampere-turns given by

$$H_c \times 79.577 \times L_m = (AT) \quad (2)$$

where H_c is in oersteds and L_m is the length of the SmCo_5 permanent magnet in meters.

For the linear analysis of permanent magnets using a scalar potential, V , the partial differential equation to be solved is given by

$$\nabla \cdot [\mu(\nabla V - \bar{H}_c)] = 0 \quad (3)$$

where μ is the slope of the normal demagnetization curve. It can be shown that the minimization of the corresponding functional,

$$F = \iiint \frac{1}{2} \mu (\nabla V - \bar{H}_c) \cdot (\nabla V - \bar{H}_c) dv \quad (4)$$

subject to prescribed boundary conditions, is equivalent to solving Eq. (3), where dv denotes an elemental volume. For a rotationally symmetric system, with the cross-section to be analyzed in r - z plane, Eq. (4) becomes

$$F = \iint \frac{1}{2} \mu \left[\left(\frac{\partial V}{\partial z} - H_{cz} \right)^2 + \left(\frac{\partial V}{\partial r} - H_{cr} \right)^2 \right] 2\pi r dz dr \quad (5)$$

where H_{cz} and H_{cr} are the z - and r -components respectively of the coercive field strength \bar{H}_c . The functional given by (5) may be minimized numerically by the finite-element method, the details of which are omitted here as they can be found in the literature.

For the linear analysis of permanent magnets using a vector potential, A , the relevant partial differential equation to be satisfied is given by

$$\nabla \times [\nu(\nabla \times \bar{A}) - \bar{H}_c] = \bar{J} \quad (6)$$

where ν , the reluctivity, is the reciprocal of the permeability, and \bar{J} is the current density. The corresponding functional is given by

$$F = \iiint \left\{ \frac{1}{2} \nu [(\nabla \times \bar{A}) - \mu \bar{H}_c] \cdot [(\nabla \times \bar{A}) - \mu \bar{H}_c] - \bar{J} \cdot \bar{A} \right\} dv \quad (7)$$

For a rotationally symmetric system with only ϕ -directed currents, Eq. (7) becomes

$$F = \iint \left\{ \frac{1}{2} \nu \left[\left(\frac{\partial A}{\partial z} + \mu H_{cz} \right)^2 + \left(\frac{\partial A}{\partial r} + \frac{A}{r} - \mu H_{cr} \right)^2 \right] - JA \right\} 2\pi r dz dr \quad (8)$$

where J and A represent the ϕ -components of \bar{J} and \bar{A} respectively. The functional given by Eq. (8) needs to be minimized numerically by the finite-element method.

Even though the properties of permanent magnet materials can sometimes be approximated by a linear relationship between B and H , it may sometimes become necessary to use a more realistic nonlinear demagnetization curve,

in which case the representation could be taken care of in the following manner:

$$\bar{B} = f(\bar{H}_{\text{eff}}) \quad (9)$$

$$\text{where } \bar{H}_{\text{eff}} = \bar{H} + \bar{H}_c \quad (10)$$

The functionals to be minimized numerically by the finite element method would then be

$$F = \iiint U_c \, dv, \text{ for scalar potential problems} \quad (11)$$

and

$$F = \iiint (U - \bar{J} \cdot \bar{A}) \, dv, \text{ for vector potential problems, where} \quad (12)$$

$$U_c(H_{\text{eff}}) = \int_0^{H_{\text{eff}}} B \, dH_{\text{eff}} \quad (13)$$

and

$$U(B) = \int_0^B H_{\text{eff}} \, dB \quad (14)$$

Piecewise linear representation of the demagnetization curve for digital computer use could be accomplished as in Reference [10].

