IRis

Northeastern University

Electrical and Computer Engineering Faculty Publications

Department of Electrical and Computer Engineering

November 01, 1976

Accelerating the magnetic field iterative solutions

Mulukutla S. Sarma
Northeastern University

James C. Wilson
Carrier Air-Conditioning Corp., Syracuse, N.Y.

Recommended Citation

Sarma, Mulukutla S. and Wilson, James C., "Accelerating the magnetic field iterative solutions" (1976). *Electrical and Computer Engineering Faculty Publications.* Paper 16.

This work is available open access, hosted by Northeastern University.

ACCELERATING THE MAGNETIC FIELD ITERATIVE SOLUTIONS

Mulukutla S. Sarma
Department of Electrical Engineering
Northeastern University
Boston, Massachusetts 02115

James C. Wilson
Carrier Air-Conditioning Corp.
Syracuse, New York

ABSTRACT

Recent years have witnessed considerable research activity in the application of digital-computer methods for the determination of the electromagnetic fields in electrical devices through the solution of Maxwell's equations, while taking full account of the magnetic saturation. Slow rate of convergence towards a meaningful solution and consequent demand for prohibitively large computer time are some of the chief drawbacks of the numerical iterative procedures. Many efforts have been made to find satisfactory methods of accelerating the convergence and thereby minimize the computer time needed for the solution. Various methods such as application of relaxation factors, block-relaxation procedures, and alternating-direction iterative techniques have been developed. It is the purpose of this paper to present a new method of computing an appropriate optimum relaxation factor for the reluctivity at each grid point (in 2-dimensions) or lattice point (in 3-dimensions) depending on the value of the local flux density and its location on the magnetization characteristic.

INTRODUCTION

Based on the well-known Maxwell's equations, one develops [1] the relevant partial differential equations to be satisfied in different regions of a magnetic field problem to be analyzed. Because of the complicated contours and the nonlinearity of the material, an explicit solution of such equations can not be found and one has to resort to a numerical solution[2]. The partial differential equations are transformed into the difference form for numerical work using finite difference techniques. The difference expressions may be developed either by directly using the partial differential equations and the first two terms of Taylor series, or by applying Ampere's law. In a three-dimensional numerical analysis[3] the lattice lines and the rectangular parallelopipeds replace the grid lines and the meshes of the two-dimensional analysis, respectively. It seems more natural to specify reluctivity (reciprocal of permeability) that is common for an area of a mesh in a two-dimensional

case, and similarly as being common for the volume of a parallelopiped in a three-dimensional case, than at the center point of a lattice line. The three-dimensional lattice is a natural extension of the two-dimensional net, mesh, or grid. A suitable grid system in two dimensional geometries or a convenient lattice system in case of three-dimensional problems is laid out for further numerical analysis.

The magnetic permeability of the ferromagnetic material is a function of the magnetic induction. The relationship is usually given by a single-valued magnetization curve, called the B-H characteristic of the magnetic material, while neglecting the hysteresis effects. An approximate representation of this characteristic, requiring a minimum of computer storage and time, for numerical purposes becomes essential. The representation yields the corresponding magnetic field intensity, H, corresponding to a given value of the magnetic induction, B.

An iterative procedure is then developed[2,3] for the numerical solution of the nonlinear partial differential equations. In each iteration, the vector potentials at each of the lattice points will be calculated, using the reluctivities calculated during the previous iteration, by successive point or line relaxation methods; later the reluctivities are recomputed using the newly calculated vector potentials and are underrelaxed. The old value of the reluctivity at a given lattice point is stored and a value is calculated by using the newly computed vector potentials; the new value of the reluctivity is computed as the sum of the old value and α times the difference between the calculated and old values; α is called the relaxation factor. An innovative method of computing an appropriate optimum relaxation factor for the reluctivity at each lattice or grid point depending on its location on the magnetization characteristic has been developed and successfully implemented by the authors for the solution of two-dimensional as well as three-dimensional nonlinear field problems. Very slow convergence of the iterative procedures and consequent demand for prohibitively large computer time are some of the chief drawbacks of such methods of solution. The new method of accelerating the convergence has resulted in significant improvement in convergence and hence a corresponding saving in the computer time required.

