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## A new approximate model for microscopy imaging

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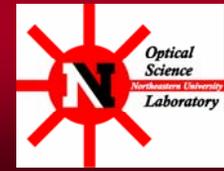
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# A New Approximate Model for Microscopy Imaging: the Product-of-Convolutions Model

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## Abstract

There has been increasing interest in recent years in techniques for microscopic examination of optically thick transparent objects. A number of phase imaging modalities have been developed to address this need. If a stack of images is acquired through focusing, the image at a given focal plane is contaminated by out-of-focus information coming from other planes [1]. There is a need to develop 3D imaging models for phase microscopes that will allow deconvolution, or more generally inverse reconstruction, techniques to be developed. Thus there is a need for an image formation model for phase microscopy that is able to maintain accuracy for thick objects but is more computationally tractable than full physical modeling. In response to this need we have developed a "product of convolutions" (POC) model. The need for the POC model arises because the Born model fails with thick objects because the field of each object plane at the image plane is calculated by a superposition of all the fields from other object planes. As a consequence, since we are adding fields rather than phases, the phase introduced by light propagating through these planes is not well reconstructed at the image plane.

## Introduction

Born model matches DIC images poorly for thick objects. We previously developed a Product of 2-D convolution (POC) model method which matches data well. The goal of the POC model is to take into consideration the phase introduced along the optical axis in the light that propagates through the object. Thus the POC method calculates the field at the image plane by adding the **fields** in the transverse planes but adds the **phases** along the optical axis.

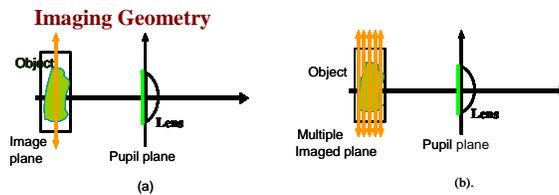
$$U(x, y, z_k) = \prod_{l=1}^k \sum_{x_l, y_l} f(x_l, y_l, t) h(x-r, y-s, z_k-t)$$

- We want to compare Born, Rytov and Product model in common framework.
- Availability of tractable accurate computational model would aid both instrument design and quantitative reconstruction, including multi-modality reconstruction (such as DIC with OQM).

## State of the Art

- Forward models of a defocused PSF function for transmitted light optics have been used using a different approaches [1].
- Computational models for three-dimensional images have been used in the analysis of biological images [2].
- A contour-finding algorithm for DIC microscopy has been used to recover quantitative information of the imaged object sectioned in stacks [3].
- Three-dimensional model using Born approximation was used for DIC imaging modeling [4].
- In previous work we used a model based on the product of two-dimensional convolutions. The model was tested on DIC images and results matched real data better than with Born model.

## Theory: Three-Dimensional Imaging Microscope



**Fig. 1.** Imaging geometry. (a) The complex field is calculated in the pupil plane and (b) propagated to the image plane. The field is calculated at pupil plane by using The Fresnel -Kirchoff integral:

$$U(x, y, z) = \frac{2ik \exp[ik(z-z_1)]}{4\pi(z-z_1)} \exp\left[ik \frac{x^2 + y^2}{2(z-z_1)}\right] \times \iint_{\text{aperture}} U_0(x_1, y_1, z_1) \exp\left[ik \frac{x_1^2 + y_1^2}{2(z-z_1)}\right] \times \exp\left[ik \frac{ix_1x}{2(z-z_1)}\right] \exp\left[ik \frac{iky_1}{2(z-z_1)}\right] dx_1 dy_1$$

**Experiment 1:** The Optical Path Length (OPL) for a binary square object is calculated at image plane by using Born and Rytov approximations.

$$\text{Optical Path Length} = \int ndz$$

### Born and Rytov approximations for Three-dimensional imaging

The total field  $U(\mathbf{r})$  is obtained for each approximation as:

• **The first Born Approximation**

$$U(\mathbf{r}) = U_0(\mathbf{r}) + U_1(\mathbf{r}) \quad \text{where} \\ U_1(\mathbf{r}) = \int g(\mathbf{r}-\mathbf{r}') o(\mathbf{r}') U_0(\mathbf{r}') d\mathbf{r}'$$

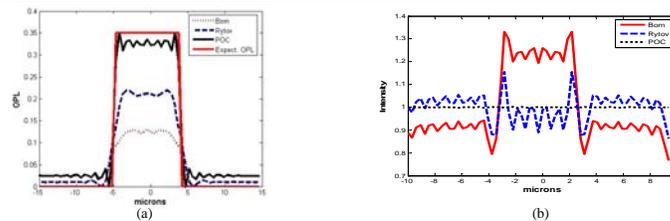
**The Rytov Approximation**

$$U(\mathbf{r}) = U_0(\mathbf{r}) U_1(\mathbf{r}), \text{ where } U_1(\mathbf{r}) = e^{\phi_1(\mathbf{r})} \text{ and} \\ \phi_1(\mathbf{r}) = -\frac{1}{U_0(\mathbf{r})} \int g(\mathbf{r}-\mathbf{r}') U_0(\mathbf{r}') o(\mathbf{r}') d\mathbf{r}'$$

**Experiment 2:** Phase comparison Born, Rytov and POC models for an infinite xy binary phase object.

Wavelength (microns)	Width of the object (microns)	Index of refraction difference(dn)	Stack size (dz)
0.550	[5 10 15 20 25 30]	0.5	0.5

**Experiment 3:** OPL comparison for a finite xy transparent binary object. Index of refraction difference between background and object equal to 0.036 and object thickness equal to 10 microns.



**Figure 4:** OPL comparison (a), Intensity at image plane comparison (b)

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## Results

Expected OPL	Born	Rytov	POC
0.1	0.087	0.09	0.1
0.52	0.16	0.6	0.5
0.87	0.5	1	0.83
1.22	0.5	1.44	1.215

**Table 1.** OPL at the center of a square object at the image plane using thickness equal to 5, 15, 25 and 35 microns

## Conclusions/Future Work

- A comparison using synthetic data of our model with Rytov and Born models following the imaging geometric used in [2] has been shown.
- The work presented here illustrates the ability of the POC model to represent the phase of uniform transparent infinite xy and finite xy objects better than either Born or Rytov approximations models as the thickness of the object increases.
- Future work will include a proof that our model correspond to a approximation solution of the wave equation as has been shown for Born and Rytov approximations.

## References

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