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AN EFFICIENT NUMERICAL APPROACH FOR THE STABILITY ANALYSIS OF A CLASS OF LTI SYSTEMS WITH ARBITRARY NUMBER OF DELAYS

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Abstract: A practical and numerically efficient algorithm is presented for determining the stability robustness map of a general class of higher order linear time invariant (LTI) systems in the parametric space of *arbitrary* number of independent delays. The stability of this class of dynamics is of particular interest in low-gain design and multi-agent control problems. The backbone of the new algorithm is inspired by a *scaled* frequency sweeping technique which enables the exhaustive determination of stability switching hypersurfaces in the parametric space of delays, without requiring nested loops. Two case studies are presented in order to demonstrate the strength of the new algorithm.

Keywords: Multiple Delays, Crossing Curves, Stability

Table 1. Nomenclature.

Subscript +, −	Positive, Negative (real or integer) numbers
\mathbb{Z}, \mathbb{R}	Set of integer, real numbers
$i\mathbb{R}$	Imaginary axis
$\mathbb{C}_-, \mathbb{C}_+$	Complex numbers with negative, positive real part
$s \in \mathbb{C}$	Laplace variable in the set of complex numbers
$\boldsymbol{\tau} \in \mathbb{R}_+^\ell$	ℓ -dimensional delay vector with positive real entries
LTI	Linear Time Invariant
MTDS	Multiple Time Delay System
MIMO	Multi-Input Multi-Output
ℓ -D	ℓ -dimensional
$\Re(s), \Im(s)$	Real, Imaginary part of s
$ x , x \in \mathbb{R}$	absolute value of x
$ y , y \in \mathbb{C}$	magnitude of y , $ y = \sqrt{(\Re(y))^2 + (\Im(y))^2}$

1. INTRODUCTION

The stability of a class of linear time invariant (LTI) multiple time delay system (MTDS) is studied in the parametric space of ℓ -dimensional delays over the respective characteristic equation given by:

$$f(s, \boldsymbol{\tau}) = P_0(s) + \sum_{j=1}^{\ell} P_j(s) e^{-\tau_j s} = 0, \quad (1)$$

where $P_j(s) \neq 0$ are polynomials in s with real coefficients and with degrees $n_j \in \mathbb{Z}_+, j = 0, \dots, \ell$, and $n_j < n_0, j = 1, \dots, \ell$, qualifying (1) as the representative of retarded type dynamics. The entries of the delay vector $\boldsymbol{\tau}, \tau_j \in \mathbb{R}_+$ are constant but uncertain $j = 1, \dots, \ell$, where $\ell \in \mathbb{Z}_+$ is *arbitrary*, but *given*. Furthermore, delays τ_j are in general rationally independent from each other, thus the complex structure of (1) with ℓ multiple delays is maintained.

The main objective in this paper is to obtain the stability robustness map of the dynamics represented by (1) against the uncertainties in delays, $\boldsymbol{\tau} \in \mathbb{R}_+^\ell$. Practically, this is especially of interest for the stability and control synthesis of interconnected MIMO dynamics in which states are inevitably available in different time-scales (independent delays). Such problems arise in consensus protocols, rendezvous and synchronization of multi-agents, (Lin and Antsaklis, 2006). Furthermore, studying the stability robustness of (1) is of interest in understanding stabilization of multiple integrators using multiple delays, (Niculescu and Michiels, 2004), (the case when $n_j = 0, j = 1, \dots, \ell$ in (1)) as well as in control law implementation for pole-placement (Michiels and Roose, 2001; Mazenc, Mondie and Niculescu, 2003; Lin, 1999; Saberi, Stoorvogel and Sanuti, 2000) and low-gain design (Michiels and Roose, 2001; Mazenc, Mondie and Niculescu, 2003) (where similar characteristic equa-

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