REPRESENTATION OF THE MAGNETIZATION CURVE FOR NUMERICAL PURPOSES

The single-valued magnetization curve is essentially divided into three main regions as shown in

Figure 1

a) For values of the magnetic flux density less than a particular value, B_{min}, in the unsaturated part, the curve is approximated by a straight line. The constant slope of the line is the permeability μ_ℓ.

b) For values of the magnetic induction beyond a particular value, B_{max}, the characteristic is approximated again as a straight line and its slope with respect to the point (B_{max}, H_{max}) is given by the permeability of empty space or air, μ_o. H for for the range $B > B_{max}$ is given by

$$H = H_{max} + (B-B_{max})\ \nu_o \tag{1}$$

c) In the remaining range of the magnetic induction, $B_{min} \leq B \leq B_{max}$, the representation is made according to the method described in Reference 4. It is divided into a convenient number of equal intervals of width ΔB. m denotes the running index such that

$$(m-1)\ \Delta B = B(m) \tag{2}$$

A table may then be laid out giving the values of m, B(m), and H(m) corresponding to the B-H characteristic under consideration. The values of m corresponding to B_{min} and B_{max} are denoted by N_{min} and N_{max}, respectively. If B_p denotes the flux density corresponding to which H_p is to be found, one has

$$\frac{B_p}{\Delta B} + 1 = M \tag{3}$$

Denoting the integral part of M by m, one can write, using the properties of similar triangles

$$\frac{\Delta B}{H(m+1) - H(m)} = \frac{B_p - B(m)}{H_p - H(m)} \tag{4}$$

So, it follows that

$$H_p = H(m) + \{B_p - B(m)\}\ \frac{H(m+1) - H(m)}{\Delta B} \tag{5}$$

or, making use of Eq. (2), one has

$$H_p = H(m)\{m - \frac{B_p}{\Delta B}\} + H(m+1)\ \{\frac{B_p}{\Delta B} - m+1\} \qquad (6)$$

or, using Eq. (3), one gets

$$H_p = H(m)\ \{m - M+1\} + H(m+1)\ \{M-m\} \qquad (7)$$

Hence, the corresponding reluctivity ν_p is given by

$$\nu_p = H_p/B_p \qquad (8)$$

For the usual magnetization characteristics that occur in practice, a value of o.1 Wb/m^2 for ΔB, and about 20 intervals between B_{min} and B_{max} are quite satisfactory.

COMPUTATION OF THE RELAXATION FACTOR FOR THE RELUCTIVITIES

A method of computing an appropriate relaxation factor for the reluctivity at each grid or lattice point depending on the value of the flux density and its location on the magnetization characteristic yields better rate of convergence than the one using a constant relaxation factor such as 0.1. The details of such a method are given below, while it was originally developed and used by Wilson[5].

Consider the expression

$$\nu^{n+1} = (\nu_p^{n+1} - \nu^n)\alpha + \nu^n \qquad (9)$$

where ν_p^{n+1} denotes a provisional value of reluctivity computed during the (n+1)th iteration; ν^{n+1} is the corrected reluctivity obtained during the (n+1)th iteration; ν^n is the reluctivity from the (n)th iteration; and α is the underrelaxation factor. For small changes in reluctivity, one can write

$$d\nu^{n+1} \simeq \nu^{n+1} - \nu^n = (\nu_p^{n+1} - \nu^n)\alpha \simeq \alpha d\nu_p^{n+1} \qquad (10)$$

From the B-H characteristic, it follows that

$$d\nu_p^{n+1} = \frac{d\nu}{d|B|}\ d|B|^{n+1} \simeq \frac{d\nu}{d|B|}\ (|B|^{n+1} - |B|^n) \qquad (11)$$

Hence,

$$\nu^{n+1} - \nu^n \simeq \alpha\ \frac{d\nu}{d|B|}\ (|B|^{n+1} - |B|^n) \qquad (12)$$

At sufficiently low levels of saturation,

$$\frac{d\nu}{d|B|} < \varepsilon, \text{ for some } \varepsilon \tag{13}$$

the relaxation factor of unity yields an acceptable rate of convergence.

A method of computing α, which will generally maintain this rate of convergence for higher saturation levels, will now be developed. For this it is important that successive corrections in $|B|$ should be of the same order of magnitude, independent of saturation. The derivative $\frac{d\nu}{d|B|}$ can change rather greatly as the level saturation is increased. Hence, the relaxation factor α can be specified as:

$$\alpha = 1 \text{ for } \frac{d\nu}{d|B|} < \varepsilon \tag{14}$$

$$\alpha = \varepsilon \frac{d|B|}{d\nu} \text{ for } \frac{d\nu}{d|B|} > \varepsilon \tag{15}$$

For generally improved rate of convergence it is necessary to determine the function of ν, which represents the lower bound of $\frac{d|B|}{d\nu}$, in order to obtain conservative estimates for α.