TYPICAL CASES ANALYZED

[A] For the prediction of flux distribution in and around an aerospace reluctance-type speed-sensing device a two-dimensional model of its cross-section is considered. With only z-directed currents, to analyze the field problem in xy-plane, the equation to be satisfied is given by

$$\frac{\partial}{\partial x} \left(\nu \frac{\partial A}{\partial x} \right) + \frac{\partial}{\partial y} \left(\nu \frac{\partial A}{\partial y} \right) = -J \quad (15)$$

The flux densities are given by

$$B_x = \frac{\partial A}{\partial y} \quad \text{and} \quad B_y = -\frac{\partial A}{\partial x} \quad (16)$$

An iterative procedure⁷ is developed consisting of successive point relaxation of the vector potentials with an optimum over-relaxation factor, a block relaxation method for accelerating the convergence, relaxation of reluctivities with an optimum under-relaxation factor, and a constant monitoring of the convergence of the sum of the absolute values of the residuals as well as of the absolute value of the maximum residual.

Next, in order to take the rotationally symmetric nature of the model, the cross-section in r-z plane with only ϕ -directed currents is considered while the equation to be satisfied is given by

$$\frac{\partial}{\partial r} \left[\frac{\nu}{r} \frac{\partial}{\partial r} (rA) \right] + \frac{\partial}{\partial z} \left(\nu \frac{\partial A}{\partial z} \right) = -J \quad (17)$$

or

$$\frac{\partial}{\partial r} \left(v \frac{\partial A}{\partial r} \right) + \frac{\partial}{\partial r} \left(\frac{vA}{r} \right) + \frac{\partial}{\partial z} \left(v \frac{\partial A}{\partial z} \right) = -J \quad (18)$$

The flux densities in this case are given by

$$B_r = - \frac{\partial A}{\partial z} \quad \text{and} \quad B_z = \frac{1}{r} \frac{\partial}{\partial r} (rA) = \frac{\partial A}{\partial r} + \frac{A}{r} \quad (19)$$

Both the forms given by Eqs. (17) and (18) have been studied from the viewpoints of difference algorithms and the convergence of the overall solution on the digital computer. The analysis with rotational symmetry does show some significant differences in the resultant flux distribution as compared to the analysis in cartesian coordinate system. To take into account various configuration changes, while aiming at design optimization and calculating various quantities of interest, computer programs have been generalized with sufficient flexibility.

[B] The methods of analysis are successfully applied to the case of aerospace alternators, whose rotors are made up of SmCo_5 or Alnico permanent magnets. Both configurations, one involving the radial magnetization and the other of circumferential magnetization, have been studied. Considering the cross-section (r - θ plane) in polar coordinates of one pole-pitch, (half a pole-pitch will do in the no-load case of analysis because of symmetry.) With only z -directed current densities, the equation to be solved is given by

$$\frac{1}{r} \left[\frac{\partial}{\partial r} \left(r v \frac{\partial A}{\partial r} \right) \right] + \frac{1}{r} \left[\frac{\partial}{\partial \theta} \left(v \frac{1}{r} \frac{\partial A}{\partial \theta} \right) \right] = -J \quad (20)$$

and the corresponding flux densities are given by

$$B_r = \frac{1}{r} \frac{\partial A}{\partial \theta} \quad \text{and} \quad B_\theta = - \frac{\partial A}{\partial r} \quad (21)$$

The permanent-magnet region is replaced by that of a non-linear material with appropriate piecewise-linear magnetic characteristic, while locating an excitation coil around the magnet region of appropriate ampere-turns.

From the converged numerical solution of the vector potentials at all the grid points in the model, contours of constant vector potential are plotted with the aid of a digital-computer program developed specifically for that purpose. Computer programs have been made sufficiently flexible to study various alternator configurations and different possibilities for design optimization.

[C] Numerical methods of analysis have also been successfully applied to the cases of magnetic particle detectors, servomotors and magnetic bearings.

Modeling techniques for simulating permanent magnets in computer-aided analysis are presented in this paper, both for the finite-difference and finite-element methods. It is also shown how to deal with various geometries and inherent symmetry that may be a characteristic of the problem being analyzed. With the aid of a special computer program developed, flux distribution plots have been obtained for various cases studied. By making the computer programs sufficiently flexible for a given class of problems, various configurations could be studied with the ultimate aim of optimized designs, size and weight reduction, and improved sensitivity, stability and overall reliability. While the discussions in this paper are limited to two-dimensional magnetostatic field problems, the methods of analysis could be extended to eddy current as well as three-dimensional field problems with some additional effort.

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