Referring to Eq. (6), one finds:

$$\frac{B}{\Delta B} = \frac{(m-1)\ H_{m+1} - mH_m}{H_{m+1} - H_m - \nu.\Delta B} \tag{16}$$

from which

$$\frac{dB}{d\nu} = \frac{[(m-1)\ H_{m+1} - mH_m]}{[H_{m+1} - H_m - \nu.\Delta B]^2} \cdot (\Delta B)^2, \text{ for } (m-1)\Delta B < B < m.\Delta B$$

$$= \frac{B^2}{(m-1)\ H_{m+1} - mH_m} \tag{17}$$

The derivative $\frac{dB}{d\nu}$ attains its minimum value at

$$B = (m-1)\ \Delta B$$

at which point

$$\min \left\{\frac{dB}{d\nu}\right\}_m = \frac{(m-1)^2\ (\Delta B)^2}{(m-1)\ H_{m+1} - mH_m} \tag{18}$$

and

$$\nu_m = \frac{H_m}{(m-1)\ (\Delta B)} \tag{19}$$

Thus the lower bound of $\frac{d|B|}{d\nu}$ versus ν in the range $B_{min} < B < B_{max}$ is represented as a function of m.

The representation of the magnetization curve for the range $B > B_{max}$ is given by Eq. (1):

$$H = H_{max} + (B - B_{max})\ \nu_o \tag{1}$$

or

$$B = \frac{B_{max}\ \nu_o - H_{max}}{\nu_o - \nu}\ , \text{ for } B > B_{max} \tag{20}$$

Hence,

$$\frac{dB}{d\nu} = \frac{B_{max}\ \nu_o - H_{Nmax}}{(\nu_o - \nu)^2}\ , \text{ for } B > B_{max} \tag{21}$$

The behavior of $\frac{d|B|}{d\nu}$ for the range $B > B_{min}$ will be

$$\frac{dB}{d\nu} \geq \begin{cases} \dfrac{(m-1)^2\ (\Delta B)^2}{(m-1)\ H_{m+1} - mH_m}\ , & \text{for } (m-1)\ \Delta B < B < m\Delta B \\ & \text{and } m = N_{min}, \ldots, N_{max} \\ \dfrac{B_{max}\ \nu_o - H_{Nmax}}{(\nu_o - \nu)^2}\ , & \text{for } B > B_{max} \end{cases} \tag{22}$$

The above two representations will have a common point of intersection, at which value of ν_I can be obtained by drawing a graph between $(\log_{10} \nu)$ and $(-\log_{10}\frac{d|B|}{d\nu})$.

For the range reluctivities in the neighborhood of the unsaturated value of ν_ℓ, it has been found that the relaxation factor of unity yields a good rate of convergence. In the range of reluctivities for which α is less than unity, one has

$$\frac{d|B|}{d\nu} \simeq \begin{cases} \left(\frac{K}{\nu}\right)^{1.3}, & \text{for } \nu_\ell < \nu < \nu_I \\ \frac{K'}{(\nu_o - \nu)^2}, & \text{for } \nu_I < \nu < \nu_o \end{cases} \tag{23}$$

where K and K' are constants, obtained from the graph drawn between $(\log_{10} \nu)$ and $(-\log \frac{d|B|}{d\nu})$. The under-relaxation factor α can be computed as

$$\alpha = \varepsilon \frac{d|B|}{d\nu} = \begin{cases} (K_1/\nu)^{1.3}, & \text{for } \nu_\ell < \nu < \nu_I \\ K_2/(\nu_o - \nu)^2, & \text{for } \nu_I < \nu < \nu_o \end{cases} \tag{24}$$

The constants K_1 and K_2 may be evaluated from the conditions that

$$\alpha = 1 \text{ , for } \nu = \nu_\ell \tag{25}$$

and

$$(K_1/\nu)^{1.3} = \{K_2/ (\nu_o - \nu)^2\} \text{ ,for } \nu = \nu_I \tag{26}$$

The procedure described here assumes that each iteration produces only small corrections of the reluctivities. Since this may not always be the case an effective value of reluctivity is obtained by

$$\nu_{eff} = \frac{1}{2} \{\nu_{computed} + \nu_{stored}\} \tag{27}$$

and is used to compute α, independent of the magnitude of the reluctivity corrections.

CONCLUDING REMARKS

The optimum underrelaxation factor will not, in general, be a constant and a choice of a suitable underrelaxation factor must depend on the saturation level. The new procedure presented here should produce a significant improvement in the rate of convergence, compared to the one that is attainable by the use of a constant underrelaxation factor. Only computational experience can ascertain if the technique presented here does yield an optimum value for the relaxation factor at each grid or lattice point. The

method has been successfully implemented by the authors for the solution of two-dimensional[5] as well as three-dimensional[6] nonlinear field problems.

ACKNOWLEDGEMENTS

The authors would like to express their sincere thanks to Professor E.A. Erdelyi of the University of Colorado and Dr. A.L. Jokl of USAMERDC for their helpful suggestions and discussions. The presented method is a part of the outcome of the contract No. DA-44-009-AMC-787(T) given to the University of Colorado by the U.S. Army Mobility Equipment Research and Development Center, Fort Belvoir, Va.

REFERENCES

*Work supported by USAMERDC

1. M.S. Sarma, "Potential Functions in Electromagnetic Field Problems", IEEE Trans. on Magnetics, Vol. MAG-6, No. 3, pp. 513-518, Sept. 1970.
2. E.A. Erdelyi, M.S. Sarma, and S.S. Coleman, "Magnetic Fields in Nonlinear Salient Pole Alternators", IEEE Trans. on Power Apparatus and Systems, Vol. PAS-87, No. 10, pp. 1848-56, Oct. 1968.
3. M.S. Sarma, et al "End-Winding Leakage of High-Speed Alternators by Three-Dimensional Field Determination", IEEE Trans on Power Apparatus and Systems, Vol. PAS-90, No. 2, pp. 465-477, March-April 1971.
4. F.C. Trutt, E.A. Erdelyi, and R.E. Hopkins, "Representation of the Magnetization Characteristic of D.C. Machines for Computer Use", IEEE Trans. on Power Apparatus and Systems, Vol. PAS-87, No. 3, 1968.
5. J.C. Wilson, "Theory of the Solid-Rotor Induction Machine", Ph.D. Dissertation, University of Colorado, 1969.
6. M.S. Sarma and E.A. Erdelyi, "Synthesis of High-Speed Homopolar Alternators and Theory of Solid-Rotor Electrical Machines--Phase DD, Pt. IV", USAMERDC Report, Fort Belvoir, Va., 1969.

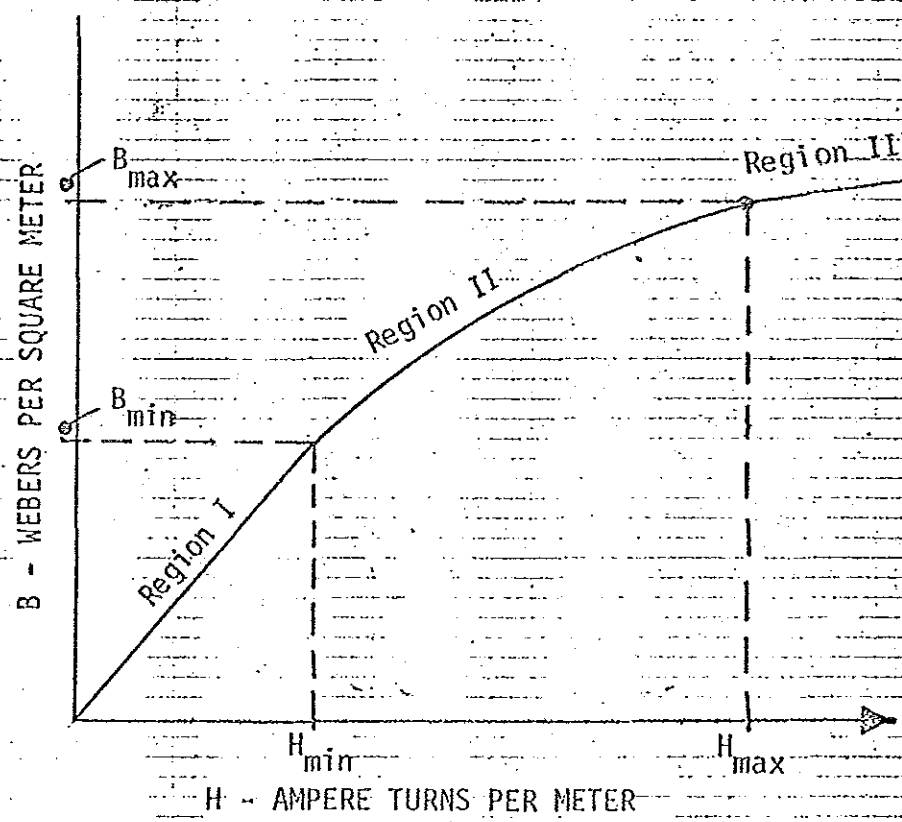

Figure 1. Single-valued Magnetization Characteristic